An Empirical Study on Statistical Properties of GSM Telephone Call Arrivals

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***Abstract* — We investigate the statistical properties of both originated and terminated call arrivals in sets of real GSM telephone traffic data (TIM, Italy), emphasizing results obtained by the Modified Allan Variance (MAVAR), a widely used time-domain quantity with excellent capability of discriminating power-law noise. The call arrival rate exhibits a diurnal trend, with peak hours in the morning and late afternoon. Besides this diurnal change, the number of call arrivals in a second is found perfectly uncorrelated to the number of arrivals in other seconds and Poisson distributed, with good consistency by χ^2-test evaluation. Uniform and accurate whiteness of call arrivals per second is verified in all hours, regardless the time of the day. In all series analyzed, the empirical statistics of both originated and terminated call arrivals proved excellent consistency with the ideal Poisson model with variable rate $\lambda(t)$. This study may be valuable to researchers concerned about realistic modelling of traffic in planning and performance evaluation of cellular networks.**

***Index Terms* — GSM, modified Allan variance, traffic model, traffic measurement (communication), Poisson random process.**

I. Introduction

Classic theory of telephone traffic was developed since the '60s, when networks were exclusively wired and offering only the circuit-switched Plain Old Telephone Service (POTS). No significant changes intervened thereon [1]—[4].

Wireless mobile telephony introduced a new scenario, given the peculiar behaviour of mobile users. Algorithms for dimensioning network resources do rely on faithful statistical characterization of traffic. Hence, there is a need for accurate statistical models of mobile telephone traffic.

In classic telephone traffic theory, call arrivals to local exchanges are modelled as a Poisson process, at least over short intervals to assume stationary arrival rate, since the user population served by the exchange is large and with negligible correlation among users. This assumption has been often retained also with mobile users: in literature, incoming calls in cellular networks are mostly Poisson modelled, with both call holding and interarrival times assumed with exponential distribution.

Nevertheless, it has been argued that this Poisson assumption might not be valid in wireless cellular networks for a number of reasons. First, cells partition the user population in small sets, each served by a small number of channels. Moreover, congestion and repeated call attempts are major causes of traffic peaks. Finally, user mobility during calls (handover) adds further complexity to the problem. Hence, not surprisingly, traffic characterization in wireless cellular networks has been attracting much attention in literature since few years.

In most cases, researchers focused on characterizing the channel or call holding time, whenever possible based on empirical data. Often, the channel holding time has been modelled with negative exponential distribution. Nevertheless, several other works contradicted this simple assumption. In papers [5][6], the probability distribution that better fits empirical data, by the Kolmogorov-Smirnov test, was found to be a sum of lognormal distributions.

The channel holding time is affected by user mobility. With exponentially-distributed call holding time, the merged traffic from new and handoff calls is Poissonian if and only if the cell residence time is exponentially distributed too [7]. For generally-distributed cell residence time, the channel holding time distribution was derived analytically in [8][9]. The channel holding time distribution was also studied in [10], when the cell residence time has Erlang or Hyper-Erlang distribution.

As for the correlation between call arrival times, the distributions of the channel idle time and of the call interarrival time in a Public Access Mobile Radio (PAMR) cellular system were investigated in [11]. In that work, the former distribution was approximated by the Erlang-*j*,*k* function and the latter resulted different from the Poissonian negative exponential.

Recently, paper [12] provided a further study of real GSM telephone traffic data. Call holding and interarrival times were found to be best modelled by the lognormal-3 function.

In summary, several studies contradicted the ubiquitous likelihood of the classic Poisson model for telephone traffic offered to cellular networks and suggested that call arrivals may be significantly time-correlated, due to access congestion and user mobility. However, we note that the Poisson model is still adopted in almost all works, when cellular network performance is evaluated. Questions may arise, therefore, on the practical relevance of this simplifying assumption.

In this paper, we analyze a few sets of real GSM telephone traffic data, collected by the mobile telecommunications operator Telecom Italia Mobile (TIM), Italy, in the same campaign as in [12]. Instead of modelling the empirical distributions of the call holding and interarrival times, we investigate directly possible time correlation of call arrivals in the time domain by means of the Modified Allan Variance (MAVAR), an efficient tool with demonstrated excellent capability of discriminating power-law noise.

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II. The Modified Allan Variance

The MAVAR was originally conceived for frequency stability characterization of precision oscillators in the time domain [13]—[17], by modifying the definition of the two-sample variance (a.k.a. Allan variance). MAVAR was designed with the goal of discriminating noise types with power-law spectrum $\sim f^{\alpha}$ ($\alpha \in \Re$, $\alpha > -5$) recognized very commonly in frequency sources. It is widely used in clock characterization.

Recently, MAVAR was also proposed as analysis tool of self-similar and long-range dependent (LRD) traffic. It was demonstrated to feature superior accuracy in estimation of the Hurst parameter H and of the exponent α, coupled with good robustness against nonstationarity in data analyzed [18]—[20].

This section just recalls some basic MAVAR properties. For details and demonstration of statements, in particular [13][20] may be suggested as first readings among cited papers.

A. Definition

Given an infinite sequence $\{x_k\}$ of samples of signal $x(t)$, spaced with sampling period τ_0, MAVAR is defined as

$$\text{Mod}\,\sigma_y^2(\tau) = \frac{1}{2n^2\tau_0^2}\left\langle\left[\frac{1}{n}\sum_{j=1}^{n}\left(x_{j+2n} - 2x_{j+n} + x_j\right)\right]^2\right\rangle \quad (1)$$

where $\tau = n\tau_0$ is the observation interval and the operator $\langle\cdot\rangle$ denotes infinite-time averaging.

In practice, given a finite set of N samples x_k spaced by τ_0 over $T=(N-1)\tau_0$, the standard estimator is used [13]

$$\text{Mod}\,\sigma_y^2(n\tau_0) = \frac{\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}\left(x_{i+2n} - 2x_{i+n} + x_i\right)\right]^2}{2n^4\tau_0^2(N-3n+1)} \quad (2)$$

with $n=1, 2, \ldots, \lfloor N/3 \rfloor$. A fast recursive algorithm exists [13].

Exact computation of confidence intervals is not immediate [21]—[24]. However, they are not negligible only for long τ, where few terms are averaged.

B. Application to Estimation of Fractional Noise Parameters

As customary in characterization of phase and frequency noise in precision oscillators [25], we deal with random processes $x(t)$ with one-sided power spectral density modelled as

$$S_x(f) = \begin{cases} \sum_{i=1}^{P} h_{\alpha_i} f^{\alpha_i} & 0 < f \le f_\text{h} \\ 0 & f > f_\text{h} \end{cases} \quad (3)$$

where P is the number of noise types considered in the model, α_i and h_{α_i} are parameters (α_i, $h_{\alpha_i} \in \Re$) and f_h is the upper cutoff frequency. Such random processes are commonly referred to as *power-law noise* or *fractional noise*.

Power-law noise with $-4 \le \alpha_i \le 0$ has been revealed in practical measurements of various physical phenomena, including phase noise of precision oscillators [13][17][25] and Internet traffic [18][19][26].

By considering separately each term of the sum in (3) and letting $P = 1$, $\alpha = \alpha_i$ ($-5 < \alpha \le 0$), MAVAR is found to obey a simple power law of the observation time τ, i.e.

$$\text{Mod}\,\sigma_y^2(\tau) \sim A_\mu \tau^\mu \quad (4)$$

where $\mu = -3-\alpha$ [13][17][20][25]. If $P>1$, it is immediate to generalize (4) to summation of powers.

Therefore, if $x(t)$ obeys (3) and assuming sufficient separation between components, a log-log plot of MAVAR looks ideally as a broken line made of P straight segments, whose slopes μ_i give the exponent estimates $\alpha_i = -3-\mu_i$ of the power-law noise components prevailing in distinct ranges of τ.

In [18][20], these estimates were demonstrated to be very accurate, even better than those obtained by the logscale diagram, which is one of the best and most widely adopted methods for analyzing LRD traffic. Moreover, nonstationary components of various kinds in the analyzed sequence (viz. polynomial drifts, periodic terms and steps) affect MAVAR negligibly or in a well recognizable way [20]. In particular, data offset and linear drift are cancelled in MAVAR results. Hence, we adopted MAVAR as main tool to analyze traffic traces.

III. GSM Traffic Data Sets and Analysis Criteria

We analyzed sets of real GSM telephone traffic data, collected by Telecom Italia Mobile (TIM) to billing and traffic monitoring purposes. Data recorded are the arrival time and duration of GSM voice calls originated or terminated in RM82D1, a large cell located in Fiumicino (30 km from Rome, Italy). Time scale is discrete, with 1-second intervals, as standard in network management (i.e., events are cumulated throughout each second).

Unfortunately, no information has been recorded to trace user mobility between cells. However, we notice that the large cell size makes handover unlikely. The negligible impact of handovers and the absence of congestion, as observed forth in Fig. 2, would lead to expect that call arrivals tend to behave as Poisson events, in some contrast with results [12]. Since modelling empirical distributions of interarrival times can be tricky and error-prone, although commonly pursued, we sought directly possible time correlation of call arrivals by MAVAR.

Data sets analyzed were collected continuously over 24 hours in 6 days: 18 July 2003, 11 Aug. 2003, 24 Oct. 2003, 26 Oct. 2003, 23 Jan. 2004 and 25 Jan. 2004. To summarize, we analyzed 6×2 traffic files (originated and terminated calls), each one further divided in 24 segments (one per hour), listing calls with arrival time and duration (time quantized to 1 s).

Processing this raw traffic data, we produced four new sets of 24×6 traffic data series, namely for a given time frame:

- $\mathbf{x}_\mathrm{O}$ and $\mathbf{x}_\mathrm{T}$, whose items $x_{\mathrm{O}k}$ and $x_{\mathrm{T}k}$ are the number of originated and terminated, respectively, new calls in the k-th second (i.e., call arrival rate in the k-th second);
- $\mathbf{n}_\mathrm{O}$ and $\mathbf{n}_\mathrm{T}$, whose items $n_{\mathrm{O}k}$ and $n_{\mathrm{T}k}$ are the number of originated and terminated, respectively, simultaneous active calls in the k-th second.

Then, we sought possible time correlation of call arrivals in the time domain, by computing MAVAR on series $\{x_{\mathrm{O}k}\}$ and $\{x_{\mathrm{T}k}\}$. Moreover, we evaluated the applicability of a Poisson model to describe the statistical properties of call arrivals.

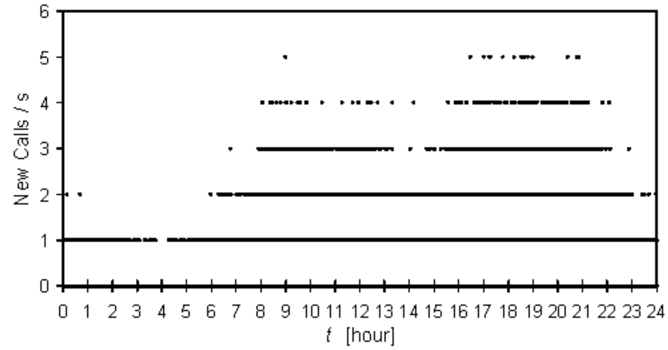

Fig. 1: Originated call arrivals $\{x_{Ok}\}$ (24 Oct. 2003, T=24 h, N=86400, τ_0=1 s).

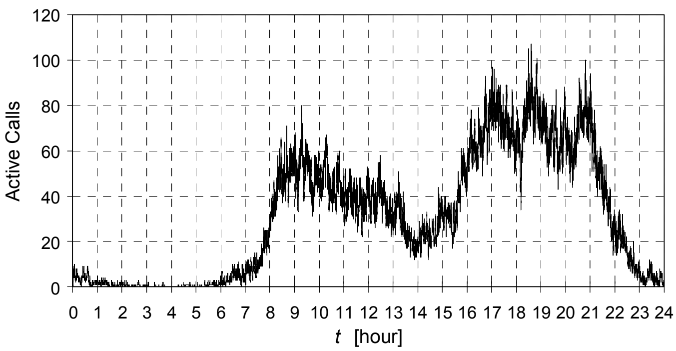

Fig. 2: Originated active calls $\{n_{Ok}\}$ (24 Oct. 2003, T=24 h, N=86400, τ_0=1 s).

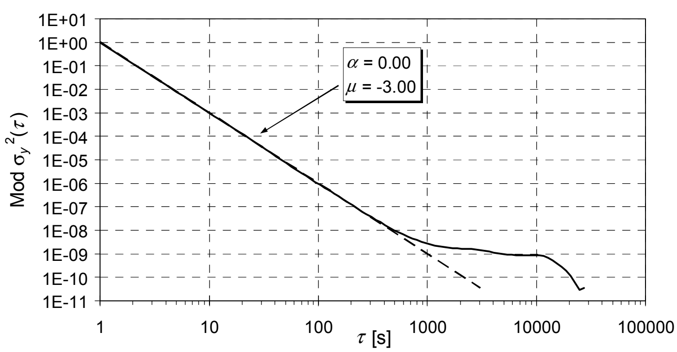

Fig. 3: MAVAR of sequence $\{x_{Ok}\}$ (24 Oct. 2003, T=24 h, N=86400, τ_0=1 s).

IV. Analysis Results

All sequences $\mathbf{x_O}$ and $\mathbf{x_T}$ examined are strongly nonstationary and exhibit a diurnal trend, with peak hours in the morning and late afternoon. For example, Figs. 1 and 2 plot originated call arrivals $\{x_{Ok}\}$ and simultaneous active calls $\{n_{Ok}\}$, respectively, recorded on 24 Oct. 2003. Fig. 2 shows no evidence of congestion. A similar trend was observed in all other days.

A. Study of Time Correlation

We computed MAVAR on the 6×2 series $\{x_{Ok}\}$ and $\{x_{Tk}\}$ over the whole measurement period $T = 24$ h ($N = 86400$, $\tau_0 = 1$ s). For instance, Fig. 3 plots MAVAR computed on the same series of Fig. 1. Here, let us notice that Mod $\sigma_y^2(\tau)$ is almost perfectly linear for $\tau <$ ~500 s, with slope $\mu \cong -3.0$, corresponding to $\alpha \cong 0.0$. This means that in the short term (i.e., on observation intervals up to few hundreds of seconds), the deviation of the data sequence from a linear trend is purely *random white* with excellent approximation (i.e., with no memory), whilst average drifts of order ≥2 are negligible (data offset and linear drift are cancelled in MAVAR results).

For $\tau >$ ~500 s, conversely, Mod $\sigma_y^2(\tau)$ departs from the linear trend, capturing the diurnal variation of the arrival rate evident in Fig. 1. For long τ, the higher slope reflects the slower wander of the data sequence in the long term. Note also the poor statistical confidence of Mod $\sigma_y^2(\tau)$ for longest τ.

In all 24-hours sequences, for both originated and terminated calls, we observed similar behaviour with little variation. In all cases, Mod $\sigma_y^2(\tau)$ is linear for $\tau < 10^2 \div 10^3$ s, with slope $\mu \cong -3.0$ ($\alpha \cong 0.0$, maximum deviation ±0.02, evaluated by linear regression): hence, the number of call arrivals in a second has been always found perfectly uncorrelated to the number of arrivals in other seconds; non-negligible time-correlation may be found only averaging on long intervals (say, at least 500 s), due to the diurnal variation of the average arrival rate.

B. Poisson Model of Call Arrivals

From such results, it comes natural to infer that both originated and terminated call arrivals may be modelled as a classic Poisson random process with variable arrival rate $\lambda(t)$, which follows a diurnal trend such as that in Fig. 1. Given the data set of a particular day, $\lambda(t)$ can be estimated and becomes, therefore, a deterministic term in the Poisson model of the random arrival process. However, the pseudoperiodic arrival rate $\lambda(t)$ changes randomly day by day, although to a limited extent. Statistical characterization of $\lambda(t)$ is beyond the scope of this paper and, however, would need measuring traffic data over many more days (i.e., years) to be significant.

MAVAR results ensure the absence of correlation between samples of series $\{x_{Ok}\}$ and $\{x_{Tk}\}$, but give no insight on intervals $\tau_0 \le 1$ s. If the call arrival process is ideal Poisson, then time correlation is null even on infinitesimal intervals and the number of arrivals x_k in τ is a random variable distributed as

$$P(x_k = i) = \frac{(\lambda\tau)^i}{i!} e^{-\lambda\tau} \tag{5}$$

with both mean m_x and variance σ_x^2 equal to $\lambda\tau$.

Hence, we studied the distribution of samples x_{Ok} and x_{Tk} in all traffic series. Due to the severe nonstationarity of sequences, the distribution of the N=84000 samples over the whole period (T=24 h) does *not* obey (5) (τ=1 s). Also their mean and variance are different. Likewise, the call interarrival time (computable from original raw data) is far from being exponentially distributed, if observed over the whole period $T = 24$ h (studying the distribution of such variables over intervals in which the traffic is manifestly nonstationary is a gross methodological mistake, yet not uncommon).

The right approach, by good practice in telephone traffic engineering, is restricting evaluation of statistics to peak hours, where stationarity holds at best and the number of calls is maximum (quantization effects are minimized, see Fig. 1). Thus, we computed mean, variance and distribution of samples x_{Ok} and x_{Tk}, in all six days, separately in four peak-hour intervals 9.00-11.00, 11.00-13.00, 16.00-18.00, 18.00-20.00 ($N = 7201$). In all cases, mean and variance resulted nearly equal ($\sigma_x^2/m_x = 1\pm0.04$) and the distribution very close to (5).

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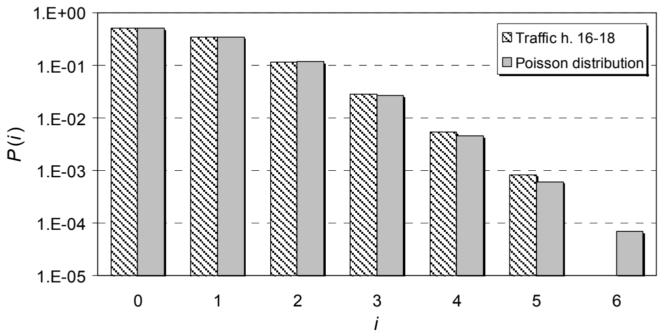

Fig. 4: Distribution of samples x_{Ok} (16.00-18.00, 24 Oct. 2003, T=2 h, N=7201, τ_0=1 s) compared to the Poisson distribution with same mean.

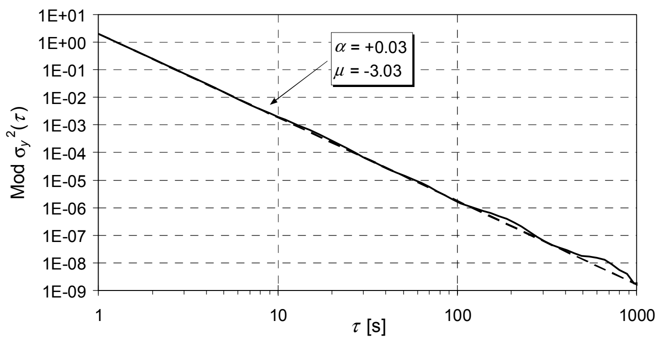

Fig. 5: MAVAR of $\{x_{Ok}\}$ (19.00-20.00 24 Oct. 2003, T=1 h, N=3600, τ_0=1 s).

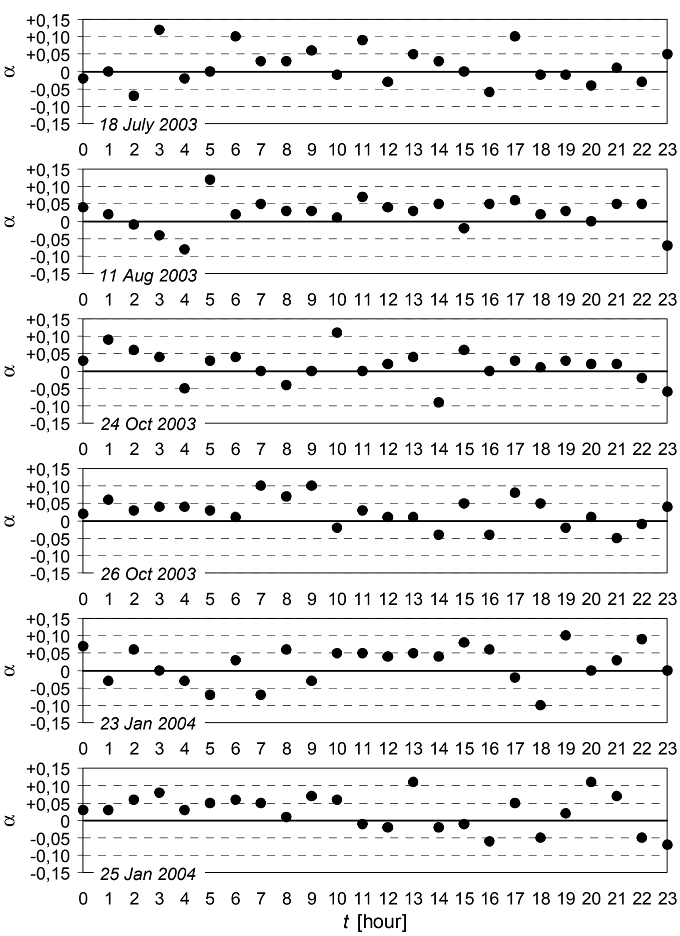

Fig. 6: Values of α over the 24 hours, estimated by MAVAR in originated call arrivals/s sequences $\{x_{Ok}\}$ (T=1 h, N=3600, τ_0=1 s, $\tau \leq$ 100 s).

Table 1: Mean, variance and χ^2–test (to same-mean Poisson distribution) of subsequences $\{x_{Ok}\}$ recorded on 24 Oct. 2003 (τ_0=1 s).

Time Interval	N	m_x	σ_x^2	σ_x^2/m_x	$Q(\chi^2)$
0.00 - 24.00	86400	0.334	0.408	1.221	0
9.00 – 14.00	18001	0.467	0.468	1.001	0.96
9.00 - 11.00	7201	0.558	0.540	0.968	0.80
11.00 - 13.00	7201	0.472	0.462	0.978	0.998
16.00 - 18.00	7201	0.680	0.700	1.029	0.95
18.00 - 20.00	7201	0.720	0.739	1.026	0.53

Numerical results obtained on sequence $\{x_{Ok}\}$ of 24 Oct. 2003 are summarized in Table 1 (cf. Fig. 1). In Fig. 4, moreover, the normalized distribution of samples x_{Ok} in interval 16.00-18.00 of the same day is compared to the Poisson distribution (5) having same mean. The two distributions match very accurately, even where confidence is little.

To have a quantitative measure of how the Poisson distribution and the empirical distributions of samples x_{Ok} and x_{Tk} do match, we evaluated the standard *chi-square test* [27]. In all days, most values of $Q(\chi^2)$ probability in peak-hour intervals are above 0.80. Values of 24 Oct. 2003 are shown in Table 1.

In conclusion, our analysis of traffic data, by both MAVAR and χ^2-test, demonstrates excellent consistency between the empirical statistics of new call arrivals and the ideal Poisson model with variable rate $\lambda(t)$.

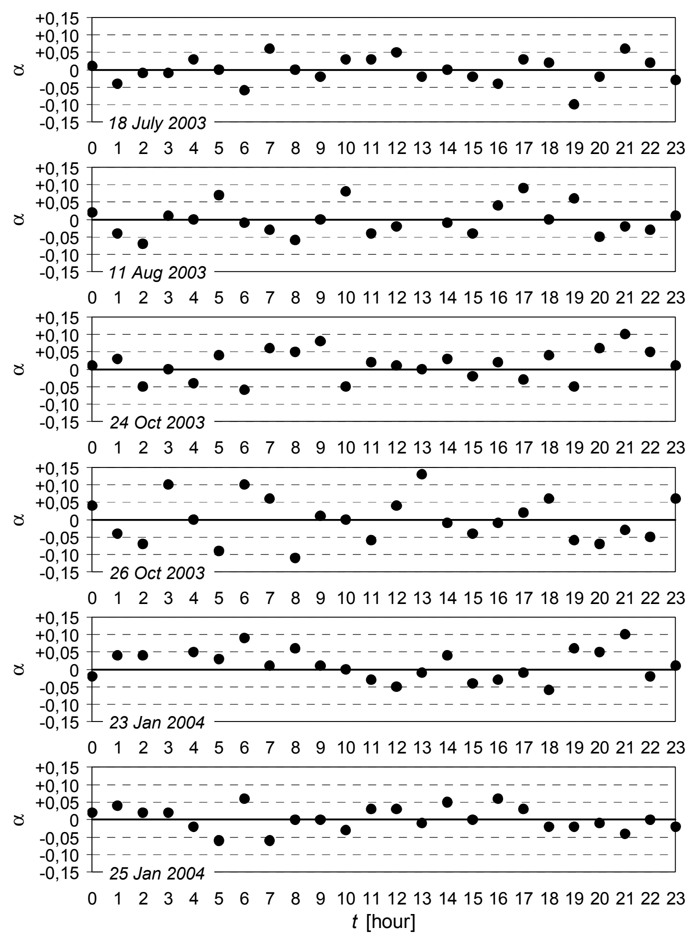

Fig. 7: Values of α over the 24 hours, estimated by MAVAR in terminated call arrivals/s sequences $\{x_{Tk}\}$ (T=1 h, N=3600, τ_0=1 s, $\tau \leq$ 100 s).

C. More about Stationarity: Whiteness

Since traffic exhibits a diurnal rate change, we investigated whether also the whiteness of short-term random fluctuations (cf. Sec. IV.A) is affected similarly. To this aim, we analyzed by MAVAR the 1-hour series separately. From each of the 24×6×2 sequences, we estimated the exponent α of the underlying fractional noise dominant in the short term, by linear regression of Mod $\sigma_y^2(\tau)$ on interval $\tau \leq 100$ s.

Fig. 5 plots MAVAR computed on originated call arrivals/s, recorded on 19.00-20.00, 24 Oct. 2003, $T = 1$ h, $N = 3600$, $\tau_0 = 1$ s). As expected, Mod $\sigma_y^2(\tau)$ follows nearly a linear trend on its whole length ($\mu \cong -3.03$, $\alpha \cong +0.03$ for $\tau \leq 100$ s).

The sequences $\{\alpha_{i,j}\}$ ($i = 1, \ldots, 6; j = 1, \ldots, 24$) of values estimated in each hour of the 6 days are plotted in Figs. 6 and 7, for originated and terminated calls, respectively. We notice that no periodicity is evident and that estimated values $\alpha_{i,j}$ have small uncertainty around their mean $m_\alpha \cong 0$. Hence, call arrival sequences $\{x_{Ok}\}$ and $\{x_{Tk}\}$ result uniformly and accurately white in all hours, regardless the time of the day.

V. Conclusions

Several studies contradicted the ubiquitous likelihood of the classic Poisson model for call arrivals in cellular networks, e.g. due to access congestion and user mobility. Though, this model is still commonly adopted in most simulation works.

In this paper, we investigated the statistical properties of both originated and terminated call arrivals in a set of real GSM telephone traffic data. MAVAR results have been emphasized. Main findings are summarized as follows.

- All traffic series examined are strongly nonstationary and exhibit a diurnal pseudoperiodic trend, with peak hours in the morning and late afternoon (cf. Figs. 1, 2).
- Besides the diurnal variation of the arrival rate, the number of arrivals in a second has been found uncorrelated to the number of arrivals in other seconds (Secs. IV.A, IV.C).
- Restricting evaluation of statistics to peak hours (~1 h), to ensure stationarity, the number of call arrivals in a second has been found having same mean and variance ($\sigma_x^2/m_x = 1 \pm 0.04$) and distributed, with good consistency by χ^2-test, as the ideal Poisson probability distribution (Sec. IV.B).
- Uniform and accurate whiteness of arrivals/s has been verified in all hours, regardless the time of the day (Sec. IV.C).

To conclude, call arrivals proved excellent consistency with the Poisson model with diurnal variable rate $\lambda(t)$, as expected considering that handovers and congestion are negligible.

These results confirm, at least to the limited extent of these empirical data, that the Poisson model may be still adequate to describe realistically telephone traffic offered to cellular networks, unless focusing specifically on issues as small user population, access congestion and frequent handovers.

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