Routing, Scheduling and Channel Assignment in Wireless Mesh Networks: optimization models and algorithms

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Abstract

Wireless Mesh networks (WMNs) can partially replace the wired backbone of traditional wireless access networks and, similarly, they require to carefully plan radio resource assignment in order to provide the same quality guarantees to traffic flows.

In this paper we study the radio resource assignment optimization problem in Wireless Mesh Networks assuming a time division multiple access (TDMA) scheme, a dynamic power control able to vary emitted power slot-by-slot, and a rate adaptation mechanism that sets transmission rates according to the signal-to-interference-and-noise ratio (SINR). The proposed optimization framework includes routing, scheduling and channel assignment. Quality requirements of traffic demands are expressed in terms of minimum bandwidth and modeled with constraints defining the number of information units (packets) that must be delivered per frame.

We consider an alternative problem formulation where decision variables represent compatible sets of links active in the same slot and channel, called configurations. We propose a two phases solution approach where a set of configurations is first selected to meet traffic requirements along the best available paths, and then configurations are assigned to channels according to device characteristics and constraints. The optimization goal is to minimize the number of used slots, which is directly related to the global resource allocation efficiency. We provide a lower bound of the optimal solution solving the continuous relaxation of problem formulation. Moreover, we propose a heuristic approach to determine practical integer solutions (upper bound). Since configuration variables are exponentially many, our solution approaches are based on the Column Generation technique. In order to assess the effectiveness of the proposed algorithms we show the numerical results obtained on a set of realistic-size randomly generated instances.

Keywords: optimization, routing, scheduling, channel assignment, radio resource assignment, power control, rate control, mixed integer linear programming, column generation.

1 Introduction

Wireless Mesh Networks (WMNs) have emerged recently as a new network architecture able to extend the coverage and increase the capacity of wireless access networks [1, 2]. WMNs are a promising solution to provide both indoor and outdoor broadband wireless connectivity in several environments without the need for costly wired network infrastructures. The network nodes in WMNs, named mesh routers, provide access to mobile users, like access points in Wireless Local Area Networks (WLAN) or base stations in cellular systems, and they relay information hop by hop, like routers, using the wireless medium. Mesh routers are fixed and usually do not have energy constraints. Therefore, WMNs are characterized by infrequent topology changes mainly due to node failures.

WMNs are being considered within several wireless technologies, including IEEE 802.11 WLAN [3], IEEE 802.16 Wireless Metropolitan Area Networks (WMAN) [4] and next generation cellular systems [5]. In all cases, WMNs partially replace the wired backbone network and should be able to provide similar services and quality guarantees. For cellular systems in particular, but also for WMAN, the backbone network is usually devised to provide an almost static resource assignment to traffic flows between base stations and network gateways. This approach allows to simplify the radio resource management at the interface between the network and the mobile users and to provide quality of service guarantees.

Therefore, traffic engineering methodologies able to provide bandwidth guarantees to traffic flows and to optimize transmission resource utilization appears to be a key element in these scenarios. Advanced multiple access schemes based on time division, power control mechanisms, and adaptive modulation and coding techniques are the most appropriate tools for defining radio resource management algorithms able to reserve the required rate to traffic flows and to achieve high network efficiency. These tools are already available for IEEE 802.16 networks and are commonly considered for next generation cellular systems [6], including Long Term Evolution Advanced (3GPP-LTE-Advanced) [7]. For all these system the radio resource optimization of mesh scenarios both based on centralized and distributed algorithms is of paramount importance.

In a wireless environment, the network topology depends on the position of nodes and propagation conditions. We can assume that a link between two nodes (i, j) exists if transmitting a signal at maximum power in i, the Signal-to-Noise Ratio (SNR) in j is sufficiently high to correctly decode the signal. To achieve high network efficiency, parallel transmissions on more than one link must be considered by the scheduling scheme. However, parallel transmissions generate interference at receiving nodes that may affect the correct decoding and limit the overall system capacity [8, 9, 10]. Therefore, some constraints on the resource reuse must be considered in order to guarantee correct network operations.

A simple model that has been proposed for reuse constraints is based on a conflict graph [9]. In the conflict graph there is a link (l, g) if the received signal level in g is above a carrier sense threshold when a signal is transmitted by l. If i is transmitting to j, no other node h can transmit if link (h, j) is in the conflict graph. That is, the node j is within the range of node h, even if node h's transmission is not intended for j (secondary interference). Moreover, assuming that nodes cannot transmit and receive at the same time, no other node k can transmit to i on link (k, i) (primary interference). Within this context, scheduling problem has some similarities with the graph coloring problem and several solutions have been proposed for both point-to-point and broadcast/multicast transmissions [9, 12, 13, 14, 15, 16, 17]. In order to take into account topology changes even more general schemes can be considered [18, 19].

Obviously, the model based on conflict graph does not consider that the effect of several interfering transmissions is cumulative. Therefore, the carrier sense threshold must be set to a quite low value and the resulting scheduling policies are not very efficient [20]. A more accurate model takes into account the cumulative effect of interference evaluating the Signal-to-Interference and Noise Ratio (SINR) at receivers [10]. Within this model a set of parallel transmissions on a set of links is admissible if the SINR at all receivers is above a given threshold [11].

Since SINR value depends on the power emitted by all transmitting nodes, power control mechanisms can improve the resource reuse adjusting powers in order to set the SINR at receivers just above the threshold

[21]. In this case a set of parallel transmissions is feasible if it is possible to find a power assignment that satisfies power limitations and provides the required SINR. Several scheduling approaches that jointly assign transmission time slot and emitted power have been proposed considering different objective and constraints [22, 23, 26, 27].

A further improvement can be achieved considering also transmission rate control within a cross-layer approach. Modern adaptive modulation and coding schemes allow to adapt transmission rates to the actual propagation and interference conditions. According to the value of the SINR the best transmission rate that provides an error rate sufficiently low can be selected. Since in the considered scenario SINR values are determined by the set of parallel transmissions and the transmission powers, it appears quite reasonable to consider transmission scheduling, power control and rate control at the same time [28, 29, 30, 31, 32].

Routing paths in the network are another parameter we can tune in order to optimize network performance: routing determines the sequence of transmitting nodes and the number of packets transmitted by each node. Transmitting nodes and the number of their transmissions influence the generated interference. Therefore, the scheduling optimization is affected by routing decisions, and this dependence leads to consider a joint routing and scheduling optimization.

Finally, with the development of new and on-the-shelf wireless equipment, the use of multiple radio interfaces in each node is now considered a common solution in WMNs. In addition, wireless technology standards provide a RF spectrum with a set of non-overlapping channels wireless interfaces can be tuned on. Multiple orthogonal channels permit the full utilization of the wireless medium through non-interfering simultaneous communications on different channels. Obviously, two interfaces can communicate only if they are tuned on the same channel, this requires a careful channel assignment in order to increase the global capacity without disconnecting the network. Effects of multiple interfaces on network capacity in presence of multiple channels have been theoretically analyzed [33] and several solutions of the channel assignment problem have been proposed [34, 35].

In this paper we study the joint routing and scheduling optimization problem in WMNs assuming a time division multiple access (TDMA) scheme, a dynamic power control able to vary emitted power slot-by-slot, and a rate adaptation mechanism that sets transmission rates according to the SINR. The dynamic and static channel assignment problems are also faced. In the static case every interface is forced to be tuned on the same channel for all the frame duration, in the dynamic one, instead, this constraint is relaxed permitting a per-slot channel rearrangement. We divide the whole optimization process in two phases. In the first phase we find an optimum time slot assignment for a scenario with a single shared channel, taking into consideration both routing and scheduling. In the second one, starting from the first phase solution, we build up a complete solution for the multi-channel multi-radio scenario.

Traffic quality constraints are expressed in terms of minimum required bandwidth. Since the time frame defined by the TDMA scheme is fixed, the bandwidth requirement can be translated into the number of information units (packets) that must be transmitted on each link per frame. Moreover, according to the discrete set of available transmission rates, the number of packets that can be transmitted per time slot directly depends on the SINR at receivers.

In order to get more insights into the characteristics of the problem and the effect of different transmission techniques, we consider three different versions of the problem with increasing complexity. In the first one we assume fixed power and rate, in the second one variable power and fixed rate, and finally in the third one variable power and rate. Given a number of available slots, our goal is to provide an assignment of time slots to links such that bandwidth constraints are satisfied and the number of available time slots is not exceeded. Such a problem can be tackled looking for the minimum number of needed time slots: if it is smaller than the number of available slots, a feasible and compact assignment is generated. The solution obtained using the minimum number of slots is also the most efficient from a resource usage point of view.

The solution approach we propose is based on a problem formulation where decision variables represent compatible sets of links active in the same time slot. Since variables are exponentially many, we use Column Generation to solve the continuous relaxation of problem which provide a lower bound of the optimal solution. In several cases the solution given by the column generation is even integer. Moreover, exploiting column generation results, we propose some heuristics to provide good solutions in reasonable time when the lower bound is not integer.

Adding channel assignment to the problem enables the activation of multiple compatible sets of links on different orthogonal channels. We show that the dynamic case, where interfaces can change channel slot-by-slot, is very efficient and the problem formulation is very similar to bin-packing problems. As for the static case, it is more involved but our solution approach, based on the admissibility version of the problem, provides efficient solutions in terms of resources exploitation.

The paper is organized as follows. In Section 2 we revise common approaches to the routing, scheduling and channel assignment problems, and comment on related work. In Section 3 we present our joint routing and scheduling problem formulations and introduce the proposed solution approach. In Section 4 we describe the channel assignment problem and present our solution algorithms for dynamic and static cases. In Section 5 we first show some numerical results on a small instance which allow us to get some insights into the behavior of proposed schemes. Then we present more general numerical results on random generated instances. Finally, in Section 6 we provide concluding remarks.

2 Radio resource management approaches

2.1 Scheduling, routing and channel assignment

The efficiency of transmission scheduling schemes depends on the number of parallel transmissions in the network which is obviously limited by the interference level. According to the well known classification proposed in [10] two possible interference models can be considered for multi-hop wireless networks, namely the *Protocol Interference Model* and the *Physical Model*. The *Protocol Interference Model* describes interference constraints according to a conflict graph but does not allow to take into account the cumulative effect of interference, while the *Physical Model* directly considers SINR constraints at receivers.

In this last model, the network can be described with a directed graph $G(V, E)$, where vertices are wireless routers and edges physical links. The SINR level at receiver j when a signal is transmitted by i is given by:

$$SINR_j = \frac{p_i G_{ij}}{\eta_j + \sum_{l \neq i,j} p_l G_{lj}} \tag{1}$$

where p_i is the power emitted by i , G_{ij} is the gain of the radio channel between i and j, and η_j is the thermal noise at receiver j. When a single modulation rate is considered, SINR constraints can be modeled just requiring that $SINR_j \geq \gamma$ for all $j \in V$. Several different scheduling schemes using this interference model have been proposed. Usually, additional constraints preventing nodes from transmitting and receiving at the same time are added, assuming that nodes are equipped with a single radio element and that a single channel is available.

Transmission rate control is another element that can be considered for the optimization of the radio resource utilization. It requires a cross-layer approach that is able to directly control the transmission rate through the use of adaptive modulation and coding schemes at the physical layer and the estimation of propagation and interference conditions. When multiple rates are considered, SINR constraints can be modeled requiring that $SINR_j \geq \gamma_w$ for all $j \in V$, where γ_w is the minimum required SINR when rate w is adopted.

The traffic load on each link in the network depends on the routing mechanism that selects available paths for all traffic demands. Routing protocols commonly considered for ad hoc networks are usually divided in two main categories: reactive protocols and proactive protocols. Reactive protocols select on-demand a path from source to destination, i.e. only when a transmission request is processed. This mechanism permits path adaptation to frequent topology changes but it may require a considerable route setup delay. Proactive protocols continuously maintain a valid route between each node pair by exchanging periodical signaling messages. This allows a prompt data delivery but at the cost of a high signaling overhead. Moreover, some hybrid approaches try to combine features of both categories to achieve better performance in terms of topology adaptation without requiring too much signaling overhead. Finally, a different viewpoint of the routing problem is given by geographical routing approaches. Here, data are greedily forwarded to the destination on the basis of geographical information about both relaying nodes and destination. All these schemes are based on the computation of a shortest path according to a routing metric. Quality of service parameters, such as available bandwidth, can be considered as well.

Even if routing schemes proposed for ad hoc networks can be used also in WMNs, enhanced approaches based on traffic engineering and optimization techniques are enabled by their peculiar characteristics. The shortest path is a routing assumption which is made for the sake of simplicity, however, it may not lead to the best possible optimum due to the imposed, but not necessarily needed, constraint of selecting only shortest paths. In many cases, it can even generate heavy-loaded bottleneck links. In WMNs we can drop the shortest path simplification and target at the real achievable optimum through global optimization approaches.

Since WMN wireless routers are fixed and with an unlimited power supply, topology changes are quite infrequent. Being WMNs wireless backbones of access networks, bandwidth guarantees to traffic flows (usually to/from gateways with the fixed network) need to be considered. Bandwidth requests are typically slowly varying over time and can be guaranteed with an almost static resource assignment, like in fixed backbone networks. Therefore, we consider a centralized network planning approach able to jointly optimize routing ad resource assignment based on traffic requests and propagation conditions.

In WMNs, multi-channel and multi-radio capabilities are usually considered as powerful features to mitigate interference effects. They permit to spread the entire traffic load across a set of orthogonal channels, increasing the network throughput. Transmissions on different channels do not interfere, therefore SINR constraints have to be considered only for radio interfaces tuned on the same channel that share the same capacity. Generally speaking, if we increase the number of channels, we can decrease the number of interfaces per channel and increase the capacity share per interface. However, channel assignment to interfaces affects also network topology as two nodes can communicate only if they have at least one interface tuned on the same channel. The channel assignment goal is the maximization of the network throughput, considering both interference effects and the network connectivity.

There are two types of channel assignment algorithms that can be adopted for WMNs: centralized and distributed. Centralized algorithms assume the whole knowledge of the network status. For this reason, they can provide an optimum assignment: they usually perform better than distributed ones. Distributed algorithms, instead, use local information at each node to make decisions and usually provide sub-optimal solutions.

Obviously, the interference model is a key element also for the channel assignment. If only topology information is considered, channels can be assigned according to transmission conflicts and the problem reduces to an edge coloring problem. In order to provide efficient and fair solutions, also the traffic load on links must be taken into account to properly estimate the impact of channel assignment to available capacity.

If a more accurate interference model based on SINR is considered, channel assignment and transmission scheduling can be jointly optimized in order to efficiently utilize available resources. In this scenario, a further important issue is the ability of radio interfaces to dynamically switch from one channel to another enabling slot-by-slot resource optimization thanks to a dynamic per-slot channel assignment. In many cases, however, technology and hardware limitations may prevent the fast switching from one channel to another. In this last case, we still have room for optimizing channel assignment, however we need to ensure that the channel assigned to each interface remains the same in all slots, that is, we must rely on a static channel assignment.

2.2 Related work

Optimization approaches to different types of scheduling problems have been considered within the general framework of ad hoc and multi-hop wireless networks. We believe that WMNs, with their peculiar characteristics, represent the only practical scenario where high computation times usually required by these approaches can be tolerated for an almost static radio resource planning. In this section, however, we revise previous works on problems similar to the one considered in this paper, even if they do not specifically address WMNs. For the sake of brevity we do not mention here papers considering the interference model based on conflict graphs, but only those adopting SINR constraints.

In [22] a scheduling and power control strategy is proposed where the general objective is the network throughput maximization. The proposed solution approach is based on the decomposition of the whole problem into two sub-problems. The first one aims at finding the maximum subset of parallel transmissions with nodes separated by at least a given distance, it is used to simplify the second sub-problem of transmission scheduling. The power-control scheduling algorithm proposed in [23] is based on a two steps approach as well, however, the set of links obtained in the first phase is usually unfeasible and it is pruned in the second phase through a greedy algorithm that directly considers SINR constraints.

Several Mixed Integer Linear Programming (MILP) models for fixed-power scheduling optimization are presented in [11], and some of these are solved by column generation methods. In [39] a scheduling optimization model aiming at maximizing the throughput is presented within a power control framework. The solution approach consists in iteratively solving the continuous relaxation of the MILP formulation. At each iteration some links are fixed and others are added ensuring that simultaneous transmissions on all links are compatible.

In [24] the authors show, using a dual approach, that the rate adaptation problem and the scheduling problem can be decomposed and solved individually; they are only coupled by costs associated with queues. In [25], two of the authors propose joint scheduling, power and rate optimization models and solve them with a column generation approach. This paper further extends those models and algorithms adding the routing and channel assignment optimization.

Joint scheduling and routing optimization has been considered in some papers on ad hoc networking. In [26] the routing problem is solved through a shortest path approach defining a link metric based on the number of interference conflicts caused by each link. The scheduling and power control problems are then jointly solved with a greedy approach. The impact of the power control on the performance of multi-hop wireless networks is deeply investigated in [27], which considers the joint routing and scheduling problem and assumes power levels constant over time.

In [28] the joint routing, scheduling, and power control problem is considered taking into account also long term minimum rate requirements for traffic flows. A two steps approach oriented towards the power minimization is considered, the first phase focuses on the scheduling and power control only assuming data rates are a continuous linear function of SINR. A similar problem is considered in [31] where, however, rate requirements are guaranteed within the frame period and not only in the long term. The solution approach is based on greedy heuristics. A MILP formulation for the joint scheduling, power control, and rate control problem is proposed in [30]. The goal here is throughput maximization and the solution approach is based on a greedy algorithm. In [32] the joint routing, scheduling and power control problem is studied considering a non-linear optimization problem where the achievable transmission rate is given by the Shannon capacity. The Shannon capacity is also adopted in [40] where the problem is solved in two phases, the first devoted to routing only, accounting for energy efficiency as well, and the second to scheduling and power control. In [29] an opportunistic scheduling scheme is proposed considering time-varying channel conditions. Emitted powers and data rates are dynamically adapted according to scheduling decisions and channel state.

In [41] the joint routing and scheduling are optimized to maximize a non-linear utility function that can be used, for example, to model the overall throughput taking also into account fairness issues. Variable power and rate are considered and the problem is solved exploiting Lagrangian duality. In [37] the authors consider a reduced set of all possible fixed-power transmission scenarios. This set is populated with a greedy algorithm that maximizes the number of simultaneously active links for each scenario. Scenarios in the set are sequentially activated, each of them for a time interval to be optimized in order to minimize the total activation time. Traffic is given as a demand matrix, every demand must be routed within the activation time. Finally, in [38] a genetic approach is presented for the joint routing and link scheduling optimization.

As mentioned before, multi-radio devices can potentially provide a remarkable efficiency improvement to WMNs. In [34] the channel assignment problem for the studied architecture is split into two sub-problems: neighbor to interface binding and interface to channel binding. A solution approach based on multi-channel conflict graphs is proposed, it guarantees that the sum of the expected per-link traffic load within interference range does not exceed each channel capacity. In [35], the general goal is to assign channels to network interfaces in such a way that the resulting available bandwidth is at least equal to the expected traffic load. The proposed solution approach is based on a greedy heuristic. In [36], a greedy algorithm to optimize transmission scheduling and channel assignment is proposed. It considers the availability of power control and directional antennas. The assignment of transmissions and channels in every slot is based on a saturation metric, that is, how SINR at each receiver is close to the minimum threshold for the correct decoding.

In this paper we consider an optimization approach to the radio resource management problem in WMNs which includes routing, scheduling, power control, rate adaptation, as well as channel assignment. To the best of our knowledge, this is the first work considering all these issues in a common framework and providing a complete solution methodology. The proposed optimization models are an extension of the models proposed by two of the authors in [25], where only scheduling, power control, and rate adaptation were considered. The optimization process is divided in two strictly-integrated phases, the first devoted to finding the optimum time slot assignment for a scenario with a single shared channel, and the second one, where, starting from the first phase solution, a complete solution for the multi-channel multi-radio scenario is built up. The proposed problem formulation considers an exponential number of decision variable; therefore, column generation is adopted to solve the continuous relaxation of the problem which provides a lower bound of the optimal solution. In several cases the column generation solution is even integer and provides the optimum also of the original problem. For the other cases we propose some heuristics which give good solutions usually quite close to the lower bound. Summarizing, the main contribution of our work is the following:

- definition of an optimization framework for WMNs including routing, scheduling, power control, rate adaptation and frequency assignment;
- calculation of a lower bound of the optimal solution by solving the continuous relaxation of the problem through column generation;
- proposal of some heuristics to obtain practical integer solutions of the problem in reasonable time.

3 Joint routing and scheduling problem

In this section we present two different formulations of the joint routing and scheduling problem for the single-channel scenario. The extension to the multi-channel case is presented in the next section.

The first ILP (Integer Linear Programming) formulation is quite simple as it directly considers per traffic-demand time slot assignment variables as well as rate and power variables. We use this formulation to describe the problem in a way that is rigorous but easy to follow. We also used it to test the performance of state-of-the-art ILP solvers with different instance sizes. Unfortunately, we found that this problem formulation cannot be applied to solve realistic-size instances in reasonable time with ILP solvers.

For this reason we considered an alternative formulation, which considers as decision variables the set of compatible links that can be activated at the same time. Since there are exponentially many variables, this formulation is specially devised for a solution approach based on column generation.

3.1 Linear number of variables model

Let us consider first the problem in which the power of each link is equal to a fixed value P, and then extend it to the general case. Each node pair (o,t) in the set of demand pairs $\mathcal{D}$ requires to send D_{ot} packets from node o to node t. Denote with $\mathcal{N}$ the set of devices belonging to the network and with $\mathcal{L}$ the set of links, represented by ordered node pairs. The minimum value of required SINR is denoted by γ. Let G_{ij} be the gain of the channel between devices i and j, $i,j \in \mathcal{N}$. The problem can be modeled with two sets of binary variables, z_{ijk}^{ot} and x_k, with $i,j \in \mathcal{N}$, $(o,t) \in \mathcal{D}$, and $1 \leq k \leq K$, where K is an upper bound on the number of available slots. These variables are defined as follows:

$$z_{ijk}^{ot} = \begin{cases} 1 & \text{if device } i \text{ transmits to device } j \text{ one packet of the demand} \\ & \text{from } o \text{ to } t \text{ in time slot } k \\ 0 & \text{otherwise} \end{cases}$$

$$x_k = \begin{cases} 1 & \text{if at least one link is active in slot } k \\ 0 & \text{otherwise} \end{cases}$$

and the resulting problem formulation is:

$$\min \sum_{k=1}^{K} x_k, \tag{2}$$

subject to:

$$z_{ijk}^{ot} \leq x_k \qquad \forall (o,t) \in \mathcal{D}, (i,j) \in \mathcal{L}, k = 1, \ldots, K \tag{3}$$

$$\sum_{(o,t)\in\mathcal{D}} \left(\sum_{(i,j)\in\mathcal{L}} z_{ijk}^{ot} + \sum_{(j,i)\in\mathcal{L}} z_{jik}^{ot} \right) \leq 1 \qquad \forall i \in \mathcal{N},\ k = 1, \dots, K \tag{4}$$

$$\sum_{k\in 1,\dots,K} \left(\sum_{(i,j)\in\mathcal{L}} z_{ijk}^{ot} - \sum_{(j,i)\in\mathcal{L}} z_{jik}^{ot} \right) = \begin{cases} D_{ot} & i = o \\ -D_{ot} & i = t \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}, \forall (o,t) \in \mathcal{D} \tag{5}$$

$$PG_{ij} \geq \gamma \sum_{(o,t)\in\mathcal{D}} z_{ijk}^{ot} \left(\eta + \sum_{(f,l)\in\mathcal{D}} \sum_{\substack{(h,m)\in\mathcal{L}: \\ h\neq i}} PG_{hj} z_{hmk}^{fl} \right) \qquad \forall (i,j) \in \mathcal{L}, k = 1, \dots, K \tag{6}$$

$$z_{ijk}^{ot} \in \{0,1\} \qquad \forall (o,t) \in \mathcal{D}, (i,j) \in \mathcal{L}, k = 1, \dots, K \tag{7}$$

$$x_k \in \{0,1\} \qquad \forall k = 1, \dots, K \tag{8}$$

The objective function (2) aims at minimizing the number of used time slots. Constraints (3) force variable x_k to one if at least one link is active in time slot k. Equations (4) impose that each device can be active in at most one link in each slot, while constraints (5) are flow balancing constraints, where it is assumed that one

Table 1: Table of notations for the formulation with a linear number of variables

$\mathcal{D}$	Set of demand pairs
D_{ot}	Number of packets to be sent from device o to device t
G_{ij}	Channel gain between devices i and j
K	Upper bound on the number of available time slots
$\mathcal{L}$	Set of links
$\mathcal{N}$	Set of nodes
p_{ik}	Variable for PC and PRC problems. It represents the power transmitted by node i during time slot k
P	Power value for FP problem
P_{max}	Maximum transmission power for PC and PRC problems
T_w	Number of packets per slot which can be sent with rate w
$\mathcal{W}$	Set of transmission rates
x_k	Binary variable for FP, PC, and PRC problems. It is 1 when at least one link is active in time slot k
y_{ijk}^{ot}	Integer variable for PRC problem. It represents the number of packets of the demand from devices o to t transmitted on link (i,j) during time slot k
z_{ijk}^{ot}	Binary variable for FP and PC problems. It is 1 when device i transmits to device j one packet of the demand from device o to device t in time slot k
z_{ijkw}^{ot}	Binary variable for PRC problem. It is 1 when device i transmits to device j one packet of the demand from device o to device t in time slot k with rate w
γ	SINR requirement for FP and PC problems
γ_w	SINR requirement at rate w for PRC problem
η	Thermal noise

packet is sent on a link during each slot in which it is active. Equations (6), where η represents thermal noise, guarantee that SINR requirements are satisfied in each active time slot at each receiver.

The model can be easily extended to include power control introducing continuous non-negative variables p_{ik}, representing the amount of power transmitted by node i in time slot k. The objective function (2) and constraints (3)-(5), together with binary constraints (7) and (8), hold also for this case, while constraints (6) need to be modified as follows:

$$p_{ik}G_{ij} \geq \gamma \sum_{(o,t)\in\mathcal{D}} z_{ijk}^{ot} \left(\eta + \sum_{\substack{h\in\mathcal{N}: \\ h\neq i}} G_{hj}p_{hk} \right) \quad \forall (i,j) \in \mathcal{L}, k = 1, \ldots, K \tag{9}$$

Moreover, two sets of constraints must be added in order to set the minimum and maximum limits of the power variables and to link power and transmission variables:

$$p_{ik} \leq P_{max} \sum_{(o,t)\in\mathcal{D}} \sum_{(i,j)\in\mathcal{L}} z_{ijk}^{ot} \qquad \forall i \in \mathcal{N}, k = 1, \ldots, K \tag{10}$$

$$p_{ik} \geq \gamma\eta \sum_{(o,t)\in\mathcal{D}} \sum_{(i,j)\in\mathcal{L}} \frac{z_{ijk}^{ot}}{G_{ij}} \qquad \forall i \in \mathcal{N}, k = 1, \ldots, K \tag{11}$$

Constraints (10) impose variables p_{ik} to be less or equal to P_{max} if node i is transmitting in slot k and to zero otherwise. Constraints (11) force variables p_{ik} to be greater than the minimum value required to reach the required SINR threshold in case there is no interference and node i is active in slot k.

If also transmission rates are included in the model, a set of parameters $\mathcal{W}$ representing available rates must be introduced as well as a parameter T_w representing the number of packets per slot which can be sent when transmitting at rate w. Multiple minimum thresholds, γ_w, must be considered for the SINR according to the selected rate w, and the new rate dimension must be added to decision variables z_{ijkw}, that now are defined as:

$$z_{ijkw}^{ot} = \begin{cases} 1 & \text{if } i \text{ transmits to } j \text{ at least a packet of the demand} \\ & \text{from } o \text{ to } t \text{ in slot } k \text{ with rate } w \\ 0 & \text{otherwise,} \end{cases}$$

New variables z_{ijkw}^{ot} replace variables z_{ijk}^{ot} in all the constraints. In addition, a set of integer variables y_{ijk}^{ot} is introduced to indicate the number of packets of the demand from o to t transmitted on the link (i,j) during time slot k. Constraints (5) are modified as follows:

$$\sum_{k\in 1,\ldots,K} \left(\sum_{(i,j)\in\mathcal{L}} y_{ijk}^{ot} - \sum_{(j,i)\in\mathcal{L}} y_{jik}^{ot} \right) = \begin{cases} D_{ot} & i = o \\ -D_{ot} & i = t \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{N}, \forall (o,t) \in \mathcal{D} \tag{12}$$

and a new set of constraints limiting the maximum number of transmitted packets according to the selected rate w must be added:

$$y_{ijk}^{ot} \leq \sum_{w\in\mathcal{W}} T_w z_{ijkw}^{ot} \qquad \forall (o,t) \in \mathcal{D}, (i,j) \in \mathcal{L}, k \in 1, \ldots, K \tag{13}$$

Finally, constraints (4) must be modified considering all possible rates:

$$\sum_{(o,t)\in\mathcal{D}}\left(\sum_{(i,j)\in\mathcal{L},w\in\mathcal{W}} z^{ot}_{ijkw} + \sum_{(j,i)\in\mathcal{L},w\in\mathcal{W}} z^{ot}_{jikw}\right) \leq 1 \qquad \forall i\in\mathcal{N},\ k=1,\ldots,K \tag{14}$$

and in constraints (6) γ_w must replace γ.

Although the above problem formulations are non linear due to SINR constraints, they can be linearized (see Section 3.3) and then solved using ILP solvers, like CPLEX [42]. Unfortunately, it turns out that only small size instances can be solved with this kind of models: we observed that solving a problem with only five devices and one packet to be sent per link requires more than four hours. For this reason a different formulation is proposed in next section.

3.2 Column generation model

To formulate the problem in a more tractable way we use a different set of decision variables. Let us define a *configuration* as a compatible set of links, *i.e.* a set of links that can be all active together without violating the SINR requirement. We denote with $\mathcal{S}$ the set of all possible compatible configurations, and with $\mathcal{S}_{ij}$ the set of configurations in which link (i,j) is active. We associate an integer decision variable λ_s to each configuration $s\in\mathcal{S}$; λ_s represents the number of slots in which s is activated. Differently from previous formulation, it is easy to observe that the number of decision variables grows exponentially with the problem size. As only one configuration can be activated per time slot, the total number of times available configurations are activated is equal to the number of used slots. Therefore, the problem of minimizing the number of slots can be modeled as follows:

$$\min\sum_{s\in\mathcal{S}}\lambda_s \tag{15}$$

subject to:

$$\sum_{(i,j)\in\mathcal{L}} f^{ot}_{ij} - \sum_{(j,i)\in\mathcal{L}} f^{ot}_{ji} = \begin{cases} D_{ot} & i=o \\ -D_{ot} & i=t \\ 0 & \text{otherwise}\end{cases} \qquad \forall i\in\mathcal{N},\forall(o,t)\in\mathcal{D} \tag{16}$$

$$\sum_{s\in\mathcal{S}_{ij}} T^s_{ij}\lambda_s \geq \sum_{(o,t)\in\mathcal{D}} f^{ot}_{ij} \qquad \forall(i,j)\in\mathcal{L} \tag{17}$$

$$\lambda_s\in\mathbb{Z}^+ \qquad \forall s\in\mathcal{S} \tag{18}$$

$$f^{ot}_{ij}\in\mathbb{Z}^+ \qquad \forall(i,j)\in\mathcal{L},(o,t)\in\mathcal{D} \tag{19}$$

where T^s_{ij} is a parameter that indicates the maximum number of packets that can be transmitted in a time slot on (i,j) according to the rate selected in the configuration s, while flow variables f^{ot}_{ij} represent the number of packets, originated in node o and destined to node t, going through link (i,j). Note that T^s_{ij} is always equal to 1 for problem versions with fixed rate. Objective function (15) imposes the minimization of the number of used time slots, flow balance constraints (16) define the routing paths for each demand, while constraints (17) guarantee that each link is active in at least one slot for each packet to be sent.

To complete the mathematical description of the problem, we need to provide equations describing compatible configurations. Consider a binary variable u_{ij} equal to one if link (i,j) is active in a given configuration and zero otherwise. A feasible configuration s must satisfy two kinds of constraints:

$$\sum_{(i,j)\in\mathcal{L}} u_{ij} + \sum_{(j,i)\in\mathcal{L}} u_{ji} \leq 1 \qquad \forall i \in \mathcal{N} \tag{20}$$

$$PG_{ij} \geq \gamma u_{ij} \left(\sum_{\substack{(h,m)\in\mathcal{L},\\ i\neq h}} PG_{hj} u_{hm} \right) \qquad \forall (i,j) \in \mathcal{L}. \tag{21}$$

and integrality constraints:

$$u_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{L}. \tag{22}$$

Constraints (20) guarantee that each device is involved at most in one transmission (half-duplex constraints) while constraints (21) enforce SINR constraints and ensure that traffic quality requirements are satisfied. Note that these sets of constraints are very similar to those considered for the problem formulation presented in Section 3.1, and that constraints (21) are non linear (bi-linear in particular).

Also in this case the model can be modified to include in the problem the power control and the rate adaptation. For the case of variable power and fixed rate we just need to introduce non-negative continuous variables p_i defining emitted power by node i and to modify constraints (21) as follows:

$$p_i G_{ij} \geq \gamma u_{ij} \left(\sum_{\substack{h\in\mathcal{N},\\ i\neq h}} p_h G_{hj} \right) \qquad \forall (i,j) \in \mathcal{L} \tag{23}$$

Moreover, we need to limit admissible values of power variables according to a maximum and a minimum threshold and to link u and p variables:

$$p_i \leq \sum_{(i,j)\in\mathcal{L}} P_{max} u_{ij} \qquad \forall i \in \mathcal{N} \tag{24}$$

$$p_i \geq \gamma\eta \sum_{(i,j)\in\mathcal{L}} \frac{u_{ij}}{G_{ij}} \qquad \forall i \in \mathcal{N}. \tag{25}$$

Finally, for the case of variable power and rate, we have also to select an available rate for each active link of a configuration. Therefore we define a binary variable u_{ijw} for each link (i,j) and each possible rate $w \in \mathcal{W}$ such that u_{ijw} is equal to 1 if link (i,j) is active in the considered configuration with rate w, and is equal to zero otherwise. Denoting by γ_w the SINR thresholds corresponding to available rates, the equations describing a compatible configuration now become:

$$\sum_{(i,j)\in\mathcal{L}} \sum_{w\in\mathcal{W}} u_{ijw} + \sum_{(j,i)\in\mathcal{L}} \sum_{w\in\mathcal{W}} u_{jiw} \leq 1 \qquad \forall i \in \mathcal{N} \tag{26}$$

$$p_i G_{ij} \geq \gamma_w u_{ijw} \left(\sum_{\substack{h\in\mathcal{N},\\ i\neq h}} p_h G_{hj} \right) \qquad \forall (i,j) \in \mathcal{L}, w \in \mathcal{W} \tag{27}$$

$$p_i \leq \sum_{w \in \mathcal{W}} \sum_{(i,j) \in \mathcal{L}} P_{max} u_{ijw} \qquad \forall (i,j) \in \mathcal{L} \tag{28}$$

$$p_i \geq \gamma\eta \sum_{w \in \mathcal{W}} \sum_{(i,j) \in \mathcal{L}} \frac{u_{ijw}}{G_{ij}} \qquad \forall (i,j) \in \mathcal{L} \tag{29}$$

$$u_{ijw} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{L}, w \in \mathcal{W} \tag{30}$$

$$p_i \geq 0 \qquad \forall i \in \mathcal{N} \tag{31}$$

Note that the column generation approach allows us to separate the routing and scheduling problem from the admissible configuration problem. The first problem, in fact, needs only to be fed up with feasible configurations, that is, possible slots, disregarding the assumptions and procedures they are generated with. The admissible configuration problem includes all technological and practical aspects about the definition of sets of simultaneously active transmissions, and it is completely independent of the previous one.

Table 2: Table of notations for the formulation with column generation

$\mathcal{D}$	Set of demand pairs
D_{ot}	Number of packets to be sent from device o to device t
f_{ij}^{ot}	Integer variable for FP, PC, and PRC problems.
	It represents the number of packets of the demand
	from devices o to t going through link (i,j)
G_{ij}	Channel gain between devices i and j
$\mathcal{L}$	Set of links
M	Big-M value for linearization
$\mathcal{N}$	Set of nodes
p_i	Variable for PC and PRC problems.
	It represents the power transmitted by node i
P	Power value for FP problem
P_{max}	Maximum transmission power for PC and PRC problems
$\mathcal{S}$	Set of all possible compatible configurations
$\mathcal{S}_{ij}$	Set of configurations where link (i,j) is active
T_{ij}^s	Maximum number of packets that can be transmitted
	in a time slot on link (i,j) according to the rate of configuration s
T_w	Number of packets per slot which can be sent with rate w
u_{ij}	Binary variable for FP and PC problems.
	It is 1 when link (i,j) is active in a given configuration
u_{ijk}	Binary variable for PRC problems.
	It is 1 when link (i,j) is active in a given configuration with rate w
γ	SINR requirement for FP and PC problems
γ_w	SINR requirement at rate w for PRC problem
η	Thermal noise
λ_s	Integer variable for FP, PC, and PRC problems.
	It represents the number of slots where configuration s is activated
σ_{ij}	Dual variable related to constraints (17)

3.3 Solution approach

To solve the proposed problem formulation we propose an approach which allows us to compute a lower bound of the optimal solution and then an upper bound (practical solution) through heuristics. The lower bound is obtained by solving to the optimum the continuous relaxation of the problem through a column generation procedure, while the upper bound is provided by solving the integer problem with the set of variables considered for the lower bound. We will show in the result section that in several cases the lower and the upper bounds coincide and therefore the solution provided by the heuristic algorithm is also the optimal one.

Consider the continuous relaxation of the problem (15)-(19), *i.e.* the problem described by (15)-(17) in which λ_s and f_{ij}^{ot} variables are continuous, and denote such problem with P. As P is the continuous relaxation of problem (15)-(19), the optimal solution of P provides a lower bound of the minimum number of needed slots. Since the number of variables in problem P is huge, it is not possible to enumerate them all. To solve P and provide a lower bound on the number of slots, we apply a column generation approach. In the column generation only a subset of variables is considered. The problem P involving a subset of variables, called *master problem* (MP), is solved: the solution is optimal for the MP but it may not be optimal for the original problem P as only a subset of variables is considered. Then, we need a procedure, called *pricing*, to check whether the solution found is optimal also for P or to find out the variables to be added to the set to improve the solution. The pricing procedure is based on the properties of the dual problem of P.

We recall that, given a solution of the problem P, if the dual variables related to such solution are feasible for the dual problem then the given solution is optimal for P. Besides, each variable (constraint) of P is associated to a constraint (variable) of the dual problem. Given a primal variable, if the associated dual constraint is violated, the considered variable has a negative reduced cost and therefore can produce an improvement in the objective function if it is added to the set of the considered variables. Thus the aim of the pricing procedure is to verify whether the dual variables associated to the primal found solution are feasible for the dual problem and, if they are not, to build a feasible configuration such that the related dual constraint is violated. The variable associated to such configuration must then be added to the considered set and MP is to be solved again. The continuous relaxation optimum is reached when no configuration can be found such that the related dual constraint is violated.

Consider first the problem with fixed power P. Denoting with σ_{ij} the dual variable related to constraint (17), the dual constraint associated to a given configuration s is

$$\sum_{\substack{(i,j): \\ s \in \mathcal{S}_{ij}}} \sigma_{ij} > 1 \tag{32}$$

To solve the pricing problem we must look for a configuration s satisfying (20), (21) and (22) with the additional constraint

$$\sum_{(i,j)\in\mathcal{L}} \sigma_{ij} u_{ij} \geq 1 \tag{33}$$

The pricing problem is not a problem aiming at maximizing a given objective function, but, instead, it consists in finding an admissible solution (configuration) that satisfies the entire constraint set. Furthermore, since constraint (33) has been included in the set, found configuration improves P's objective function.

If such a solution exists, the variable related to such configuration must be added to the set. Otherwise, column generation stops as no improving configuration can be found. In conclusion, the pricing problem is then formulated as an Integer Linear Problem (ILP) that looks for the first admissible solution that satisfies (20), (21), (22) and (33).

In case of power control the pricing problem is subject to (20), (22), (23), (24), (25), (31), and (33), while, if both power and rate control are envisaged the pricing problem constraints are (26), (27), (28), (29), (30), (31) and the following modified constraint

$$\sum_{(i,j)\in\mathcal{L}} \sum_{w\in\mathcal{W}} T_w \sigma_{ij} u_{ijw} > 1 \tag{34}$$

In both last cases the pricing problem is formulated as a mixed integer problem (MIP).

The pricing problem as formulated above turns out to be non-linear for all the proposed problems. The formulation can be linearized and the ILP (or MIP in case of power and rate control) pricing problem can be solved to optimality with commercial solver as CPLEX. We proposed two ways to linearize constraints (21) – or (23) or (27), respectively –. In a similar way constraints (6) can be linearized as well. The first linearization can be applied to all the proposed problems, with fixed power, with power control and with power and rate control. The linear formulation of the traffic quality constraint is:

$$PG_{ij} + M(1-u_{ij}) \geq \gamma \left(\eta + \sum_{\substack{(h,m)\in\mathcal{L},\\ i\neq h}} PG_{hj} u_{hm} \right) \quad \forall (i,j)\in\mathcal{L}, \tag{35}$$

where M is a constant such that:

$$M \geq \gamma \left(\eta + \sum_{\substack{(h,m)\in\mathcal{L}:\\ i\neq h}} PG_{hj} u_{hm} \right) \qquad \forall (i,j)\in\mathcal{L}. \tag{36}$$

Different values of M satisfying equations (36) have been proposed and tested: according to computational results the value of M has been set as follows:

$$M = \gamma \left(\eta + G_{max} P \frac{|\mathcal{N}|}{2} \right),$$

where $G_{max} = \max_{(i,j)\in\mathcal{L}} G_{ij}$.

A second linearization can be applied only to the problem in which power and rate control are not considered. A continuous nonnegative variable y_{ij} is defined for each link (i, j) such that

$$y_{ij} = \begin{cases} \eta + \sum_{\substack{(h,m)\in\mathcal{L}:\\ i\neq h}} PG_{hj}u_{hm} & \text{if } u_{ij} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Using such variable, constraints (21) can be rewritten as

$$PG_{ij}u_{ij} \geq \gamma\, y_{ij} \qquad \forall (i, j) \in \mathcal{L} \tag{37}$$

To force the correct value of variables y_{ij}, the following constraints must be added:

$$\tau_{ij}u_{ij} + \sum_{\substack{(h,m)\in\mathcal{L}:\\ i\neq h}} PG_{hj}u_{hm} + \eta - y_{ij} \leq \tau_{ij} \qquad \forall (i, j) \in \mathcal{L}, \tag{38}$$

where

$$\tau_{ij} = \eta + \sum_{\substack{h\in\mathcal{N}:\\ i\neq h}} PG_{hj}. \tag{39}$$

As an alternative to solving to optimality the pricing problem with CPLEX, it is possible to solve the problem heuristically applying greedy approaches in order to reduce the computation time. New configurations can be easily built up by sorting links and adding them one by one while constraints are satisfied.

We considered different sorting criteria: non decreasing values of G_{ij}, non decreasing values of σ_{ij}, and non increasing values of $\sigma_{ij}G_{ij}$. Then we applied two different types of greedy algorithms. The first one considers the links one by one and adds the considered link only if the constraints are not violated, while the second one, after adding a link, removes all the links not considered yet that would cause infeasibility. If the greedy approach fails in finding an improving compatible configuration, the exact solution is computed by solving ILP or MIP formulation to check whether the optimum of P has been reached or not. Preliminary computational tests have shown that in most of the cases the greedy approaches do not manage to find the optimal pricing solution and the exact approach is to be run. Thus the total computational time does not improve.

The initial set of configurations $\mathcal{S}_0$ must be able to provide an initial feasible solution for problem P. It is computed by sequential including every single pair in one-item configurations, provided SINR constraints allow to activate the considered pair.

The column generation approach provides a lower bound, as it solves the continuous relaxation P. Besides, we compute an integer feasible solution by solving the problem (15)–(19) over the final set of configurations selected by the column generation. The obtained solution is a heuristic one, as we consider only a subset of variables, heuristically chosen. We remark that the subset of variables generated to optimally solve the continuous relaxation does not guarantee the optimality when integer P is considered.

As shown, the whole approach – column generation and heuristics for the integer solution – can be applied to all the different versions of the problem.

4 Channel assignment

The overall efficiency of WMNs can be considerably increased using multi-radio devices in a multi-channel scenario as parallel transmissions on different orthogonal channels do not interfere [33]. In our optimization framework based on a TDMA approach the increased efficiency is reflected in a reduced number of time slots required to serve all traffic demands. To model and solve the global problem of optimizing routing, scheduling and channel assignment, we exploit the features of the problem formulation proposed in the previous section that is based on sets of compatible parallel transmissions, called configurations. In this second optimization phase we assign configurations to channels taking into account constraints due to device characteristics.

Each network node is equipped with a given number of wireless interfaces and each of them can be tuned on a single channel selected from a set of orthogonal channels. According to the hardware and software characteristics of nodes and wireless cards, two channel assignment strategies can be considered:

- *Dynamic assignment.* Channel assignment to interfaces can be changed on a slot-by-slot basis. We assume here that wireless interfaces can switch very quickly from one channel to another with a negligible switching delay.
- *Static assignment.* Wireless interfaces are statically tuned on a channel and the assignment cannot be changed for the entire frame duration.

Routing and scheduling optimization algorithms presented in previous section generate a single-channel frame in which each activated configuration is assigned to a slot in order to satisfy traffic demands. Starting from this solution we aim at obtaining a multi-channel frame solution assigning a channel to every activated configuration. In fact, configurations already contain sets of feasible simultaneously active links where transmissions can be carried out in parallel on the same channel. Properly assigning different channels to active configurations and then configurations to slots in the multi-channel frame, we minimize the number of total required slots and, at the same time, we implicitly satisfy SINR constraints.

Clearly, the channel assignment is subject to some constraints. In a multi-channel time slot we cannot activate a number of configurations greater than the number of available orthogonal channels. In addition, a node cannot be active during a time slot in a number of channels greater than the number of its available interfaces as wireless cards are half-duplex. Finally, in the static assignment case we need to add constraints to ensure that channels used by each node remain the same for the entire frame duration.

For the channel assignment problem we propose formulations for the dynamic and static cases. Let $\mathcal{N}$ denote the set of network nodes, $\mathcal{S}$ the set of multi-channel time slots, and $\mathcal{C}$ the set of configurations to be allocated into the multi-channel frame. Problem parameters are the following: I the maximum number of per-node interfaces, O the number of orthogonal channels, and A_{ci} that is equal to 1 if node i appears in

Table 3: Table of notations for channel assignment formulations

A_{ci}	Binary parameter equal to 1 if device i appears in configuration c
$\mathcal{C}$	Set of configurations to be allocated into a multi-channel frame
I	Number of per-node interfaces
$\mathcal{N}$	Set of devices
O	Number of orthogonal channels
$\mathcal{O}$	Set of orthogonal channels. $\lvert\mathcal{O}\rvert = O$
$\mathcal{S}$	Set of multi-channel time slots
r_{if}	Binary variable for static assignment. It is 1 when device i uses channel f
t_s	Binary variable for static and dynamic assignment. It is 1 when time slot s is used in the resulting frame
y_{cs}	Binary variable for dynamic assignment. It is 1 when configuration c is assigned to time slot s
y^f_{cs}	Binary variable for static assignment. It is 1 when configuration c is assigned to time slot s with channel f

configuration c and to 0 otherwise. Note that a node can appear at most once in a configuration because of half-duplex constraints.

For the dynamic assignment case, the sets of binary decision variables are:

$$y_{cs} = \begin{cases} 1 & \text{if configuration } c \text{ is assigned to slot } s \\ 0 & \text{otherwise} \end{cases}$$

and

$$t_s = \begin{cases} 1 & \text{if slot } s \text{ is used in the resulting frame} \\ 0 & \text{otherwise} \end{cases}$$

and the problem is formulated as:

$$\min \sum_{s \in \mathcal{S}} t_s \tag{40}$$

$$\sum_{s \in \mathcal{S}} y_{cs} = 1 \qquad \forall c \in \mathcal{C} \tag{41}$$

$$\sum_{c \in \mathcal{C}} y_{cs} \leq O \qquad \forall s \in \mathcal{S} \tag{42}$$

$$\sum_{c \in \mathcal{C}} y_{cs} A_{ci} \leq I \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{43}$$

$$y_{cs} \leq t_s \qquad \forall c \in \mathcal{C}, s \in \mathcal{S} \tag{44}$$

$$t_s \in \{0, 1\} \qquad \forall s \in \mathcal{S} \tag{45}$$

$$y_{cs} \in \{0, 1\} \qquad \forall c \in \mathcal{C}, s \in \mathcal{S} \tag{46}$$

Note that in the above formulation, channels are not explicitly identified. We just need to ensure that the number of used channels is compatible with problem parameters thanks to the ability of wireless interfaces to switch from one channel to another. Constraints (41) guarantee that each configuration is assigned to a slot. Constraints (42) and (43) force respectively the maximum number of available orthogonal channels and maximum number of per-node interfaces. Constraints (44) regulate multi-channel time slot activation, while constraints (45) and (46) define variable domains. A feasible channel assignment can be always obtained assigning one of the available channels to each configuration in a multi-channel time slot, as constraints (42) guarantee the process correctness.

In the static assignment case we have to tackle a more constrained and complex problem and we need to explicitly identify channels assigned to configurations to ensure that channel assignment to nodes does not change from slot to slot. To this purpose we use two new binary variable sets, in addition to t_s:

$$y_{cs}^f = \begin{cases} 1 & \text{if configuration } c \text{ is assigned to slot } s \text{ with channel } f \\ 0 & \text{otherwise} \end{cases}$$

and

$$r_{if} = \begin{cases} 1 & \text{if node } i \text{ uses channel } f \\ 0 & \text{otherwise} \end{cases}$$

The static assignment problem is formulated as:

$$\min \sum_{s \in \mathcal{S}} t_s \tag{47}$$

$$\sum_{s \in \mathcal{S}} \sum_{f \in \mathcal{O}} y_{cs}^f = 1 \qquad \forall c \in \mathcal{C} \tag{48}$$

$$\sum_{c \in \mathcal{C}} y_{cs}^f \leq 1 \qquad \forall s \in \mathcal{S}, \forall f \in \mathcal{O} \tag{49}$$

$$\sum_{c \in \mathcal{C}} \sum_{f \in \mathcal{O}} y_{cs}^f \leq |\mathcal{O}| \qquad \forall s \in \mathcal{S} \tag{50}$$

$$r_{if} \geq \sum_{s \in \mathcal{S}} y_{cs}^f A_{ci} \qquad \forall i \in \mathcal{N}, \forall c \in \mathcal{C}, \forall f \in \mathcal{O} \tag{51}$$

$$\sum_{f \in \mathcal{O}} r_{if} \leq I \qquad \forall i \in \mathcal{N} \tag{52}$$

$$y_{cs}^f \leq t_s \qquad \forall c \in \mathcal{C}, s \in \mathcal{S}, f \in \mathcal{O} \tag{53}$$

$$y_{cs}^f \in \{0, 1\} \qquad \forall c \in \mathcal{C}, \forall s \in \mathcal{S}, \forall f \in \mathcal{O} \tag{54}$$

$$r_{if} \in \{0, 1\} \qquad \forall i \in \mathcal{N}, \forall f \in \mathcal{O} \tag{55}$$

where $\mathcal{O}$ is the set of orthogonal channels. Constraints (48), (50), (52) and (53) have respective equivalents in the dynamic assignment problem. We add constraints (49) which state that in each multi-channel time slot we can assign at most one configuration per channel and constraints (51) that allow to count the number of channels assigned to a node within the frame. The number of assigned channels is equal to the number of interfaces to be installed.

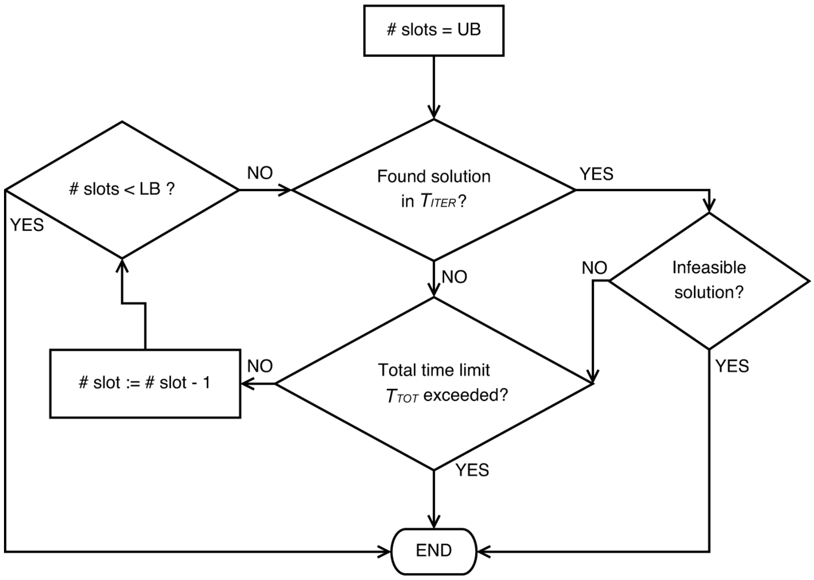

Figure 1: Static channel assignment algorithm.

4.1 Assignment algorithms

We adopt the same solution approach for the two assignment problems. We present here the case of the static channel assignment, for the dynamic channel assignment is sufficient to replace the problem formulation (47)-(55) with the correspondent formulation (40)-(46).

We have implemented a heuristic algorithm based on the admissibility version of the problem (47)-(55). This admissibility version is a problem where the objective function (47) is removed as well as constraints (53) and t_s variables, and a fixed number of multi-channel time slots is introduced. The goal is to find a feasible solution that exactly uses the given number of multi-channel time slots and satisfies all the constraints. Clearly, the complexity of this version is not greater than the optimization version, and we observed it runs in reasonable time with the solver.

The flow chart of our heuristic algorithm is reported in Fig. 1. We define the upper bound on the number of multi-channel time slots of the final frame as $UB = \frac{|\mathcal{C}|}{I}$ and the relative lower bound as $LB = \frac{|\mathcal{C}|}{|\mathcal{O}|}$. The upper bound is due to the maximum number of per-node interfaces: a solution where in each time slot only I configurations are present is always feasible because a node is active at most in I channels per slot. The lower bound, instead, is motivated by the fact that, relaxing constraints on the maximum number of interfaces, we can assign at most $|\mathcal{O}|$ configurations in a slot.

The algorithm starts solving the admissibility problem with a number of multi-channel time slots equal to UB and, if a feasible solution is found, this number is iteratively decreased by one. Otherwise, if the problem with that number of time slots is infeasible, execution is stopped. At this point, if the previous iteration ended with a feasible solution, its number of time slots is also optimal. Since finding the optimal solution in this way could require a computation time almost equivalent to that required by the solver, we limit the execution

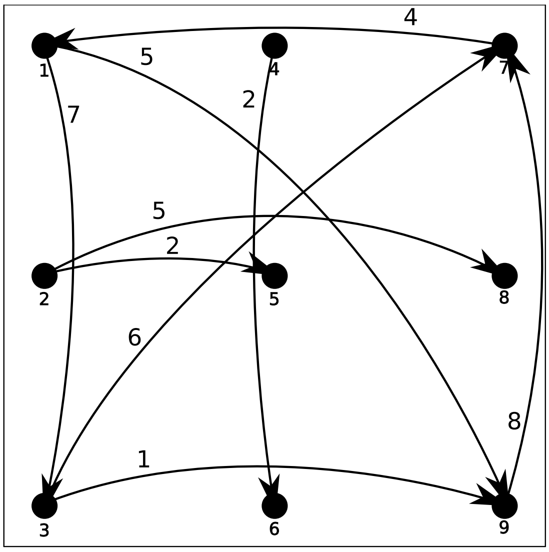

Figure 2: Grid instance: nodes positions and traffic demands

time of both the single iteration and the total algorithm to obtain at least a sub-optimal solution within a given time.

5 Numerical results

The joint routing and scheduling optimization algorithm, as well as the channel assignment algorithms, have been tested on several randomly generated instances and some ad-hoc instances.

Problem formulations have been implemented using modeler AMPL [43] and solver CPLEX 11 [42]. Computational tests have been run on a Intel Pentium 4 at 3 GHz and with 2 GB RAM under Linux. In Section 5.1 the behavior of proposed algorithms is analyzed solving a small size instance in order to get some insights into the impact of routing, and power and rate control strategies on the optimal solution, while in Section 5.2 computational results on randomly generated instances are reported.

We consider, where not differently indicated, the following parameters. SINR thresholds are $2, 2.8, 7.1, 15.9$ respectively for $1, 2, 4, 8$ packets per time slot, η has been set to 10^{-11}mW. Channel gain G_{ij} between a node pair is computed as $G_{ij} = d_{ij}^{-3}$, where d_{ij} is the distance between the nodes.

5.1 Small instance

We start our numerical analysis considering a small instance based on a grid with 9 devices deployed in a 700m square area, as shown in Fig.2. The figure also shows the demand pattern and the number of per-demand packets to be transmitted. Maximum transmission power is set so that a receiver can correctly decode a transmission from a device at a distance at most equal to half the diagonal of the area, with zero interference and using the lowest rate. As already mentioned, even for this small size instance solving to optimality the models described in Section 3.1 would require more than four hours, thus the problem has been solved

Table 4: Grid instance results for the fixed power case.
Column *Configuration* indicates the number of the selected configuration among the set of generated configurations, column *Active links* indicates active link pairs in the configuration, while column *# slots* counts the number of times the selected configuration appears in the solution. Column *Demand* indicates the source-destination pair of the traffic demand, while column *Links* indicates links used to route the demands and the number of packets to be transmitted through each link.

(a) Scheduling

Configuration	Active links *(tx,rx)*	# slots
1	(1,5)	1
2	(2,4)	11
3	(2,5)	2
4	(2,6)	6
6	(4,2)	2
8	(4,8)	5
16	(5,9)	1
26	(1,2) (8,7)	3
28	(1,2) (9,8)	8
36	(2,3) (4,7)	4
38	(2,3) (8,7)	3
39	(3,2) (7,4)	4
41	(3,2) (8,7)	2
42	(3,6) (4,1)	1
46	(4,1) (6,9)	3
49	(4,7) (6,9)	2

(b) Routing

Demand	Links *(tx,rx)[# pkts]*
1,9	(1,2)[4] (2,6)[4] (6,9)[4] (1,5)[1] (5,9)[1]
3,7	(3,2)[6] (2,4)[6] (4,7)[6]
2,8	(2,4)[5] (4,8)[5]
4,6	(4,2)[2] (2,6)[2]
1,3	(1,2)[7] (2,3)[7]
3,9	(3,6)[1] (6,9)[1]
9,7	(9,8)[8] (8,7)[8]
7,1	(7,4)[4] (4,1)[4]
2,5	(2,5)[2]

applying the column generation approach.

We consider first the fixed power case. In Table 4(a) and Table 4(b) the scheduling and routing results are shown. Table 4(a) shows activated configurations together with the number of slots to which each configuration is assigned, while Table 4(b) shows traffic routing, reporting for each traffic demand the links of selected paths and the corresponding number of packets.

The obtained solution, from both the integer problem and the continuous relaxation over the final set of configurations, uses 58 time slots. The total computation time is very small, about 0.06s. Note that with fixed powers the number of active links in each configuration is small because of interference constraints. This effect influences routing strategy as well and we get short paths in term of hops and only one demand split over two different paths.

The results obtained for the variable power case are shown in Tables 5(a) and 5(b). In Table 5(a) we also report the transmission power adopted on each link. After the column generation we obtain a frame of 40 time slots. The problem relaxation solved over the final configurations set gives a solution equal to 39.17, so our algorithm finds again the optimum. Computational time is 1.07 s.

With power optimization we have configurations with many active links due to reduced interference levels. In fact, transmission powers are set in order to just reach the SINR threshold at the receiver, and the overall emitted power is then reduced. As a result the total number of slots decreases with respect to the fixed

Table 5: Grid instance results for the variable power case.

Column *Configuration* indicates the number of the selected configuration among the set of generated configurations, column *Active links* indicates active link pairs in the configuration along with the used power, while column *# slots* counts the number of times the selected configuration appears in the solution. Column *Demand* indicates the source-destination pair of the traffic demand, while column *Links* indicates links used to route the demands and the number of packets to be transmitted through each link.

(a) Scheduling

Configuration	Active links *(tx,rx)[power(mW)]*	# slots
22	(1,4)[3.34] (2,3)[1.78] (9,8)[1.65]	2
29	(2,3)[1.69] (4,7)[1.69] (9,8)[2.40]	11
34	(2,5)[1.63] (9,8)[1.19]	2
35	(3,2)[2.40] (4,1)[1.69] (8,9)[1.69]	2
40	(3,6)[2.40] (4,1)[1.69] (8,9)[1.69]	2
65	(1,4)[2.43] (3,6)[2.43] (8,7)[1.70]	7
67	(1,2)[2.30] (6,9)[1.57] (7,8)[2.31]	6
68	(2,1)[1.70] (6,9)[1.74] (7,4)[2.43]	2
69	(3,2)[1.67] (7,4)[2.43] (8,9)[1.81]	2
70	(1,2)[1.70] (8,7)[1.90] (9,6)[2.43]	2
73	(2,1)[9.57] (8,7)[9.57]	1
74	(2,1)[1.70] (6,9)[1.72] (7,8)[2.43]	1

(b) Routing

Demand	Links *(tx,rx)[# pkts]*
1,9	(1,2)[1] (2,3)[1] (3,6)[1] (6,9)[1] (1,4)[4] (4,7)[4] (7,8)[4] (8,9)[4]
3,7	(3,6)[2] (6,9)[2] (9,8)[2] (8,7)[2] (3,2)[4] (2,1)[4] (1,4)[4] (4,7)[4]
2,8	(2,3)[5] (3,6)[5] (6,9)[5] (9,8)[5]
4,6	(4,7)[2] (7,8)[2] (8,9)[2] (9,6)[2]
1,3	(1,2)[7](2,3)[7]
3,9	(3,6)[1](6,9)[1]
9,7	(9,8)[8](8,7)[8]
7,1	(7,4)[4](4,1)[4]
2,5	(2,5)[2]

power case. We observe that longer paths with low-power short hops are used more often as they are usually more effective than a single high-power long hop. In addition, multiple paths per traffic demand are more common.

Finally, the results for the variable power and rate are shown in Tables 6(a) and 6(b). In Table 6(a) the selected transmission rate is reported along with the transmission power. Problem (15)–(19) solved on the final configurations set provides a solution with 13 time slots. Differently from previous cases, the continuous relaxation of the problem gives a lower bound equal to 9.75, thus we cannot state that the solution is optimal.

We can observe that rate control has a great impact on the solution as almost all the configurations have a single active link. Obviously, there is a trade-off between the transmission parallelism and the efficiency of single transmission at high rate that may shift the solution to one side or the other depending on the available rates, the corresponding SINR thresholds, and the number of possible parallel transmissions. With our instance and parameter settings the efficiency of higher transmission rates definitely outperforms that of transmission parallelism. Rate control impacts routing strategy as well: all but one of traffic demands are served at the highest rate and are routed along the shortest path between source and destination. The single demand served with lower rates is routed along a longer path in order to exploit the remaining capacity of already activated links and partially occupied by the other demands.

Based on the configurations considered during the routing and scheduling optimization, we applied channel assignment techniques described in Sec. 4.1. We set the maximum number of wireless interfaces per node equal to 3 and consider a set of 6 orthogonal channels. The results obtained are shown in Table 7. For the sake of brevity we report in Table 8 the multi-channel frame for the variable power and rate case only.

Table 6: Grid instance results for variable power and rate case.
Column *Configuration* indicates the number of the selected configuration among the set of generated configurations, column *Active links* indicates active link pairs in the configuration along with both the used power and the selected rate, while column *# slots* counts the number of times the selected configuration appears in the solution. Column *Demand* indicates the source-destination pair of the traffic demand, while column *Links* indicates links used to route the demands and the number of packets to be transmitted through each link.

(a) Scheduling

Configuration	Active links *(tx,rx)[power(mW),rate(# pkts)]*	# slots
33	(1,5)[17.7,8]	2
63	(2,5)[6.25,8]	1
83	(3,5)[17.7,8]	1
110	(4,7)[1.70,2] (9,6)[1.70,2]	1
114	(5,1)[17.7,8]	1
118	(5,3)[17.7,8]	1
123	(5,7)[17.7,8]	1
124	(5,8)[6.25,8]	1
125	(5,9)[17.7,8]	1
142	(7,5)[17.7,8]	1
147	(8,7)[6.25,8]	1
152	(9,8)[6.25,8]	1

(b) Routing

Demand	Links *(tx,rx)[# pkts]*
1,9	(1,5)[5] (5,9)[5]
3,7	(3,5)[6] (5,7)[6]
2,8	(2,5)[5] (5,8)[5]
4,6	(4,7)[2] (7,5)[2] (5,1)[2] (1,5)[2] (5,9)[2] (9,6)[2]
1,3	(1,5)[7] (5,3)[7]
3,9	(3,5)[1] (5,9)[1]
9,7	(9,8)[8] (8,7)[8]
7,1	(7,5)[4](5,1)[4]
2,5	(2,5)[2]

Table 7: Channel assignment results.

	# slots (# Configurations)	Dynamic Model			Static Heuristics		
		# slots	Time[s]	Channels	# slots	Time[s]	Channels
Fixed power	58	15	0.83	6	15	3682.8262	4
Variable power	40	13	0.43	6	14(Opt)	0.3680	3
Variable power and rate	13	4	0.04	6	4(Opt)	0.0680	5

We observe that, as expected, the number of slots is greatly reduced with respect to the single channel scenario. Moreover, the number of slots used with the dynamic and static strategies are almost the same. In two cases out of three the algorithm provided a certified optimal solution also for the static assignment.

5.2 Random instances

The proposed solution approach has been tested also on a set of randomly generated instances with 10, 15, 20 and 25 nodes. The instances have been generated by randomly locating the devices over a 1000m x 1000m area. Also settings of other parameters have been kept as above. Traffic demands are defined by randomly selecting the node pairs and the number of packets between 1 and 25. 15 different instances with the same number of nodes have been generated and then results averaged for each class of instances.

In Table 9 results on instances with fixed power, variable power and variable power and rate are given for different network sizes. For each class of instances, indicated in the first column, columns from three to five provide column generation data, while columns from six to nine report results provided by the final set of configurations.

We denote with I_MP the optimal solution of the continuous master problem on the initial set of configuration $\mathcal{S}_0$, with LB the lower bound, *i.e.*, the optimal solution of P computed by column generation, while

Table 8: Grid instance multi-channel frame, variable power and rate case.
Each column represents a slot of the multi-channel frame. Column *Conf* indicates the selected column among the ones of the single-channel solution, while column *chan* indicates the channel assigned to the selected configuration in the multi-channel slot.

(a) Dynamic channel assignment

Slot1		Slot2		Slot3		Slot4	
Conf	chan	Conf	chan	Conf	chan	Conf	chan
33	6	83	1	118	3	33	6
123	5	114	4	124	4	63	5
125	2	147	3			110	1
						142	4
						152	2

(b) Static channel assignment

Slot 1		Slot 2		Slot 3		Slot 4	
Conf	chan	Conf	chan	Conf	chan	Conf	chan
83	2	110	5	33	2	33	2
125	1	118	2	63	6	123	6
		124	1	114	1	152	1
		142	6	147	4		

Table 9: Results for random instances. FP stands for fixed power case, PC for power control, PRC for power and rate control

		Column generation			Final set			
		impr [%]	T [s]		error [%]		T [s]	
Size	Probl	av	av	max	av	max	av	max
10	FP	28.40	0.44	0.68	0.00	0.00	0.02	0.04
	PC	39.80	0.57	0.86	0.00	0.00	0.03	0.04
	PRC	22.90	2.26	5.08	1.50	2.70	0.73	5.90
15	FP	39.60	16.53	45.08	0.00	0.00	0.12	0.24
	PC	49.30	14.77	84.28	0.00	0.00	0.20	0.35
	PRC	26.70	72.67	287.76	2.60	12.50	5400.60	21600.00
20	FP	49.10	518.44	990.14	0.00	0.60	0.84	2.02
	PC	57.50	628.94	2527.84	0.00	0.60	1.83	5.90
	PRC	39.00	2235.35	5196.82	3.60	10.70	20400.83	21600.00
25	FP	51.06	5422.71	14168.55	0.00	0.00	2.40	6.65
	PC	59.60	12736.68	27622.81	0.00	0.00	5.57	29.55
	PRC	44.23	17850.34	28410.53	6.90	21.43	21600.00	21600.00

with FH the heuristic solution computed by solving the integer master problem over the final set of configurations. The third column gives the average error of the initial master problem solution with respect to the final lower bound, $\left(\frac{I\_MP-LB}{LB}\right)$, *i.e.*, the improvement in the solution of the continuous problem P obtained by column generation procedure. Columns four and five report the average and the maximum computational time spent in the column generation procedure. Columns six and seven give the average and maximum error of the final heuristic solution with respect to the lower bound $\left(\frac{FH-LB}{LB}\right)$, while the last two columns report the FH computational time, average and maximum values.

We start analyzing the algorithm behavior in the fixed power case. The final heuristic solution is actually always the optimal one; in fact it provides the same number of slot required by the continuous relaxation. The column generation procedure improves the initial solution obtained with one-pair configurations and fills the gap to the optimum. The computational time is negligible for small size instances but increases with the problem size (instance size is basically determined by the number of links that can be potentially activated).

With the variable power case, the algorithm behavior is similar to the fixed power case. Even in this case the final heuristic solution is actually always optimal, but the initial heuristic solution is worse, therefore, the column generation procedure has a better improving effect. The problem is more difficult to solve, and the

Table 10: *Greedy-Add* heuristic results

Size	Probl	CG	Greedy-Add					
		# slots	100 conf.		500 conf.		1000 conf.	
			# slots	gap [%]	# slots	gap [%]	# slots	gap [%]
10	FP	224.33	252.36	12.49	248.52	10.78	248.52	10.78
	PC	186.33	245.47	31.74	240.19	28.91	240.19	28.91
	PRC	96.11	147.41	53.37	146.60	52.54	144.89	50.76
15	FP	167.08	190.00	13.72	179.28	7.30	177.69	6.35
	PC	141.23	176.86	25.22	164.69	16.61	162.80	15.27
	PRC	75.46	89.50	18.60	82.32	9.08	81.75	8.33
20	FP	157.15	208.43	32.63	183.84	16.98	175.53	11.69
	PC	130.92	198.50	51.62	169.02	29.10	163.14	24.61
	PRC	63.92	109.00	70.51	80.94	26.62	78.17	22.29
25	FP	129.67	164.80	27.09	144.14	11.16	140.87	8.64
	PC	107.00	146.20	36.64	123.54	15.46	115.43	7.88
	PRC	46.93	81.19	72.99	56.59	20.58	53.14	13.22

Table 11: *Matching-Remove* heuristic results

Size	Probl	CG	Matching-Remove					
		# slots	100 conf.		500 conf.		1000 conf.	
			# slots	gap [%]	# slots	gap [%]	# slots	gap [%]
10	FP	224.33	299.78	33.63	289.56	29.07	289.56	29.07
	PC	186.33	281.22	50.92	273.78	46.93	271.68	45.80
	PRC	96.11	249.09	159.17	243.97	153.84	237.68	147.30
15	FP	167.08	253.77	51.89	248.65	48.83	248.51	48.74
	PC	141.23	247.57	75.30	238.04	68.55	235.54	66.78
	PRC	75.46	219.77	191.23	214.56	184.33	214.28	183.96
20	FP	157.15	290.71	84.99	283.97	80.70	280.90	78.74
	PC	130.92	281.61	115.09	272.46	108.11	269.18	105.61
	PRC	63.92	255.61	299.87	245.83	284.57	242.65	279.60
25	FP	129.67	261.57	101.72	253.78	95.72	250.07	92.85
	PC	107.00	258.38	141.47	248.88	132.60	244.05	128.09
	PRC	46.93	234.97	400.64	229.08	388.10	226.05	381.63

computational time increases.

The case with variable power and rate is the most complex among proposed problems: due to the increased number of pricing iterations and to the fact that the pricing problem is more difficult to solve than previous ones, the total computational time increases remarkably. Results show that the performance of the algorithms with respect to the bound gets worse, however, the gap does not exceed 8%. One of the reasons for the higher error is that the frame is much shorter than in the other cases (see comments below on columns four and five of Table 12), thus a difference of few slots with respect to the optimum value has a higher impact. We observe, however, that column generation helps to reduce the error in particular when instance sizes increase.

In order to get insights into the effectiveness of our approach based on column generation, we compare in Table 10 and Table 11 results achieved by two different heuristic algorithms. These algorithms have been selected from those available in literature, and then modified to make a fair comparison. Indeed, previous works do not consider all aspects we do in our formulation. Therefore, starting from the key idea of previously proposed algorithms we have extended them to fit into our framework. This allows us to compare the effectiveness of each approach considering a common set of problem features. In practice, we use these two heuristic algorithms as generators for the set of configurations, in place of the column generation procedure.

The first algorithm, inspired by the algorithm in [37], consists of a greedy procedure that tries to maximize the number of concurrently transmitted packets during a slot. It builds up a configuration by iteratively including the link that can transmit at the highest rate without violating SINR and half-duplex constraints. When only one rate is available or more links can use the highest rate, the link to be inserted is randomly chosen. The second algorithm is derived from the work in [31]. The configuration generation process starts finding a maximum matching among network nodes. Then, the link with the largest interference on the other links in the matching set is iteratively removed, until SINR constraints are satisfied. The links of a valid set are activated in such a way that the resulting configuration has the largest total number of transmitted packets. We call the first algorithm *Greedy-Add* and the second *Matching-Remove*.

Results in Table 10 and Table 11 show the percentage gap between the length of frames created using the set of configurations generated with the proposed Column Generation (CG) procedure and with heuristic algorithms. Heuristic algorithms have been tested with generated sets of 100, 500, and 1000 configurations. In order to show the minimum achievable gain of our approach, we make the comparison between the upper bound given by FH and a lower bound computed by solving the continuous master problem over the set of configurations generated with heuristic algorithms. Results show that heuristic algorithms are outperformed by the column generation procedure. Indeed, the use of the column generation, in particular the information given by the dual variables of the master problem, is able to guide the generation process toward configurations that can effectively contribute to shorten the frame length. Dual variables provide a valuable tool to jointly consider traffic demands and their routing paths when creating each configuration. This helps to explore a solution space that, otherwise, would be too large to be faced with heuristics. Moreover, moving from a set with 500 configurations to one with 1000 configurations does not substantially improve results. That is an evidence of the fact that further increasing the size of the heuristic set of configurations (beyond 1000 items) will not produce significantly better solutions.

In Table 12 we evaluate, from column 4 to column 6, the advantage of using more advanced transmission techniques and the joint routing optimization. We define the parameter ψ as the ratio between the number of slots in a frame and the total number of packets to be routed in the network. Roughly speaking, ψ represents the average number of slots needed to transmit a packet. The value of ψ decreases as advanced transmission techniques, like power control and rate adaptation, are introduced. It also interesting to note that ψ decreases when the instance size increases: the algorithm can choose many more alternative paths exploring a larger solution space. Moreover, from column 7 to column 9, we compare the results with the proposed problem formulations with results obtained with a simple min-hop routing and the optimization of the scheduling only (scheduling optimization has been performed with the same column generation approach). We observe that the approach based on min-hop routing requires very low computational effort, but the joint routing and scheduling optimization can greatly improve the network efficiency reducing the frame length (more than 60%) with respect to min-hop case in big instances. This is more evident when the instance sizes increase

Table 12: Frame length with routing optimization and with min-hop routing

Size	# pkts	Probl	# slots FH	ψ	Time [s]	# slots min-hop	Time [s]	Saving [%]
10	175.18	FP	224.33	1.45	0.47	250.78	0.19	10.54
		PC	186.33	1.20	0.62	216.56	0.24	13.96
		PRC	96.11	0.62	3.24	141.22	0.58	31.94
15	181.00	FP	167.08	1.05	17.34	204.54	0.79	18.32
		PC	141.23	0.89	16.15	179.77	0.77	21.44
		PRC	75.46	0.48	5248.36	120.31	2.71	37.28
20	190.87	FP	157.15	0.97	484.28	226.00	4.20	30.46
		PC	130.92	0.81	612.20	201.08	2.17	34.89
		PRC	63.92	0.40	22067.31	140.85	8.84	54.61
25	191.79	FP	144.53	0.75	2465.22	241.68	5.85	37.40
		PC	123.00	0.64	607.97	216.47	3.42	42.53
		PRC	55.11	0.29	41238.74	152.42	13.10	63.99

or advanced transmission techniques are available. As explained above, when the network size increases, an optimized routing can better exploit the alternative paths provided. The use of rate adaptation and power control techniques can take advantage of optimized routing as well. As shown in the previous sub-section, proper use of high-rate shorter paths and low-rate longer paths results in a better resources utilization in case of different traffic demands. We believe that, differently from mobile ad hoc networks, the longer computation time required by the joint optimization approach is tolerable with WMNs.

Finally, in Table 13 we present channel assignment results obtained with the same setting of parameters adopted in the previous sub-section. We note that the use of multi-radio equipment in a multi-channel scenario permits to save about the 70-80% of the frame length with respect to a single channel scenario. Therefore, the network throughput is 3-4 times higher. From the computational point of view, the instance size is basically given by the number of configurations to be assigned to multi-channel time slots; as this value increases the computational time rises up, as well as the number of static assignment instances that cannot be solved to the optimum within given time limits. In addition, note that the high number of links activated in configurations generated with power control leads to smaller slot savings when multiple channels are used. This is due to the limited number of per-node interfaces, 3 in our scenario.

Dynamic channel assignment is solved to optimality in reasonably short time. It is a less difficult and less constrained problem than static assignment. Configurations can be packed tightly and resulting frames are short. In every instance the assignment exploits almost all the 6 available channels.

Static channel assignment is more complex and, therefore, computational times increase. The number of needed slots is slight greater than in the dynamic assignment, however, with static assignment no channel switching is required. Due to tight constraints on the channel assignment to wireless interfaces we also note that, usually, not all the 6 available channels are used (see the ninth column of Table 13). In the last column of the table we report the percentage of cases in which the algorithm is able to reach the optimum within the time limit.

Table 13: Channel assignment results

Size	Probl	1 channel	Dynamic chan. assign.			Static chan. assign.				
		# slots (# confs)	# slots	Time [s]	Saving [%]	# slots	Time [s]	Used channels	Saving [%]	Opt [%]
10	FP	224.33	47.33	362.19	78.90	57.44	9811.30	4.78	74.39	0.00
	PC	186.33	47.56	125.54	74.48	54.67	7935.01	4.33	70.66	11.11
	PRC	96.11	24.67	12.66	74.34	26.78	1116.15	4.11	72.14	66.67
15	FP	167.08	38.54	109.65	76.93	45.69	5397.91	4.62	72.65	0.00
	PC	141.23	37.92	128.69	73.15	44.46	2563.82	3.62	68.52	15.38
	PRC	75.46	19.46	9.13	74.21	22.69	1002.22	3.92	69.93	60.77
20	FP	157.15	34.77	198.76	77.88	48.08	5036.36	4.31	69.41	0.00
	PC	130.92	34.54	90.49	73.62	43.77	1268.99	3.08	66.57	7.69
	PRC	63.92	14.69	9.01	77.02	19.69	890.78	4.08	69.19	69.23
25	FP	129.67	28.60	57.63	77.94	40.20	5641.62	4.93	69.00	0.00
	PC	107.00	28.47	20.87	73.40	35.93	487.32	3.07	66.42	6.67
	PRC	46.93	9.87	2.67	78.98	14.73	1048.73	4.00	68.61	66.67

6 Conclusion

In this paper we studied the joint routing, scheduling, power control, rate control, and channel assignment problem in Wireless Mesh Networks where traffic engineering methodologies able to provide bandwidth guarantees to traffic flows and to optimize transmission resource utilization are of paramount importance.

For each traffic demand we considered bandwidth constraints defining the number of packets that must be delivered per frame. Interference model is based on the SINR at receivers. The number of packets transmitted per time slot is selected according to a discrete set of possible transmission rates and their corresponding SINR thresholds. Data rates and SINR thresholds are an input of the problem and no a priori model of the relation between the data rate and the SINR is considered. We adopted a multi-radio multi-channel WMN scenario where mesh devices are equipped with a set of wireless interfaces that can be tuned on available orthogonal channels. Two channel allocation policies have been considered, a dynamic one where wireless interfaces are able to switch from a channel to another on a slot-by-slot basis, and a static policy where channels are assigned permanently.

We considered problem formulations with an exponential number of variables that define feasible sets of links over which parallel transmissions can safely occur on the same channel, called configurations. Since the number of variables is huge even with small instances we adopted a solution approach based on column generation: starting from an initial set of variables the continuous relaxation is solved to the optimum and then additional variables are added exploiting duality properties. The solution of the continuous relaxation provides a lower bound, while an upper bound is obtained solving the integer problem over the same set of configurations used by the column generation procedure. Then, configurations are assigned to channels considering a dynamic or static strategy by using a heuristic algorithm based on the admissibility version of the problems.

The described resource optimization approach has been tested on randomly generated instances. We used a small ad hoc instance to get some insights into the impact of different problem features to the optimal solution. Numerical results obtained on a larger set of instances show that the column generation approach provides good lower bounds and often it proves that the heuristic solution is the optimal one. Moreover, comparison results against shortest path routing and other heuristic scheduling solutions show the effectiveness of the proposed approach.

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