Joint Routing and Scheduling Optimization in Arbitrary Ad Hoc Networks: Comparison of Cooperative and Hop-by-Hop Forwarding

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Abstract

Cooperation schemes are the key elements of infrastructure-less wireless networks that allow nodes that cannot directly communicate to exchange information through the help of intermediate nodes that relay their messages towards destination. The most widely adopted approach is based on hop-by-hop forwarding at network layer along a path to destination. Cooperative relaying brings cooperation al physical layer in order to fully exploit wireless resources. The concept exploits channel diversity by using multiple radio units to transmit the same message. The underlying fundamentals of cooperative relaying have been quite well-studied from a transmission efficiency point of view, in particular with a single source and destination. Results of its performance gain in a multi-hop networking context with multiple sources and destinations are, however, less available. In this paper, we provide an optimization approach to assess the performance gain of cooperative relaying vis-a-vis conventional multi-hop forwarding for joint packet routing and transmission scheduling under general network topology. The approach extends and generalizes classical optimization schemes for non-cooperative networks. We provide numerical results demonstrating that the gain of cooperative relaying decreases when network connectivity and the

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number of traffic flows increase due to the effect of interference and resource reuse limitation. In addition to quantifying the performance gain, our approach leads to a new framework for optimizing routing and scheduling in cooperative networks.
Keywords: Cooperative Networking, Routing and Scheduling, Column Generation

1. Introduction

Infrastructure-less wireless networks where nodes directly communicate with each other have received much attention from the research community and generated a plethora of applications (including ad hoc, sensor, and mesh networks). In most of these application scenarios, direct communications among nodes are not sufficient due to the transmission range limitations and cooperative approaches are adopted. The goal is to allow nodes that cannot directly communicate to exchange information through the help of intermediate nodes that relay their messages towards destination.

The first and most common cooperation mechanism is based on the hop-by-hop forwarding along a path composed by a sequence of nodes from source to destination. At each hop a single node forwards an information packet to the next node in the path which is usually dynamically selected through a routing protocol [1]. This conventional multi-hop forwarding has been adopted in almost all practical implementations of ad hoc, sensor and wireless mesh networks and analyzed by a huge literature on the topic.

Recently, in order to overcome the capacity limits of conventional multi-hop forwarding, a new approach that exploits cooperation also at transmission layer has been considered [2, 3]. The idea, that dates back to the work of van der Meulen [4] and Cover [5], is to use multiple relays whose transmissions are combined together at the receiver. Exploiting the broadcast nature of the radio channel, transmissions can be received and then simultaneously forwarded by groups of nodes acting together as a virtual array of antennas [6, 7]. Relay nodes can decode-and-forward or just amplify-and-forward information packets [8, 9].

Similarly to multiple antenna systems, with cooperative relaying there are some remarkable advantages in terms of channel capacity that have been extensively analyzed [10–12] considering also distributed coding schemes [13–15], retransmission mechanisms [16], and power allocation methods [17, 18]. Moreover, due to the fading effect of the radio channel caused by obstacles and multi-path propagation, cooperative relaying can provide some spacial diversity gain [8, 19], even though there is an obvious trade-off between capacity and diversity gains [20].

If the positive effect of cooperative relaying is easy to understand in single source-destination scenarios, the case of multiple sources and destinations is more complex to assess [21, 22] and requires the design of cooperation protocols for multiple access, channel state information and routing [23–25]. In general, it is expected that the signaling overhead is higher than traditional hop-by-hop forwarding due to the increased coordination information. However, even neglecting the effect of increased overhead, we argue that the potential gain of cooperative relaying from a network capacity perspective is affected by interference and resource reuse limitations.

The evaluation at system level of the performance gain of cooperative relaying in wireless multi-hop network has received so far little attention in the literature. It requires measuring the overall efficiency in using radio resources when serving multiple traffic flows between different source-destination pairs in the presence of interference. Actually, due to the difficulty of modeling interference, network capacity evaluation is not an easy task even with the conventional multi-hop forwarding scheme for which mainly asymptotic results exist [26]. A first work in [**?**] considers cooperative relaying in a network with two sources and two destinations and evaluates the channel capacity. Differently in this paper, we consider arbitrary ad hoc network topologies and focus on the evaluation of network capacity. In [28], ad hoc network scenarios with multi-user interference are considered and the behavior of the amplify-and-forward scheme is analyzed characterizing the interference from a statistical perspective. Instead in our work, we consider the decode-and-forward scheme and evaluate the network efficiency in using radio resources when simultaneous transmissions can be coordinated and interference controlled. In [29] the gain of cooperative relaying in large networks with multiple sources and destinations and amplify-and-forward relays is evaluated and asymptotic results are provided modeling interference with a conflict graph according to the protocol model. In this paper we propose a different approach modeling interference with the more accurate physical model based on the signal-to-interference and noise ratio (SINR) and providing a method for evaluating arbitrary networks and defining optimal scheduling and routing schemes.

From a network perspective, in addition to radio channel capacity, there is another important element that determines the overall efficiency: the reuse level of radio resources in the network. Parallel transmissions of nodes are possible if they are sufficiently apart and the interference at their receivers is low enough to allow correct decoding of information packets. Even if cooperative relaying can increase the radio channel capacity, or equivalently the transmission range, it may negatively affect the reuse level. Transmissions from multiple relays can increase interference limiting the number of parallel transmissions. Moreover, since nodes are usually equipped with a single transceiver, at a give time they can be involved in only one cooperation group for transmitting or receiving a packet, further limiting resource reuse. In the extreme case all nodes cooperate to relay the packet of a source-destination pair, no parallel transmission can occur in the network (Figure 1).

In this paper, we provide an optimization approach to compare the performance of cooperative relaying with that of conventional multi-hop forwarding considering a decode-and-forward mechanism. Our approach allows to capture both the channel quality improvement of cooperative relaying, expressed in terms of increased transmission range, as well as the resource reuse, modeling the effect of interference with a threshold on the Signal-to-Noise and Interference Ratio (physical model). Since both packet routing and transmission scheduling

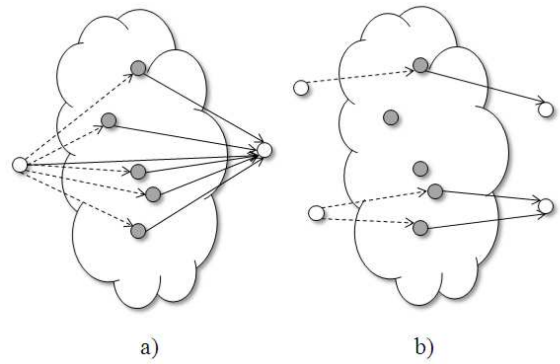


Figure 1: Cooperation and resource reuse tradeoff: a) all nodes cooperate for a single transmission, b) partial cooperation with parallel transmissions.

greatly impact the overall network capacity, we jointly optimize them considering a spacial time division multiplexing approach in order to make the comparison fair. Our model considers arbitrary network topologies and optimizes the path selection and the transmission scheme at each hop (with or without cooperation). Therefore, in the considered scenario the conventional multi-hop forwarding is just a special case of the cooperative relaying and we can measure the relative gain in network efficiency selecting the best level of cooperation. The proposed approach extends and generalizes classical optimization schemes for non-cooperative networks [30–32].

The paper is organized as follows. In Section 2 we present the system model and the related assumptions. In Section 3 we propose an integer programming formulation of the cooperation optimization problem, while in Section 4 we present a solution approach based on column generation. Computation results on some example networks and on a larger set of randomly generated networks are provided in Section 5. Finally, Section 6 concludes the paper.

2. System Model

2.1. Preliminaries

We use a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ to represent the physical topology of a wireless network, where $\mathcal{N}$ and $\mathcal{L}$ denote the sets of nodes and communication links, respectively. Denote by P_v the transmission power of $v \in \mathcal{N}$, g_{vw} the radio propagation gain between v and w, and η the effect of thermal noise. A link (v, w) exists if and only if its signal to noise ratio (SNR) meets a threshold γ. The condition reads:

$$SNR_{vw} = \frac{P_v g_{vw}}{\eta} \geq \gamma. \tag{1}$$

With time division multiple access, radio transmissions are organized using time slots, each of which may accommodate one or several parallel transmissions. Transmission between two nodes generates interference at other receivers, and limits the amount of parallel transmissions.

A widely used performance metric in scheduling transmissions is the number of time slots required to deliver traffic demand which is directly related to the overall efficiency in using radio resources.

In comparing cooperative networking with conventional forwarding, this metric captures the effect of resource reuse that through parallel transmissions reduces the number of slots, and the effect of range extension of cooperative relaying that reduces the required number of transmissions.

To this end, we consider a demand set $\mathcal{D}$; each element $d \in \mathcal{D}$ is a single packet associated with a tuple $\langle o^d, t^d \rangle$, where o^d are t^d are the source and destination of the packet, respectively. Without cooperation, transmission of a single packet involves exactly one sender and one receiver, and the transmission takes place over one link in $\mathcal{L}$. Under cooperation, multiple nodes may simultaneously transmit or receive the same packet, and a receiver can combine signals from multiple senders, which may or may not have links to the receiver in the physical topology $\mathcal{G}$.

Definition 1. *A wireless network $\mathcal{G}$ has a* κ**-level of cooperation***, if, during the same time slot, at most κ nodes are allowed to transmit the* **same packet** *to a set of one or more receiving nodes, and each receiver can combine the packets from the κ transmitters.*

Our comparative study of network performance seeks the answer to the following question: What are the minimum numbers of time slots required to deliver the given set of packets with and without cooperation, respectively? This leads to an optimization problem in routing and scheduling the packet set $\mathcal{D}$ in network $\mathcal{G}$ under a κ-level of cooperation, where a non-cooperative network corresponds to $\kappa = 1$.

2.2. Communication Graph with Cooperation

For $\kappa > 1$, graph $\mathcal{G}$ is not sufficient for modeling cooperative transmission. We develop a graph concept, which we refer to as κ-Cooperation graph, to generalize the original topology $\mathcal{G}$.

Definition 2. *For graph $\mathcal{G}$, the* κ**-Cooperation Graph** $G_{\mathcal{G},\kappa} = (V, A)$*, is the auxiliary graph representing all possible transmissions in $\mathcal{G}$ permitted by a κ-level of cooperation. A node $i \in V$ represents a non-empty subset of $\mathcal{N}$ having cardinality up to κ, and a link $(i, j) \in A$ represents simultaneous transmission of one packet from all nodes in the i's node set in $\mathcal{N}$ to j's node set in $\mathcal{N}$.*

Henceforth, we simplify the notation of the κ-cooperation graph to G, when there is no ambiguity. For the sake of clarity, we use v and w to denote nodes in the original graph $\mathcal{G}$, and i and j nodes in the cooperation graph G. Nodes and links in G are also referred to as *super-nodes* and *super-links*. Let $\Gamma(i)$ denote the set of nodes in $\mathcal{N}$ associated with super-node i in V, and $\Lambda(i, j)$ the set of links associated with super-link $(i, j) \in A$: $\Lambda(i, j) = \{(v, w) \mid v \in \Gamma(i), w \in \Gamma(j)\}$. Note that the size of G grows exponentially in κ. When $\kappa = 1$, the κ-cooperation graph G reduces to the original topology $\mathcal{G}$. The concepts of super-node and super-link are illustrated in Figure 2. In this example, each of the two super-nodes i and j contains two nodes in $\mathcal{N}$, and the super-link (i, j) represents transmissions from all nodes in $\Gamma(i)$ to all nodes in $\Gamma(j)$.

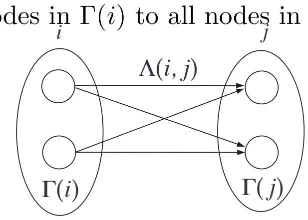


Figure 2: An illustration of super-nodes and super-link.

In Figure 2, the four transmissions do not necessarily correspond to links in the original graph $\mathcal{G}$. This is because the SNR condition takes a new form in the cooperation graph G. For a super-link $(i, j) \in A$ to exist, the following SNR condition applies to all receivers of the super-link, i.e., for all $w \in \Gamma(j)$:

$$SNR_{iw} = \frac{\sum_{v \in \Gamma(i)} P_v g_{vw}}{\eta} \geq \gamma. \qquad (2)$$

The numerator in (2) models the fact that the nodes $\Gamma(i)$ are transmitting the same packet and hence all contributing to improving SNR. For this reason, a super-link can be established, as a result of cooperation, even if some or possibly none of the transmissions of this super-link is part of of the link set in the original topology.

STE: Note that for $k > 1$ the cooperation graph is not symmetric: there might exist pairs of nodes v and w such that v is reachable from w, but not viceversa.

2.3. Classes of Super-Links

The super-nodes in G vary in the cardinality of associated subsets of nodes in the original graph. Based on this cardinality, we define four classes of super-links.

1. **One-to-one**. These are the links in the original physical topology, i.e., links in $\mathcal{L}$. Super-link (i, j) is of this class if it satisfies the condition: $\Gamma(i) = \{v\}$, $\Gamma(j) = \{w\}$, and $(v, w) \in \mathcal{L}$.

2. **One-to-many**. A super-link of this class corresponds to a group of links in $\mathcal{L}$ originating from the same node in $\mathcal{N}$. Thus $(i, j) \in A$ is a one-to-many super-link if $\Gamma(i) = \{v\}$, $|\Gamma(j)| > 1$, and $(v, w) \in \mathcal{L}$ for all $w \in \Gamma(j)$. The one-to-many links are also referred to as *broadcast links*.

   A special subclass of one-to-many links are the so–called *buffering links*, in which a node behaves as it was transmitting also to itself.

3. **Many-to-one**. A super-link $(i, j) \in A$ is of class many-to-one, if $|\Gamma(i)| > 1$ and $\Gamma(j) = \{w\}, w \in \mathcal{N} \setminus \Gamma(i)$. This super-link represents simultaneous transmissions of the same packet from all nodes in $\Gamma(i)$ to the single receiver w in $\Gamma(j)$. A many-to-one link does not necessarily consist in a group of links in the original graph, i.e., $\Gamma(i)$ may contain node v for which $(v, w) \notin \mathcal{L}$, given that the SNR at w satisfies (2) by cooperation. Links in the many-to-one class are also referred to as *cooperating links*.

4. **Many-to-many**: This class of links in G represent transmissions of the same packet between multiple transmitters and receivers in the original graph $\mathcal{G}$. A super-link (i, j) is a many-to-many link if and only if $|\Gamma(i)| > 1$ and $|\Gamma(j)| > 1$. The super-link shown in Figure 2 is of this class. Similar to many-to-one super-links, a many-to-many super-link (i, j) can be created by cooperation, hence it may have transmissions between one or multiple pairs of nodes $v \in \Gamma(i)$ and $w \in \Gamma(j)$ for which $(v, w) \notin \mathcal{L}$. Many-to-many super-links are also called *broad-cooperating links* or *multicasting links*.

Example 1. *Figure 3 shows the physical topology of a wireless network with 5 nodes. Figure 4 shows the super-nodes of the corresponding 2-cooperation network. For clarity, only a few of the super-links are drawn. The blue links are of class one-to-one; these are the links incident to node 2 in the original topology. The red super-links are broadcasting links from node 3 to super-nodes* $\{1, 2\}$, $\{1, 5\}$, *and* $\{2, 5\}$, *respectively. Each of the three green super-links is a cooperating link, representing simultaneous transmissions of a packet from super-node* $\{2, 3\}$ *to a single receiver. Note that transmissions on* $(2, 1)$, $(2, 5)$, *and* $(3, 4)$ *do not correspond to physical links in Figure 3. Finally, the brown super-links are*

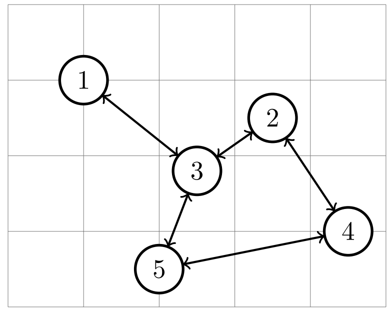


Figure 3: Topology of a very simple wireless network.

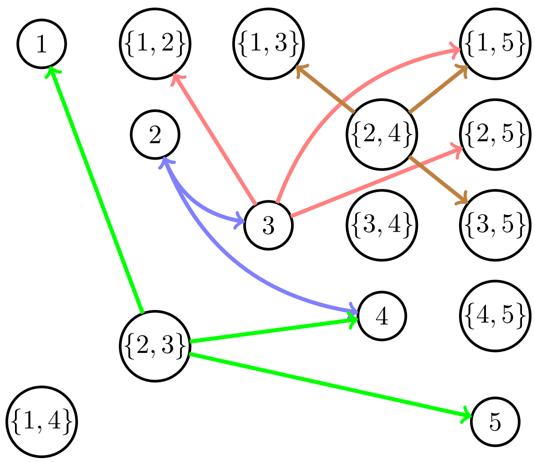


Figure 4: Super-nodes and some super-links of the 2-cooperation graph for the network of Figure 3.

multicasting links, each of which carries transmissions involving two transmitters and two receivers; some of these transmissions occur between nodes that do not have links in Figure 3. The distance in hops between a source-destination pair may benefit from cooperating and multicasting links. For example, the shortest path from node 4 to node 1 has three hops if no cooperation is allowed. The shortest path distance in the cooperation graph becomes two hops, and one of such paths is formed by 4, $\{2, 5\}$*, and 1.*

2.4. Interference Models

When several super-nodes are transmitting in the same time slot, interference must be accounted for. We consider here the the SINR physical model as follows.

SINR model.. Suppose link (v, w) is used for a transmission from v to w, and K is the set of nodes transmitting in the same time slot other than v, the transmission on (v, w) is considered successful if and only if the signal-to-interference and noise ratio (SINR) is at least γ, i.e.,

$$SINR_{vw} = \frac{P_v g_{vw}}{\sum_{u \in K} P_u g_{uw} + \eta} \geq \gamma. \tag{3}$$

Suppose now that in addition to node i, a set of super-nodes Ω are transmitting in the same time slot. The SINR condition for super-link (i, j) is that for all $w \in \Gamma(j)$, the following inequality holds,

$$SINR_{iw} = \frac{\sum_{v \in \Gamma(i)} P_v g_{vw}}{\sum_{l \in \Omega} \sum_{u \in \Gamma(l)} P_u g_{uw} + \eta} \geq \gamma. \tag{4}$$

In (4), all nodes composing the super-nodes in Ω increase the interference for super-link (i, j). Note that a node $u \in \mathcal{N}$ may appear at most once in the denominator, because the super-nodes transmitting in any time slot all must have mutually disjoint sets of nodes in the original graph.

3. An Integer Programming Formulation

Routing and scheduling of packets without cooperation, with the objective of minimizing the total number of time slots, has been studied quite extensively in the literature (see Section 1). From a computational complexity standpoint, this optimization problem is NP-hard, in particular because even if optimization is restricted to scheduling only (i.e., routing is fixed), the problem generalizes graph coloring [30]. Integer programming based on identifying *feasible configurations* admits, within a reasonable amount of computing time, an optimal or very-close-to optimal routing and scheduling solution for networks of up to moderate size [32]. Here, a feasible configuration refers to a group of transmissions that may share a time slot. Utilizing the cooperation graph, we are able to generalize this integer programming concept to networks with cooperation.

For cooperation graph G, a feasible configuration of a time slot is a group of super-links, such that the interference constraints are satisfied for all of them under simultaneous transmissions. Thus constructing a scheduling solution amounts to selecting a set of feasible configurations, and the cardinality of the set equals the total number of time slots.

Let $\mathcal{S}$ denote the collection of all feasible configurations, and $S_{ij} \subseteq \mathcal{S}$ the set of configurations containing super-link $(i, j) \in A$. Let b_i^d be a parameter that is equal to 1 if node i is the source of the packet d, -1 if it is the destination of d, and 0 otherwise. We define two sets of optimization variables.

$$\lambda_s = \text{The number of time slots allocated to configuration } s \in \mathcal{S},$$

$$\beta_{ij}^d = \begin{cases} 1 & \text{If super-link } (i,j) \text{ is used to route packet } d \text{ from } i \text{ to } j, \\ 0 & \text{otherwise.} \end{cases}$$

The λ_s are called the configuration variables, and the β_{ij}^d are the flow variables.

The problem of routing and scheduling the demand set $\mathcal{D}$ with a minimum number of time slots can be formulated as the following mixed integer linear programming model.

$$[\mathbf{P}] \ \min \sum_{s \in S} \lambda_s \tag{5}$$

$$\text{s.t.} \sum_{(i,j) \in A} \beta_{ij}^d - \sum_{(j,i) \in A} \beta_{ji}^d = b_i^d, i \in V, d \in \mathcal{D}, \tag{6}$$

$$\sum_{s \in S_{ij}} \lambda_s \geq \sum_{d \in \mathcal{D}} \beta_{ij}^d, (i,j) \in A, \tag{7}$$

$$\beta_{ij}^d \in \{0,1\}, \ (i,j) \in A, d \in \mathcal{D}, \tag{8}$$

$$\lambda_s \in \mathbb{Z}^+, \ s \in \mathcal{S}. \tag{9}$$

The objective is to minimize the total number of configurations (i.e., time slots). Constraints (6) are the flow balance equations for each packet $d \in \mathcal{D}$. Constraints (7) link the flow variables to the configuration variables: For each super-link $(i, j) \in A$, sufficiently many configurations containing (i, j) must be chosen to accommodate the total flow going through (i, j).

4. A Column Generation Approach

4.1. The Framework

The number of λ-variables in **P** equals $|S|$, which is exponential in the size of the cooperation graph G. Thus a solution algorithm requiring the generation of all possible feasible configurations a priori is not computationally feasible. Exploiting the structure of **P**, an effective solution strategy is to solve its linear programming (LP) relaxation by means of a column generation method, which provides a lower bound on the optimal number of time slots, followed by computing an optimal or near-optimal integer solution (e.g., see [33] for a textbook on column generation). By keeping a small subset $\mathcal{S}' \subset \mathcal{S}$ and expanding the subset systematically, the column generation method decomposes the LP version of **P** into a master problem and a subproblem. The former is used to find the optimal solution when optimization is restricted to the elements in $\mathcal{S}'$; the latter corresponds to a separation problem for the dual LP for checking optimality, that is, to either find additional elements that are of interest to be added to $\mathcal{S}'$, or conclude that the current solution of the master problem is optimal not only for $\mathcal{S}'$ but also for $\mathcal{S}$ (i.e., all variables λ_s for $s \in \mathcal{S} \setminus \mathcal{S}'$ are zeros at optimum).

The master problem looks very similar to **P**. To save space, we do not present it in mathematical form. The master problem differs from **P** in the following two aspects.

1. Set $\mathcal{S}$ is replaced by a subset $\mathcal{S}'$, of which $|\mathcal{S}'| \ll |\mathcal{S}|$.
2. The integrality requirements on the β- and λ-variables are relaxed.

As long as the resulting master problem contains at least one feasible solution, any $\mathcal{S}' \subset \mathcal{S}$ can serve as the starting point. That is, the subset $\mathcal{S}'$ must be such that constraints (7) are satisfied. The simplest choice consists of setting $\mathcal{S}' = A$, that is for each super-link (i,j) in A we have a configuration where the super-link (i,j) is the only active link.

4.2. The Pricing Problem for the SINR model

After solving the master problem, checking the LP optimality condition amounts to examining whether or not any $\lambda_s, s \in \mathcal{S} \setminus \mathcal{S}'$, has a negative reduced cost. Denoting the dual variables of (7) by $\overline{\pi}, (i,j) \in A$, the reduced cost of λ_s equals $1 - \sum_{(i,j)\in A: s \in S_{ij}} \overline{\pi}_{ij}$. The subproblem, also called the pricing problem, takes the following form,

$$\min_{s \in \mathcal{S} \setminus \mathcal{S}'} (1 - \sum_{(i,j)\in A: s\in S_{ij}} \overline{\pi}_{ij}). \tag{10}$$

The minimization in (10), in its turn, can be formulated by an integer linear model; thus we can solve (10) without explicitly having the set $\mathcal{S}$. A solution of the integer model corresponds to a feasible configuration, that is, a set of super-links that can be active in the same time slot. Below we define the set of binary variables used by the model, and the model itself.

$$z_{ij} = \begin{cases} 1 & \text{if super-link } (i,j) \text{ is active,} \\ 0 & \text{otherwise.} \end{cases}$$

$$[\mathbf{R1}] \ \min - \sum_{(i,j)\in A} \overline{\pi}_{ij} z_{ij} \tag{11}$$

$$\text{s.t.} \sum_{\substack{(i,j)\in A: \\ v\in\Gamma(i)}} z_{ij} + \sum_{\substack{(j,i)\in A: \\ v\in\Gamma(i)}} z_{ji} \le 1, \qquad v \in \mathcal{N}, \tag{12}$$

$$\frac{\sum_{v\in\Gamma(i)} P_v g_{vw}}{\sum_{\substack{(k,l)\in A: \\ k\neq i, l\neq j}} \sum_{u\in\Gamma(k)} P_u g_{uw} z_{kl} + \eta} \ge \gamma z_{ij}, \qquad (i,j) \in A, w \in \Gamma(j), \tag{13}$$

$$z_{ij} \in \{0,1\}, \qquad (i,j) \in A. \tag{14}$$

The objective is to find a configuration of minimum reduced cost. Constraints (12) ensure that the z-variables satisfy two conditions. First, for any node v in the original topology $\mathcal{G}$, at most one super-link, of which the starting super-node contains v, can be active, that is, v is allowed to transmit at most one packet in a time slot with or without cooperation. Second, if node v is transmitting, it can not be a receiving node of any active super-link at the same time. The SINR conditions are formulated by (13). The constraint has effect only if $z_{ij} = 1$. In this case, the inequality states that the SINR, in which interference originates from all active super-links other than (i, j), must be at least γ. Note that for any super-node k, $k \neq i$, the outer-sum in the denominator will contain at most one super-link from k, because of (12).

Constraints (13) are non-linear. A standard way of linearizing these constraints (e.g., see [34]) is to introduce a parameter, denote by M, and reformulate (13) into the following linear inequalities. The parameter M is chosen to be large enough such that the inequality has no effect if $z_{ij} = 0$.

$$\sum_{v \in \Gamma(i)} P_v g_{vw} \geq \gamma \left(\sum_{\substack{(k,l) \in A: \\ k \neq i, l \neq j}} \sum_{u \in \Gamma(k)} P_u g_{uw} z_{kl} + \eta \right) - M(1 - z_{ij}), (i, j) \in A, w \in \Gamma(j). \tag{15}$$

Computationally, the number of SINR constraints (15) has a great impact on the time required to solve the pricing problem. Recall that the number of super-links $|A|$ is exponential in the size of the original graph $\mathcal{G}$. In **R1**, there are $\sum_{(i,j) \in A} |\Gamma(j)|$ SINR constraints. Below we present an alternative model for the pricing problem using more variables and inequalities but significantly less SINR constraints.

$$q_{vw} = \begin{cases} 1 & \text{If node } v \in \mathcal{N} \text{ is transmitting to } w \in \mathcal{N}, \\ 0 & \text{otherwise.} \end{cases}$$

$$q'_{vw} = \begin{cases} 1 & \text{If transmission of node } v \in \mathcal{N} \text{ gives interference at } w \in \mathcal{N}, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_v = \begin{cases} 1 & \text{If node } v \in \mathcal{N} \text{ is transmitting,} \\ 0 & \text{otherwise.} \end{cases}$$

$$y_w = \begin{cases} 1 & \text{If node } w \in \mathcal{N} \text{ is receiving,} \\ 0 & \text{otherwise.} \end{cases}$$

$$
\begin{aligned}
&[\mathbf{R2}]\ \min - \sum_{(i,j)\in A} \overline{\pi}_{ij} z_{ij} \\
&\quad \text{s.t.}(12), \\
&\quad \sum_{\substack{(i,j)\in A:\\ v\in\Gamma(i),\\ w\in\Gamma(j)}} z_{ij} = q_{vw}, && v\in\mathcal{N}, w\in\mathcal{N}, v\neq w, && (16)\\
&\quad x_v = q_{vw} + q'_{vw}, && v\in\mathcal{N}, w\in\mathcal{N}, v\neq w, && (17)\\
&\quad q_{vw} \leq y_w, && v\in\mathcal{N}, w\in\mathcal{N}, v\neq w, && (18)\\
&\quad \frac{\sum_{v\in\mathcal{N}} P_v g_{vw} q_{vw}}{\sum_{u\in\mathcal{N}} P_u g_{uw} q'_{uw} + \eta} \geq \gamma y_w, && w\in\mathcal{N}, && (19)\\
&\quad z_{ij} \in \{0,1\}, && (i,j)\in A, && (20)\\
&\quad q_{vw}, q'_{vw} \in \{0,1\}, && v\in\mathcal{N}, w\in\mathcal{N}, v\neq w, && (21)\\
&\quad x_v, y_v \in \{0,1\}, && v\in\mathcal{N}. && (22)
\end{aligned}
$$

By (16), node v transmits to node w if and only if a super-link containing v and w as transmitting and receiving nodes, respectively, is active in the cooperation graph. Constraints (17) state that, if v is transmitting, then another node w either is a receiver or the transmission of v generates interference at w. The next set of constraints ensures $y_w = 1$ if $q_{vw} = 1$ for any v. The SINR conditions, one for each $w \in \mathcal{N}$, are formulated in (19). These constraints can be linearized in a way similar to (15).

4.3. Algorithm Summary

The computational machinery that we propose for solving the routing and scheduling problem in a cooperative network consists in the following steps.

Step 1. Define an initial subset $\mathcal{S}' \subset \mathcal{S}$.

Step 2. Solve the master problem of the LP relaxation of $\mathbf{P}$ (Section 4.1).

Step 3. Solve the pricing problem using either of the two models (Section 4.2).

Step 4. **STE:** If at least a feasible configuration $s \in \mathcal{S} \backslash \mathcal{S}'$ with negative reduced cost is found, then add the configuration(s) to the master problem and go to Step 2, otherwise go to the next step.

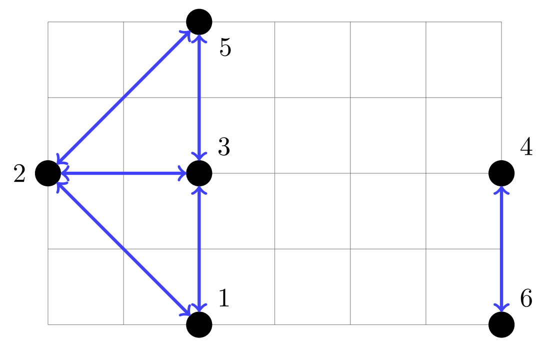
Step 5. Find an integer solution for the configurations in set $\mathcal{S}'$ and return the corresponding solution.

The last step can be performed by applying an integer linear solver, or, if this takes excessive computing time, using a heuristic algorithm. In either case, **P** is not guaranteed to be solved to optimality, because some configurations used at integer optimum may not be present in $\mathcal{S}'$ in this step. Ensuring integer optimality would require embedding the above algorithm into a branch and price scheme. Empirically, however, for all network instances used in the experiments, integer optimum is obtained and verified (by the LP value, sometimes after rounding), i.e., no branching is needed.

Among the steps in the algorithm, two of them are quite important. The first important step is to compute the initial subset of configuration $\mathcal{S}'$, and the second crucial consists in solving the pricing problem. In order to generate $\mathcal{S}'$, we use a Shortest Path Allocation (SPA) heuristic that generates for each demand the shortest path form its source to its destination, and add to $\mathcal{S}'$ a configuration for each link that appears in at least one of the generated paths. **STE:** The second step of solving the pricing subproblem is definitly the most crucial. One approach to speed up this step is to solve the pricing problem to optimality (i.e., finding the minimum reduced cost), but keeping a pool of feasible solution as diverse as possible, and adding up to 20 columns per iteration. Although the number of columns in the master problem grows rapidly, the approach pays off since much less iteration are required to reach the convergence.

5. Computational Results

We present first a set of basic examples that show the main issues in evaluating cooperative schemes. Then, in the next subsection, we report computational results on a large set of random instances.



(a) Topology with $P = 0.2$ mW, $\eta = 10^{-10}$ mW, $\gamma = 10$.

$2 \to 3, 5$	$3, 5 \to 4$	$4, 6 \to 3$	$4 \to 4, 6$

(b) Two demands: $\langle 2, 4\rangle$ and $\langle 4, 3\rangle$

Figure 5: Cooperative networking to provide additional connectivity.

5.1. Basic Examples

A clear advantage of cooperative relaying consists in allowing the communication between nodes that, due to the SNR constraint, are not connected in the original network. Figure 5 shows a small example of a network with 6 nodes and two demands $\langle 2, 4\rangle$ and $\langle 4, 3\rangle$. Remember that, in this paper, a demand represents a single packet associated with a source and a destination. The fixed parameters are the transmission power $P = 0.2$ mW, the thermal noise at the receiver $\eta = 10^{-10}$ mW, and the SNR thresholds $\gamma = 10$. Since node 4 is disconnected from both node 2 and node 3, these two demands cannot be satisfied without cooperation. If we allow a 2-level of cooperation, the demand $\langle 2, 4\rangle$ is satisfied with the configurations $2 \to \{3, 5\}$ and $\{3, 5\} \to 4$, and the demand $\langle 4, 3\rangle$ with the configurations $4 \to \{4, 6\}$ and $\{4, 6\} \to 3$. Note that the second demand uses the *buffering link* $\{4, 6\} \to 3$.

The advantage of cooperation tends to decrease as the connectivity of the network increases. To quantify the connectivity, we use two network properties: (i) the network density $\delta = \frac{\mathcal{L}}{\mathcal{N}(\mathcal{N}-1)}$, that is the percentage of links that satisfies the SNR constraint with respect to a fully connected network, and (ii) the

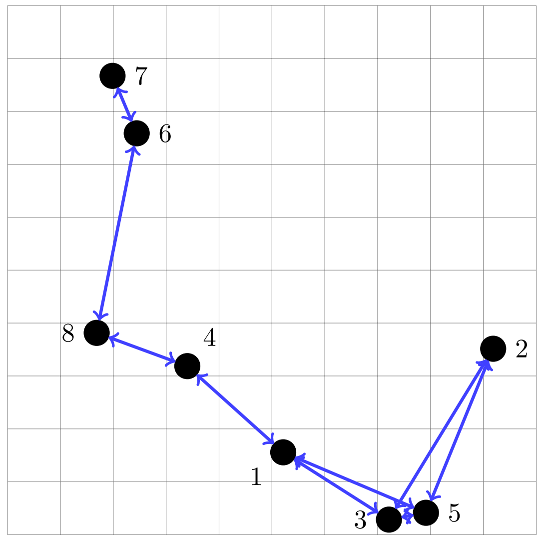


Figure 6: Topology with density $\delta = 0.14$ and diameter $\Delta = 6$, induced by parameters $P = 0.1$ mW and $\eta = 10^{-10}$ mW, $\gamma = 15$.

network diameter Δ, that is, the maximum hop distance between any pair of nodes. Figures 6 and 7 show two networks that differ only for their level of connectivity. The difference of connectivity is induced by the SNR constraint, since in the first case the nodes transmit with $P = 0.1$ mW and the second case with $P = 0.4$ mW. In this example, each pair of nodes i and j has a demand in both directions, that is $\langle i, j\rangle$ and $\langle j, i\rangle$. For the network of Figure 6, the optimal solution requires 130 time slots without cooperation, and only 110 time slots with cooperation, showing a clear advantage of cooperation. For the network of Figure 7, having higher connectivity, both no-cooperation and cooperation require 74 time slots for satisfying the traffic demands, yielding no difference in the two technologies.

The traffic load, measured as number of traffic demands, is another important factor to evaluate the effects of cooperation. In particular, increasing the number of traffic demands the advantage of cooperative relaying decreases. Figure 8 shows a significant example, on a grid network with 12 nodes and with parameters equal to $P = 0.2$ mW, $\eta = 10^{-10}$ mW, and $\gamma = 10$. We compare two

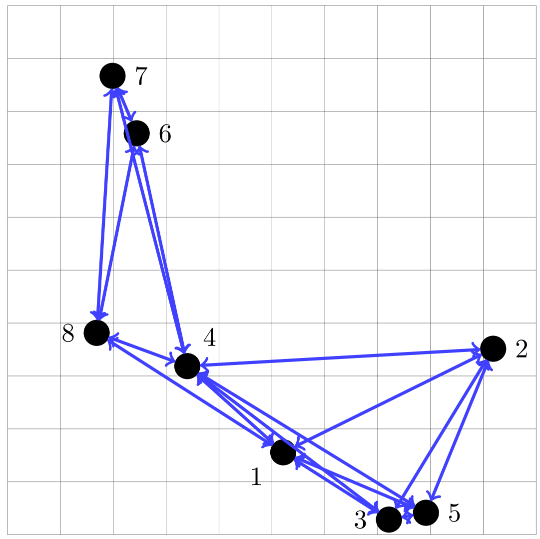
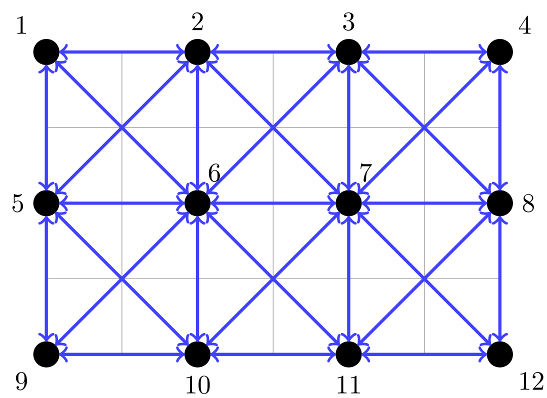


Figure 7: Topology with density $\delta = 0.30$ and diameter $\Delta = 2$, induced by parameters $P = 0.4$ mW, $\eta = 10^{-10}$ mW and $\gamma = 15$.

problem instances: the first instance has a single demand $\langle 1, 4 \rangle$, the second instance has two demands $\langle 1, 4 \rangle$ and $\langle 9, 12 \rangle$. In the first case cooperative relaying requires one time slot less to satisfy the demand. In the second case cooperative networking has an optimal solution where cooperation does not occur, yielding the same solution of the problem with traditional hop-by-hop forwarding.

5.2. Computational Evaluation

We performed a preliminary assessment of cooperative forwarding by running a series of simulations on random topologies with 13 and 15 nodes drawn on a square area of 1000 m^2. The thermal noise is set to $\eta = 2.0 * 10^{-10}$ mW and the SNR ratio to $\gamma = 10$. The propagation gain is computed as $g_{ij} = (\text{distance}_{ij})^{-3}$. The number of demands ranges from 5, 10, 20, and all pairs of connected nodes. The transmission power is set either to $P = 0.1$ mW or to $P = 0.4$ mW. For $P = 0.1$ mW the network given by the SNR ratio results to be a sparse network with average diameter equal to 3, while for $P = 0.4$ mW the network is denser, and, above all, it has an average diameter equal to or smaller than 2.



(a) Topology with $P = 0.2$ mW, $\eta = 10^{-10}$ mW, and $\gamma = 10$.

$1 \to 2$	$2 \to 3$	$3 \to 4$

(b) Single demand $\langle 1, 4 \rangle$: solution without cooperation

$1 \to \{2, 6\}$	$\{2, 6\} \to 4$

(c) Single demand $\langle 1, 4 \rangle$: solution with cooperation

$2 \to 3$ $10 \to 11$	$3 \to 4$ $11 \to 12$	$1 \to 2$ $9 \to 10$

(d) Two demands $\langle 1, 4 \rangle$ and $\langle 9, 12 \rangle$: solution with cooperation

Figure 8: Evaluating cooperative networking in function of the traffic demands.

Table 1: Evaluation on random topologies with 13 nodes drawn on a square are of 1000 m^2, with parameters $P = 0.1$ mW, γ=10, $\eta = 2 * 10^{-10}$ mW. The time is in seconds.

	No-Cooperation					Cooperation					
Ist.	$\mathcal{L}$	Iter	Time	LB_1	UB_1	$\|A\|$	Iter	Time	LB_2	UB_2	Gap
p1	38	20	3.3	43.0	43	525	369	29.4	38.0	38	-13.2%
p2	38	27	6.3	324.3	325	526	609	309.5	303.8	304	-6.9%
p3	36	22	3.3	61.0	61	520	190	20.6	57.0	57	-7.0%
p4	42	26	5.7	115.7	116	593	329	151.0	112.0	112	-3.6%
p5	34	13	2.8	168.0	168	509	316	43.0	132.0	132	-27.3%
p6	34	21	3.5	125.0	125	510	127	42.9	122.5	123	-1.6%
p7	60	32	14.6	236.0	236	876	447	1093.4	216.5	217	-8.8%
p8	38	28	5.0	138.0	138	581	433	139.5	134.7	135	-2.2%
p9	50	8	3.3	95.0	95	741	43	43.9	93.0	93	-2.2%
p10	46	23	5.5	209.0	209	715	148	74.2	165.0	165	-26.7%
Av.	41.6		5.3			609.6		194.7			-9.93%

Table 2: Evaluation on random topologies with 13 nodes drawn on a square are of 1000 m^2, with parameters $P = 0.2$ mW, γ=10, $\eta = 2 * 10^{-10}$ mW. The time is in seconds.

	No-Cooperation					Cooperation					
Ist.	$\mathcal{L}$	Iter	Time	LB_1	UB_1	$\|A\|$	Iter	Time	LB_2	UB_2	Gap
p11	50	23	5.1	73.0	73	786	171	77	73.0	73	0.00%
p12	54	19	7.1	290.0	290	812	798	1288	236.3	237	-22.36%
p13	50	34	12.4	232.0	232	696	1189	1085	215.3	216	-7.41%
p14	60	23	11.3	148.0	148	1004	399	639	140.3	141	-4.96%
p15	54	25	9.8	257.0	257	845	546	834	230.3	231	-11.26%
p16	56	32	12.8	85.8	86	885	292	377	85.7	86	0.00%
p17	84	78	75.6	162.6	163	1396	736	7200	161.0	161	-1.24%
p18	64	26	14.1	203.0	203	1020	1092	3561	189.5	190	-6.84%
p19	74	19	11.8	225.0	225	1168	251	1304	207.0	207	-8.70%
p20	62	37	17.7	236.3	237	974	610	1562	205.0	205	-15.61%
Av.	60.8		17.8			958.6		1793			-7.84%

Table 3: Evaluation on random topologies with 13 nodes drawn on a square are of 1000 m^2, with parameters $P = 0.3$ mW, γ=10, $\eta = 2 * 10^{-10}$ mW. The time is in seconds.

	No-Cooperation					Cooperation					
Ist.	$\mathcal{L}$	Iter	Time	LB_1	UB_1	$\|A\|$	Iter	Time	LB_2	UB_2	Gap
p21	70	38	26.79	150.5	151	1057	1365	7200	149	149	-1.34%
p22	64	29	16.71	184	184	1014	1848	7061	166.4	167	-10.18%
p23	52	31	11.48	230	230	820	1489	2026	203.5	204	-12.75%
p24	80	40	41.37	186.22	187	1424	861	6479	183.2	184	-1.63%
p25	80	22	16.16	200.5	201	1309	778	7200	193.5	194	-3.61%
p26	68	36	29.21	181.75	182	1068	461	2169	177	177	-2.82%
p27	98	42	30.88	161.33	162	1850	421	7200	154	154	-5.19%
p28	82	28	20.41	162.68	163	1337	736	7200	162	162	-0.62%
p29	92	34	25.27	198	198	1630	273	4083	191	191	-3.66%
p30	78	18	12.54	209	209	1267	1176	7200	174.3	175	-19.43%
Av.	76.4		23.1			1277.6		5782			-6.12%

Table 4: Comparing the average computational results by changing the number of demands and the transmission power (in mW). Higher transmission powers give denser networks.

		No-Cooperation		Cooperation		
Power	$\|D\|$	$\|\mathcal{L}\|$	Time	$\|A\|$	Time	Gap
0.1	5	42.4	0.9	626.0	16.6	-11.9%
0.1	10	47.2	2.2	683.2	123.0	-8.2%
0.1	20	41.6	2.5	609.6	147.4	-7.8%
0.1	"all"	41.6	5.3	609.6	194.7	-9.9%
0.2	"all"	60.8	17.8	958.6	1792.7	-7.8%
0.3	"all"	76.4	23.1	1277.6	5781.8	-6.1%

Table 1, 2 and 3 report detailed results for random instances, with 13 nodes and a demand between each pair of connected nodes, obtained with both no-cooperative and 2-cooperative networking. The transmission power P is set to 0.1 mW, 0.2 mW, and 0.3 mW, respectively, yielding networks of increasing density. For each instance we report the number of links $\mathcal{L}$, the number of iterations of column generation, the computation time in seconds, and the number of superlinks $|A|$. We denote LB_1 and UB_1 the lower and upper bound obtained without cooperation, and LB_2 and UB_2 in case of cooperation. The last column gives the gap between the schedule length obtain with and without cooperation, i.e., the gap is equal to $\frac{UB_1 - UB_2}{UB_2} * 100$.

Our column generation approach produces the optimal solution whenever the ceiling of the lower bound is equal to the upper bound. In general, the optimal solutions with 2-cooperative networking are around 6–11% better than those without cooperation. Note (see Table 2 instances p11 and p12) that the gain of cooperation can be as high as 22%, but as low as 0%. In addition, we can remark that increasing the transmission power, and therefore increasing the connectivity of the network the gain of cooperation decrease.

Table 4 gives addition details on the evaluation of cooperative networking. Each row of table 4 reports the average over 10 random instances. We just compare no-cooperation and cooperation in terms of size of the network and computation time. We looked at the gain of cooperation as both a function of the number of demands, and as network density. Note that increasing the number of demands, the gain of cooperation decreases. Note also that the computation time (exponentially) increase with both the number of demands and the number of links (and super-links).

Finally, Table 5 gives better insight on the advantage of using cooperation. The table reports results for networks with 15 nodes and the same technical parameters than above. This time the traffic demands was generated in a specific way: for each number of demands, i.e., 5, 10, and 20, we have generated respectively 5, 10, and 20 pairs of demands with a fix hop distance h, where $h = 2, 3, 4, 5, 6$. In practice, for $h = 6$, we have considered all the demands with

Table 5: Gain of cooperation as a function of the hop distance between pairs of demands. For each combination of number of demands $|D|$ and hop distance we report the average over 10 random instances with 15 nodes, for a total of 150 different instances.

	hops=2		hops=3		hops=4		hops=5		hops> 6	
$\|D\|$	Avg.	(stdev)	Avg.	(stdev)	Avg.	(stdev)	Avg.	(stdev)	Avg.	(stdev)
5	0.00	(0.00)	-0.11	(0.06)	-0.12	(0.09)	-0.20	0.16	-0.28	(0.19)
10	-0.03	(0.04)	-0.06	(0.05)	-0.15	(0.07)	-0.20	0.16	-0.21	(0.12)
20	-0.03	(0.03)	-0.06	(0.06)	-0.13	(0.08)	n.a.		-0.37	(0.19)

hop distance equal to or greater than 6. The idea is to analyze the behaviour of cooperation not only as function of traffic load, but also as a function of distance between the demands. The table gives the percentage gain of using cooperation against as the ratio of the schedule length obtained with and without cooperation. For instance, for the instances with 20 demands and hop distance equal to or greater then 6, the schedule length is 37% shorter than without cooperation. The table shows clearly that the higher is the distance between the pairs of demands, the higher will be the gain of cooperation. This is due to the fact that in the cooperation graph the hop distances between pair of nodes are usually smaller, for the presence of super-links that gives additional shortcuts.

6. Conclusion

In this paper we have proposed an optimization approach for evaluating the relative gain in resource efficiency of cooperative relaying with respect to conventional multi-hop forwarding. The results presented show that the overall efficiency greatly depends on the network topology and the traffic pattern since the negative effect of cooperation on the reuse level of radio resources is particulary relevant when the network is dense of links and several traffic flows must be served in parallel.

Even if our work provides interesting insights on the advantages of cooperative relaying at network level, there are several open issues that deserve to be investigated. In our future research, we plan to extend the proposed model to account for the effect of power control and account modulation and coding that can be included quite easily in the current optimization framework. Moreover, we plan to consider also amplify-and-forward approaches even if the intrinsic non-linear nature of the SINR prevents the computation of bounds using linear programming.

Acknowledgment

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