The Joint Gateway Placement and Spatial Reuse Problem in Wireless Mesh Networks

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Abstract

A Wireless Mesh Network (WMN) is composed of multiple Access Points (APs) that are connected together using the radio channel, and by a limited number of gateway APs connected to the Internet. In this paper, we address the problem of gateways placement, that consists of minimizing the number of gateways while satisfying performance requirements of the APs. Along with the placement problem, the formulation includes the jointly routing and scheduling to account for the problem of interference and to enable spatial reuse. The problem, that we coined GPSRP, allows a much more efficient use of the available resources and reduces overall gateway costs. This article presents for the first time a mathematical formulation of the problem and discusses its advantages and limitations with respect to other approaches.

1 INTRODUCTION

Wireless Mesh Networks (WMNs) present interesting properties for an Internet Service Provider (ISP) since they will provide routing flexibility and robustness for the backhaul. Current backhaul solutions are either wireline, or point-to-multipoint wireless networks. In the WMN architecture, every Access Point (AP) is a router that can act as a relay toward the user or toward the Internet. The APs that are connected to the backbone network are called gateways. Gateways are expensive; furthermore, the connection presents typically a modular character. Having less connections to the core network, but with more individual capacity, permits lowering the total cost of the backhaul. Therefore the planner will try to minimize the number of gateways in the network. However, both the number of gateways and their location will influence network performance. Thus, an important planning problem for WMN is to optimize the number and the gateway location subject to architectural and performance constraints. We identify Quality of Service (QoS) constraints on bandwidth: an AP can route its packets through multiple wireless hops, provided that the bandwidth requirements of the aggregated flows are satisfied at each link stage and at the gateway interface. In addition, it is possible to set a limitation on the maximum number of hops
packets should traverse in the WMN, in order to meet QoS constraints on link-additive parameters, such as delay and jitter. This problem is usually referred to as Gateway Placement Problem (GPP) [1, 2, 3].

Another issue that affects the performance of the system are the routing and the scheduling of the connections. This is an operation problem that has been studied by numerous authors [4, 5] in the context of throughput optimization and that has been named the Routing and Scheduling Problem (RSP). Because of wireless interference, the capacity of one wireless link cannot be considered fully available when operating on the same or overlapping channels. The interference problem can be efficiently solved through the time scheduling of the transmissions in the mesh network so that collisions are avoided and resource reuse is maximized.

Even though the RSP is an operational issue whereas the GPP is a planning problem, they are closely related to each other. In fact, well designed routing and scheduling schemes can increase the overall network capacity and allow to reduce the number of gateways to be installed, and the number and location of the gateways affect the routing and scheduling efficiency. Up to now, the two issues have been considered separately in the literature. Very recently in [6], the relationship between the two problems was explored but they have been tackled separately.

In this paper, we propose to consider the problem as a whole so that the routing and scheduling is optimized along with the gateway placement. Generally speaking, mixing together an operation problem with a planning problem into a single optimization model may appear not convenient mainly because resource allocation mechanisms (such as scheduling and routing) could adapt to traffic conditions during network lifetime. However, contrary to ad hoc networks, wireless mesh networks create fixed backhaul networks where a pre-planned resource allocation can guarantee QoS and resource reuse efficiency. Moreover, the detailed modelling of interference, that is allowed by directly taking into account routing and scheduling, provides a powerful tool that can be used to carefully select gateways without using approximate interference models [3, 11].

The goal of this paper is to present a mathematical formulation of the joint resource allocation and gateway placement problem, that we call GPSRP (Gateway Placement and Spatial Reuse Problem), and to analyze how the different issues influence the characteristics of its optimal solution. The paper is structured as follows. In Section 2 we review previous work on related problems. The proposed model is presented in Section 3. Computational results are presented and analyzed in Section 4 whereas concluding remarks and a discussion of possible further extensions of this work are presented in Section 5.

2 RELATED WORK

A fundamental Wireless Mesh Network planning problem is the Gateway Placement Problem (GPP), which consists of finding the optimal number and location of gateways in a WMN. The GPP can be considered as an in-
stance of the more general Capacitated Facility Location Problem (CFLP) which has been studied in the fields of operations research and clustering algorithms [1, 7]. Routing is a component of the GPP that is not typically present in the CFLP, since in the classical CFLP customers demands are satisfied directly by the facilities. Multi-hop routing is however a central element of the GPP, permitting increasing coverage of the installed gateways. Traffic relaying can be modelled introducing additional constraints, an example of these being Quality of Services (QoS) constraints on bandwidth and on delay used by Prasad and Wu [2] to capture the flexibility of WMN.

Traditional facility location problems have not taken into account the case where link capacity has to be respected. To model such restrictions in WMN, a Network Flow Model based on the so-called connectivity graph \( G(N,E) \) was proposed by Qiu et al. [3]. Each node is identified by the traffic demand of the Access Point and each link is characterized by a wireless link capacity. Routing variables appear in capacity constraints on wireless links. Capacity constraints in the Network Flow Model suppose that transmissions are physically separated, a realistic assumption only in multi-point-to-point networks.

Another important problem that arises in WMN is Throughput Optimization, presented in the seminal work of Gupta and Kumar [8]. The problem is solved introducing routing and scheduling variables (the RSP) as well as interference constraints. More sophisticated interference models provide accurate description of the spatial reuse of the channels in WMN. Simultaneous transmissions on the same channel are allowed only if interference is avoided and correct decoding of signal is possible at the receivers. The use of different transmission links at the same time permits increasing the network capacity, i.e. the possibility to serve traffic demands.

Also relevant to our work is the concept of conflict graph, which was introduced by Jain et al. [9]. The conflict graph \( \mathcal{F} \) is a graph whose vertices correspond to the oriented links in the connectivity graph \( \mathcal{G} \) (then there is the need to rewrite the edge set \( E \) as a set of oriented arcs \( A \)). Thus, we say that there is an edge between the vertices \( l_{ij} \) and \( l_{mn} \) of \( \mathcal{F} \) if links \( l_{ij} \) and \( l_{mn} \) of \( \mathcal{G} \) interfere with each other, that is, the two links cannot be active at the same time. Starting from the conflict graph, a conflict matrix of size \( |A| \times |A| \) is easily computed and contains binary elements \( c_{e,f} \), where \( e \) and \( f \) are links of the network and

- \( c_{e,f} \in \{0,1\}, \forall e,f \in A(\mathcal{G}), c_{e,f} = 1 \) if transmission on link \( f \) interferes with the successful transmission on link \( e \) (\( c_{f,e} = c_{e,f}, c_{e,e} = 1 \))

To model the realistic case when the cumulative effect of several transmissions hinders successful reception, [9] introduces the Boolean model, inspired by the Physical model of Gupta and Kumar [8]. A conflict is no longer modeled to act as a simple binary operator. The weighted conflict graph extends the conflict graph by assigning a weight - bounded in \([0,1]\) - for each directed arc in it. Weights for arcs leaving \( l_{ij} \) express which fraction of the maximum permissible noise at node \( j \) (that permits the successful transmission on link \( l_{ij} \)) is contributed by other transmissions.

As previously mentioned in the introduction, there has not been, to
our knowledge, any works that deal jointly with routing and scheduling and gateway placement. Recently, however, Li et al. [6] have proposed a routing and scheduling method that optimizes throughput and that is used to assess the best gateway location. There are several differences of this work with ours. First, they do not incorporate the gateway placement problem together with the scheduling and routing into the same optimization procedure, as we do. Second, the objective of the scheduling optimization is network throughput, and it is not directly related with gateway costs, which is the objective in our model. Third, the gateway placement problem is presented in a heuristic fashion and the number of gateways is fixed beforehand, this is not the case in our problem in which the model will find the number of gateways and their location.

3 MODEL DESCRIPTION

3.1 Network Model

Let $G(V,E)$ be the connectivity graph representing a WMN. $G$ is an undirected graph where each node $v \in V$ represents an Access Point. Let $N(v)$ be the neighborhood of $v$ defined as the set of nodes residing in its transmission range and that are able to transmit back to $v$. Thus, there exists a bidirectional wireless link, represented by an edge $(u,v) \in E$, between $v$ and any neighbour $u \in N(v)$.

As previously mentioned in the literature review section, to complete the description of the radio system, we need to define the so called conflict graph, denoted by $F$. The relationship between the connectivity and the conflict graph is the following: the nodes in $F$ are the arcs in $G$. It is important to consider directional links because the interference set of one link depends also on its direction, as shown in Figure 1. For computational purposes, a conflict matrix will be used to represent the conflict graph.

Figure 1: a) Interference set of link (A,B) b) Interference set of link (B,A)
3.2 Problem Description

The aim of this paper is to formulate the GPP such that sophisticated interference models from the literature can be included to provide spatial reuse optimization. The adoption of a more sophisticated interference model improves the quality of the solution and the fact that scheduling and routing are taken into account at the same time as the gateway placement will lower the costs, when compared with solutions obtained solving disjoint optimization problems.

Clearly, the applicability of the model depends on having some demand stability between the gateways and the access points. Thus, while the model would not be appropriate for the design of Ad-hoc networks, it could very well be implemented in the WMN backhaul where traffic aggregation of several access users is performed and where a relative stable resource allocation schedule in the wireless links could be thought of.

We chose as the objective of the problem the minimization of the total gateway costs because they have been identified as the largest infrastructural cost for the wireless Internet Service Provider [10]. The cost minimization is subject to QoS constraints that apply both at the links and at the interface represented by gateways. To capture the link QoS, we propose to introduce routing and scheduling constraints.

We assume that traffic flows are indivisible. In other words, routing is supposed to be non-bifurcated. This is different than the Network Model of Qiu et al. [3], which allows for multipath routing. Our suggestion of non-bifurcated routing is due to the fact that even though bifurcation benefits load balancing, the additional protocol overhead of setting overlapped routing trees can be significant. Moreover, it has been shown in the literature that bifurcated routing has a negligible impact on throughput maximization.

Let $S(t)$ be a schedule that represents a 0-1 assignment of a variable $x_{e,t}, \forall e \in A, t \geq 0$ where $x_{e,t}$ is defined as follows:

- $x_{e,t} \in \{0, 1\}, x_{e,t} = 1$ if there is a successful transmission on link $e$ during time $t$.

The interference constraints force the schedule feasibility by insuring that at any given time only links that do not interfere with each other can be scheduled to transmit.

In order to satisfy quasi-permanent traffic demands, we consider periodic schedules. We also assume that the time period is divided into time-slots used to assign the transmission opportunities to individual links. Thus it is assumed that the link scheduling will not be changed during the length of a time-slot. Let $W$ be the number of time-slots. Then the condition for periodicity can be written as: $\forall t \in \{1, 2, \ldots, W\}, i \in \mathbb{Z}, x_{e,(i+1)W+t} = x_{e,iW+t}$. The number of $x_{e,t}$ variables that describe the scheduling is $|E| \times W$.

In our model, we assume that the demand from the AP to the gateways and vice-versa is known and so is the capacity of the wireless links. The variables are the gateway locations and the scheduling. The model also uses $W$ as an input parameter.
A wireless link must be inactive to schedule the links with which it interferes; spatial reuse is however possible in separated areas of the network. The maximum throughput of a link is then a fraction of its nominal capacity which contains $W$ at the denominator. Let $G \subset V$ indicate the set of gateways of minimum cost that will satisfy traffic demands with indivisible routing without violating interference constraints. It is always possible to define a time-slotted scheduling to provide connectivity from nodes in $G$ to all others APs that adopts $W = W^*$, chosen to route the basic traffic unit. Where:

$$W^* = \left\lfloor \frac{\text{m.c.m. of the wireless link capacities}}{\text{traffic granularity}} \right\rfloor$$

The m.c.m. above being the minimum common multiple. $W^*$ will be used in our assessment of the optimization model.

### 3.3 ILP Formulation

We formulate the placement problem as an integer linear program. Let $N = V$ be the set of APs that conform the WMN. In our model, we assume that any of the existing APs can become an Internet gateway, therefore $N$ also represents the set of gateway candidate locations. We assume that $K$ is the set of possible interfaces for a gateway to connect to the Internet and that the gateway cost depends on the type of interface is used.

There are three kinds of variables in our model: location and capacity assignment variables, routing variables and scheduling variables.

- **Location and capacity assignment variables:**
  - $g^k_i \in \{0, 1\}, \forall i \in N, \forall k \in K, g^k_i = 1$ if and only if AP $i$ is a gateway connected to the backbone through interface $k \in K$.

- **Routing variables:**
  - $y^i_e \in \{0, 1\}, \forall e \in A, \forall i \in N, y^i_e = 1$ if link $e$ is used to carry the traffic flow of AP $i \in N$.

- **Scheduling variables:**
  - $x^i_{e,t} \in \{0, 1\}, \forall e \in E, \forall t \in \{1, 2, \cdots, W\}, x^i_{e,t} = 1$ if link $e$ is active in whatever direction during time-slot $t$.

Let us introduce the following parameters: $M_k$: the capacity of gateway interface of type $k$; $C_k, \forall k \in K$: the cost of the interface; $T^i, \forall i \in N$: the traffic demand at AP $i$; $\Psi^e, \forall e \in E$: the wireless link capacity. The Gateway Placement and Spatial Reuse Problem (GPSRP) can be stated as the following:

$$\min \sum_{i \in N, k \in K} C_k g^k_i$$  \hspace{1cm} (1)

Subject To

$$\sum_{e = (h,v)} y^i_{e} + y^i_{(h,0)} = \sum_{e = (v,h)} y^i_{e}, \forall i, h \in N | h \neq i$$  \hspace{1cm} (2)
\begin{equation}
\sum_{e=(i,v)} y^i_e = 1 - y^i_{(i,0)}, \quad \forall i \in N \quad (3)
\end{equation}

\begin{equation}
\sum_{e=(v,i)} y^i_e = 0, \quad \forall i \in N \quad (4)
\end{equation}

\begin{equation}
\sum_{h \in N} y^h_{(i,0)} \cdot T^h \leq \sum_{k \in K} M^i_k g^i_k, \quad \forall i \in N \quad (5)
\end{equation}

\begin{equation}
\Psi^e_w \sum_{t \in \{1, \ldots, W\}} x_{(u,v),t} \geq \sum_{i \in N} (y^i_{(u,v)} + y^i_{(v,u)}) \cdot T^i, \quad \forall \{u,v\} \in E \quad (6)
\end{equation}

\begin{equation}
x^e_{f,t} + c_{ef} x^f_{f,t} \leq 1, \quad \forall e, f \in E, t \in \{1, 2, \ldots, W\} \quad (7)
\end{equation}

In the above formulation, the wired backbone that connects to the Internet is modeled by a super-node to which we assign node number 0. (1) is the objective function dealing with the minimization of the cost of transforming an AP into a gateway. Equations (2) to (4) represent the flow conservation constraints. Equation (2) considers one flow at a time and states that, for every node except those where the traffic originates, flow balance is null. Equations (3) and (4) consider flow balance for the traffic terminations. All the traffic demand is supposed to originate from the AP.

Inequality (5) is the capacity constraint on gateways. It states that the flows routed through a gateway cannot be larger than its capacity, i.e. the capacity of the interface connected to the Internet. Inequality (6) expresses the link capacity constraint: it states that, for each link \(e\), the available capacity of the link must be greater or equal than the sum of all flows routed through that link in both directions.

Inequality (7) is the interference constraint that makes use of the coefficients of the conflict matrix \(c\). In a formulation which employs scheduling for edges rather than directional links, the coefficients would be symmetric and take the greatest value among the coefficients that distinguish direction of transmission in the asymmetric formulation. The constraint finds application also in the Boolean model for weighted coefficients, as a restrictive constraint. It is sufficient that the coefficient is weakly positive to consider that the two links interfere with each other. In the case of a binary conflict model, we look for the union of the two interference sets of the same edge in the conflict graph, as shown in Figure 2 which should be compared with Figure 1.

In terms of complexity, the GSPRP, as many design problems, is clearly NP-hard since it is reduced to an instance of the GPP. The above formulation presents \(|E| \times W\) binary scheduling variables, \(2|E| \times 2|N|\) binary routing variables and \(|K| \times |N|\) gateway placement variables. Assuming all variables as binary has proved to be the best choice in term of computational requirements when the interference constraint is expressed for couple of links.

We observe that there are two particular ways to increase spatial reuse in our model:

1. A WMN performs the backhaul of both incoming and outgoing traffic and it is easy to distinguish - for each AP \(i\) - an uplink traffic flow \(T^i_{up}\)
Figure 2: Interference set of the edge \{A,B\} in the binary interference model

directed to one gateway and a downlink traffic flow $T_{down}$ originated from one gateway and terminated at the AP.

We can write different flow conservation constraints for both uplink and downlink traffic flows. We then express scheduling variables for both directions of transmission in an edge. This is possible because we are now routing directed traffic and will introduce a major benefit: the interference sets of the two transmissions in opposite directions can be considered separately. Tighter interference sets enable more transmissions to be scheduled simultaneously, increasing the capacity of the WMN, as will be shown in the next section.

2. The second way to increase spatial reuse is through the understanding of the Boolean interference model. The Boolean model extends the binary conflict model by assessing when a set of links is able to cause cumulative interference to a third communication. One needs to check if the sum of the fractional coefficient $c$ of the set exceeds the value of 1. Then, it is easy to rewrite the interference constraints, that also assure schedule feasibility (7) as:

$$\sum_{f \in A, f \neq e} x_{f,t} \cdot c_{e,f} < 1 + R(1 - x_{e,t}), \quad \forall e \in A, \forall t \in \{1, 2, \cdots, W\}, \quad (8)$$

in which $R$ is a very large number, greater than the number of nodes in the network. In the case that $x_{e,t} = 0$, the link is inactive at the time slot indicated and so is its interference constraint. Otherwise, the constraint assures that the transmission over $e$ will not be disrupted by the interference.

A third way - rate adaptation - is already covered by the original model, at the condition that a way to distinguish different edges between the same endpoints but with different wireless link capacities is implemented. Rate adaptation is a physical layer property: several transmission rates can
be realized by the technology using different modulation rate. The two radio equipments involved in a transmission evaluate the link quality and select the best transmission rate, which is the greatest transmission rate assuring sufficient signal to noise ratio, eventually scaling down to a lower transmission rate when links that interfere are active.

4 COMPUTATIONAL RESULTS

Both the GPP and the GPSRP aim at minimum-cost integration of the WMN with the wired Internet. However, as previously stated, only the solution of the GPSRP permits to include the interference constraints during the network design phase, since it takes in consideration routing and scheduling of the WMN.

We are giving an example to illustrate the advantages of tackling the GPRSP in Figure 3 and Figure 4. The first figure shows a solution of the GPP where there is a gateway that will be located at node 5. The only constraints are that a node cannot receive and transmit at the same time and that it receives from one node at a time. In terms of operation, it is not possible to achieve the target link throughputs, because of the interference between the nodes. As a consequence, the bandwidth requirement of 1 Mbps for each AP cannot be satisfied. On the other hand, in Figure 4, we can see the solution obtained when the GPSRP is applied. It shows that the GPSRP places the gateway in a different position and makes sure that the interference constraints are respected. This degree of spatial reuse is only possible because of the time-slot allocation, as we observe that the sum of all transmissions in the interference set of the central node exceeds 9 Mbps. Without spatial reuse, the number of gateways required by the solution will double to 2.

In order to evaluate the behavior of the model, we generate random topologies of installed APs and make two simplifying assumptions. First, there is only one type of backbone connectivity, with capacity $M = 45$Mbps. Secondly, all the wireless links have the same capacity $\Psi$ and the transmission range is fixed, $R_{tr} = 250$m, with no obstacles interposed. We are looking for random deployments in a bidimensional topology of $|N|$ nodes. In particular, we look for connected topologies placing nodes randomly in a square of radius $R_{cs} \cong \sqrt{|N| \cdot \frac{R_{tr}^2}{2}}$. We then compute the symmetric conflict matrix, considering only the interference range $R_{lf} = 375$m.

In Table 1, let $Cfg_{15,25}$ be a random topology comprising 15 nodes and 25 links and let us name all other scenarios following the same naming scheme. We consider separately downlink and uplink flows: $T_{down} = 2$Mbps and $T_{up} = 1$Mbps for each AP. We define two sets of experiments, employing different wireless capacity values: $\Psi = 20$Mbps and $\Psi = 40$Mbps. If traffic granularity is 1 Mbps, $W^*$ is fixed respectively to 20 and 40. We ran the optimization (on a commercial desktop) with CPLEX. The first column of the table indicates the scenario $I$ of the GPRSP that is defined by the location and number of the $|N|$ APs. The second and fifth column indicates the optimal number of gateways obtained for scenario $I$, this is denoted by $\alpha(I)$. $\beta(I)$, in the third and
Figure 3: Solution to the GPP which considers routing. It admits no feasible schedule.

Figure 4: Solution to the GPSRP. Schedules are listed in the table.

The sixth column is the average number of hops per flow that is computed as follows.

$$\beta(I) = \frac{H(I)}{2 \times (|N| - \alpha(I))}$$  \hspace{1cm} (9)

where $H(I)$ is the total number of hops obtained in the solution of scenario $I$.

The fourth and last columns provide the execution time, in seconds.

In order to improve the quality of the solution, we ran a post-processing optimization on the results of the GPSRP. We fixed the value of the gateway placement variables and imposed the minimization of the number of hops. This post-processing optimization takes a bounded time even for large instances.

As expected, when intra-mesh capacity increases because more capacity is available on individual links, longer paths that cover more hops are possible, and fewer gateways are needed. On the contrary, in a heavily loaded network, the average number of hops can even be equal to 1, suggesting that there are no multi-hop flows in the network.

For some instances of the problem, we were not able to obtain an optimal integer solution of the GPSRP formulation respecting the time...
Table 1: Computational results for the GPSRP model, with different wireless link capacities

<table>
<thead>
<tr>
<th>Scenario (I)</th>
<th>$\Psi = 20$</th>
<th>$\Psi = 40$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha(I)$</td>
<td>$\beta(I)$</td>
</tr>
<tr>
<td>Cfg15,25</td>
<td>3</td>
<td>1.083</td>
</tr>
<tr>
<td>Cfg15,29</td>
<td>4</td>
<td>1.091</td>
</tr>
<tr>
<td>Cfg15,31</td>
<td>3</td>
<td>1.083</td>
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<tr>
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</tr>
<tr>
<td>Cfg25,62</td>
<td>7</td>
<td>1.166</td>
</tr>
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</table>

We think that the proposed time limit is reasonable for a network planning problem that has to be run only once. The best solution found is portrayed in brackets in Table 1. To reduce the computational requirements of the GPSRP we decided to treat uplink and downlink traffic as an aggregate. This decision translates into two modifications to the formulation. First, the uplink and downlink traffic aggregation produces $|N|$ flows of 3 Mbps. Secondly, the conflict matrix must be rewritten aggregating the interference sets of the directional arcs corresponding to the same edge. The meaning of this modifications is, respectively, that we have bulkier flows, and that we rule out some possible configurations of concurrent transmissions.

The value of $W^*$ was computed anew as $\left\lfloor \frac{20\text{Mbps}}{3\text{Mbps}} \right\rfloor = 6$ and as $\left\lfloor \frac{40\text{Mbps}}{3\text{Mbps}} \right\rfloor = 13$. Table 2 illustrates the decrease in computational time, but also the increase in the objective function value due to the aggregation of uplink and downlink flows. We added two columns to the table, that portray the value $\Delta$ that provides the percentage increase on the number of gateways that are obtained in this case. We can see that the advantage in spatial reuse, intra-mesh capacity and finally cost of the backhaul of maintaining separate entities for the uplink and the downlink is lost. Comparison between Tables 1 and 2, show that an increase of up to 75% in the number of gateways can be seen for some scenarios.

This shows the importance of the detailed modeling approach that we are proposing.

To demonstrate that we are able to solve the optimization problem for large instances, when downlink and uplink are aggregated Table 3 presents...
Ψ = 20

| Scenario (I) | α(I) | ∆(%) | β(I) | exec. time[s] | | Scenario (I) | α(I) | ∆(%) | β(I) | exec. time[s] |
|--------------|------|------|------|---------------| | Cfg20_39     | 4    | 0    | 1.063 | 4.07           | | Cfg20_43     | 6    | 50   | 3.59  | 5.96           |
|              |      |      |      |               | | Cfg20_47     | 6    | 20   | 3.55  | 10.11          | | Cfg20_48     | 5    | 25   | 6.56  | 13.17          |
|              |      |      |      |               | | Cfg20_60     | 8    | 60   | 1.081 | 17.60          | | Cfg25_53     | 7    | 40   | 1.74  | 26.59          |
|              |      |      |      |               | | Cfg25_54     | 7    | 40   | 1.74  | 24.48          | | Cfg25_57     | 6    | 50   | 1.053 | 274            |
|              |      |      |      |               | | Cfg25_57b    | 7    | 75   | 1.055 | 58.18          | | Cfg25_62     | 8    | 14   | 1.116 | 22.36          |

Table 2: Performance of the GPSRP model, under the assumption of aggregated downlink and uplink traffic.

some additional scenarios. Note that only wireless capacity Ψ = 40 Mbps was considered, because a lower capacity would not lead to a multi-hop network due to poor spatial reuse. Not shown in the table, we have found optimization times superior to the hour only for instances with 50 nodes.

Ψ = 40

<table>
<thead>
<tr>
<th>Scenario (I)</th>
<th>α(I)</th>
<th>β(I)</th>
<th>exec. time[s]</th>
</tr>
</thead>
<tbody>
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<td>Cfg30_55</td>
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<td>76.24</td>
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<tr>
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<td>131</td>
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<td>Cfg30_74</td>
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<td>1.257</td>
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<tr>
<td>Cfg40_109</td>
<td>5</td>
<td>1.200</td>
<td>208</td>
</tr>
<tr>
<td>Cfg40_124</td>
<td>4</td>
<td>1.305</td>
<td>393</td>
</tr>
</tbody>
</table>

Table 3: Results for larger instances of the GPSRP model, under the assumption of aggregated downlink and uplink traffic

**Link capacity**  We want to study the relationship between the wireless capacity of the links and the cost of the backhaul. Figure 5 represents the case of |N| = 25 nodes. **Please remember that we set capacity of the backbone connectivity to a fixed value M=45Mbps. In this case, at least two gateways are required to provide backhaul for all the node traffic demands.** As the wireless capacity Ψ increases, we require less gateways. In fact, the average on-air traffic increases as longer hops do not violate the link capacity constraint and you can aggregate more traffic. Let Υ(I) represent the average on-air traffic of scenario I.
Figure 5: Average on-air traffic varying the capacity of the wireless link $\Psi$ computed as follows:

$$\Upsilon(I) = \beta(I) \times T \times (|N| - \alpha(I))$$  \hspace{1cm} (10)

where $T$ is the traffic demand of the node (3 Mbps in the example, with no separation between downlink and uplink).

The wireless link capacity influences the computational cost of the model, too. Optimization times decrease when wireless capacity of links is set to a value greater than 15 times the traffic demand, even if the number of scheduling variables increases through our definition of $W^*$. To explain this, let us define the spatial reuse factor $\Gamma$ as the rate of the average on-air traffic over the link capacity:

$$\Gamma = \frac{\Upsilon}{\Psi}$$  \hspace{1cm} (11)

We can see that spatial reuse decreases as the wireless capacity increases, because longer hops determine an unbalanced distribution of the requested throughput on links. Links outside the interference set of the most loaded link typically have much less throughput to carry and remain inactive. Bottlenecks in the transmission trees become then more evident. The longest optimization times are to be found, according to graphic in Figure 6, when the number of scheduling variables is high and simultaneously the configuration poses much stress on spatial reuse, which does not coincide with the maximum of scheduling variables. Coherently, we tested performance of our model in this worst case.

**Comparison with continuous scheduling** Other models [11] avoid binary variables to represent time-slot activation by introducing the new variable:

- $x_e \in [0, 1]$, fraction of the time period during which directional link $e$ is active.
It is however difficult to write the interference constraints using $x_e$. Jun and Sichitiu [12] propose to calculate the collision domain for each link, in a way similar to the collision domain for an Ethernet network. Only one link in the collision domain is active at each time, for collision domains of activated links. The formulation of [12] assumes that each collision domain needs to forward the sum of all the traffic of its links sequentially, which is a pessimistic hypothesis in face of spatial reuse.

This restrictive interference model provides always a poor upper bound for the number of gateways: the returned solutions indicate a number of gateways that is from 25% to 100% greater than in the time-slotted GPSRP. The restrictive interference model is not effective in optimizing spatial reuse. It also takes longer computational time to solve than the GPSRP model when the ratio between the wireless link capacity and the traffic demand is in the practical range that we have tested in the previous paragraph. In fact, it remains a complex MIP (Mixed Integer Programming) problem and cannot benefit of optimal choice for time-slotting $W^*$.  

5 CONCLUSION AND FUTURE WORK

In this paper, we presented the Gateway Placement and Spatial Reuse Problem for Wireless Mesh Networks. Unique properties of WMNs have been underlined by our model. WMNs differ from ad hoc networks, characterized by variability in their topology. WMNs behave almost like wired networks since they present infrequent topology changes and node failures, with the exception that the communication channel is shared by the wireless terminals. We used the static properties of WMNs to justify the inclusion of routing and scheduling at the network planning phase. We have considered the issue of interference and spatial reuse by including
interference models in the cost minimization problem.

The gateways exercise great influence on the topology of WMNs. Traffic flows are either directed to or originated by gateways, so gateways placement and number affect the network capacity. They also determine the amount of on-air traffic, since the number of gateways in the network has an impact on the length of paths. The amount of spatial reuse in the network cannot be set automatically, but is expected to decline, when the wireless capacity of the links is high with respect to the traffic demand.

Testing has permitted to evaluate the effectiveness of our model of the Gateway Placement and Spatial Reuse Problem and, in particular:

- the respect of the interference constraints at the network planning time;
- the reduction in computational time with respect to models with continuous scheduling;
- the decrease in the cost for gateways, taking the computational burden of increased spatial reuse.

As directions for further work, we want to provide heuristics to be able to solve the GPSRP for very large instances and to explore the influence of different cost functions on spatial reuse. We also want to tackle the multi-radio planning problem.

References


