

in c , we show that the game is exactly potential, the potential function being the total cost paid by all householders, P . Such feature guarantees the existence of at least one pure Nash equilibrium (where no customer has an incentive to deviate unilaterally from the scheduling pattern he decided upon), namely the strategy that minimizes P . Furthermore, in such games, best response dynamics always converges to a Nash equilibrium.

In summary, our paper makes the following contributions:

- The proposition of a novel, fully distributed Demand Side Management (DSM) method able to reduce the peak demand of a group of residential users, which we model and study using a game theoretical framework. In our vision, the energy retailer fixes the energy price dynamically, based on the total power demand of customers; then, end customers automatically schedule their electrical appliances, reaching an efficient Nash equilibrium point.
- Detailed mathematical proofs that our proposed game is potential, and in particular *exactly potential* when the pricing scheme imposed by the energy retailer is linear in the total demand of customers.
- The demonstration of the Finite Improvement Property, according to which any sequence of asynchronous improvement steps (and, in particular, *best response dynamics*) converges to a pure Nash equilibrium.
- The numerical evaluation to show the effectiveness of the proposed game and best response dynamics in several scenarios, with real electric appliances scheduled by householders.

The paper is organized as follows. In Section II we review the most relevant works on demand management mechanisms. Section III, describes the main characteristics of the distributed system we propose to manage the energy consumption of residential users. Section IV presents our proposed game model. The distributed optimization model that we propose to reach stable (equilibrium) solutions, based on a best-response mechanism, is presented in Section V. Numerical results are presented and analyzed in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK

Demand side management methods have been deeply studied by the scientific community due to the numerous advantages achievable through this kind of mechanism [6]. In [7], a dynamic programming framework is proposed to balance user's comfort level and preferred lifestyle with the goal of minimizing the electricity bill. A similar solution is presented in [8], where an optimization model is proposed which schedules the activities with the aim of minimizing energy costs and maximizing users comfort, taking into account users requirements. Energy costs and users' comfort are also modeled in [9], which proposes a method to manage houses energy consumption with the aim of optimizing a three-term objective function, taking into account the overall energy cost, the schedule deviation with reference to the user preferences and the climate comfort. In this work, houses are assumed

to be equipped with photovoltaic (PV) generation and storage resources.

Solutions [7]-[9] are based on a single-user approach in which the energy plans of residential customers are individually and locally optimized. However, in order to obtain relevant results, the energy management problem must not be applied to single users but to *groups* of users (e.g., neighborhood or micro-grids). For this reason, some solutions are proposed in the literature to manage energy resources of groups of customers. In [10], for example, the energy bill minimization problem is applied to a group of cooperative residential users equipped with PV panels and storage devices (i.e., electric vehicle batteries). A global scale optimization method is also proposed in [11], in which an algorithm is defined to control domestic electricity and heat demand, as well as the generation and storage of heat and electricity of a group of houses. These multi-user solutions require some sort of centralized coordination system run by the operator in order to collect all energy requests and find the optimal solution. To this purpose, a large flow of data must be transmitted through the Smart Grid network, hence introducing scalability constraints and requiring the definition of high-performance communication protocols. Furthermore, the collection and transmission of users' metering data can introduce novel threats to customers' security and privacy. Finally, the coordination system should also verify that all customers comply with the optimal task schedule, since the operator has no guarantee that any user can gain, deviating unilaterally from the optimal solution. For these reasons, some *distributed* DSM methods have been proposed in which decisions are taken locally, directly by the end consumer. In this case, Game Theory represents an ideal framework to design DSM solutions. Specifically, in [12] a distributed demand-side energy management system among users is proposed. In this system, the users' energy consumption scheduling problem is formulated as a game, where the players are the users and their strategies are the daily schedules of their household appliances and loads. The goal of the game is to either reduce the peak demand or the energy bill of users. Game theory is also used in [13], in which a distributed load management is defined to control the power demand of users through dynamic pricing strategies. However, in these works, a very simplified mathematical description is used to model houses, which does not correspond to real use cases.

In this paper we propose a DSM method, based on a game theoretical approach, which overcomes the most important limitations of the works proposed in the literature and described above. Our DSM is a fully distributed system, in which no centralized coordination is required, and only a limited amount of data needs to be transmitted through the Smart Grid. For these reasons, scalability, communication, privacy and security issues are greatly mitigated. Moreover, a realistic model of household contexts is provided. Specifically, a mathematical description of home devices is provided. Devices are defined as non-preemptable activities characterized by specific load consumption profiles, determined based on real data, and

are scheduled according to users' preferences defined based on real use-case scenarios. Finally, a parallel analysis between users' and overall electric system performance achieved through the proposed demand management mechanisms is provided.

III. SYSTEM MODEL

The power scheduling system here proposed is designed to manage the electric appliances of a group of residential users consisting of \mathcal{H} houses (e.g., a smart city neighborhood). This system is used to schedule the energy plan of the whole group of users over a 24-hour time horizon based on a *fully distributed* approach, with the final goal of improving the efficiency of the whole power grid by reducing the peak demand of electricity, while still complying with users' needs and preferences. More specifically, in our model we represent the daily time as a set \mathcal{T} of time slots. Each householder ¹ $h \in \mathcal{H}$ has a set of non-preemptive electric appliances, \mathcal{A}_h , that must be executed only once during the day. Specifically, each appliance load profile is modeled as an ordered sequence of phases, \mathcal{F} , in which a certain amount of energy is consumed. We assume that the consumption l_{ahf} of a device $a \in \mathcal{A}_h$ belonging to user $h \in \mathcal{H}$ in each phase $f \in \mathcal{F}$ is an average of the real consumption of the device within the time slot's duration (see Figure 1, where 15-minute phases are used).

Each device $a \in \mathcal{A}_h$ of user $h \in \mathcal{H}$ needs to run for d_{ah} consecutive slots within a total of \mathcal{R}_{ah} slots delimited by a minimum starting time slot, ST_{ah} , and a maximum ending time slot, ET_{ah} verifying the constrain $ST_{ah} \leq ET_{ah} - d_{ah}$. These two parameters, ST_{ah} and ET_{ah} , represent the users preferences in starting each home device; they can be directly provided by users or automatically obtained through learning algorithms such as the one presented in [14]. In our model, we consider two different kinds of devices:

- *Shiftable* appliances (e.g., washing machine, dishwasher): they are manageable devices that must be scheduled and executed only once during the day. In particular, for each shiftable device $a \in \mathcal{A}_h$ of the householder $h \in \mathcal{H}$, the minimum starting time and the maximum ending time verify the constraint $ST_{ah} < ET_{ah} - d_{ah}$, hence their scheduling is a variable of the model.
- *Fixed* appliances (e.g., light, TV): they are not manageable devices that must be executed only once during the day. For each fixed device $a \in \mathcal{A}_h$ of the householder $h \in \mathcal{H}$, the minimum starting time and the maximum ending time verify the constraint $ST_{ah} = ET_{ah} - d_{ah}$, hence their scheduling is not a variable of the model and cannot be optimized.

Devices scheduling is represented by $x_{ah,t}$, which are defined for each activity $a \in \mathcal{A}_h$ of each user $h \in \mathcal{H}$, and for each time slot $t \in \mathcal{T}$; they are equal to 1 if the activity a starts in time slot t , 0 otherwise. $x_{ah,t}$ associated with shiftable appliances are variables of the problem, while

¹In this paper, we use the terms *householder* and *user* interchangeably.

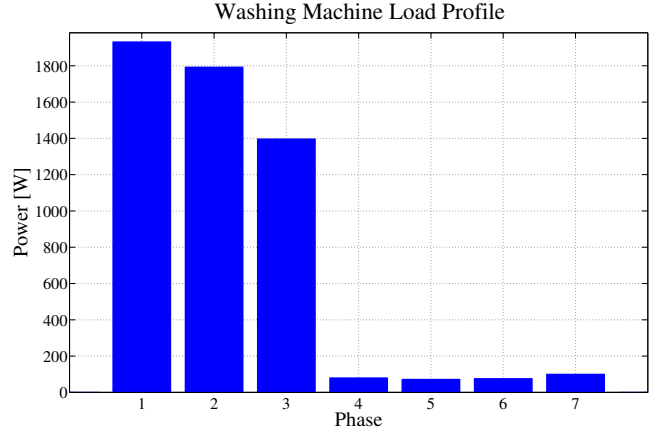


Figure 1: Example of a load profile l_{ahf} of a washing machine.

those associated with fixed appliances are only parameters and cannot be modified. In order to use home appliances, householders can buy energy from the electricity retailer. In particular, the amount of electricity bought by user $h \in \mathcal{H}$ at time $t \in \mathcal{T}$ is denoted by y_{ht} . Let $c_t(\cdot)$ denote the price of electricity at time $t \in \mathcal{T}$. In our model, we suppose that $c_t(\cdot)$ is an increasing function of the total power demand, y_t , of the group of users \mathcal{H} at time t . The objective of the power scheduling system is to minimize the daily bill of each user, by optimally scheduling house appliance activities and managing the power exchange with the network. Intuitively, based on the definition of the electricity price, by reducing the users' bills, the model is able to decrease the corresponding peak demand.

IV. POWER SCHEDULING GAME

We now describe the *power scheduling game* used to model the fully distributed DSM system we propose in our paper. In such scenario, each user $h \in \mathcal{H}$ must decide when to use his home appliances (i.e., the $x_{ah,t}$ values) and, consequently, when to buy energy from the grid (i.e. y_{ht}) in order to minimize his bill. Since the energy price is defined as a function of the total energy demand of the *whole group* of users, the power scheduling problem cannot be solved with a centralized model because of the conflict between users' goals. For this reason, a distributed approach naturally arises, where all decisions are made locally and directly by end consumers. Specifically, in this paper we model the power scheduling problem as a game, since game theory naturally models interactions in distributed decision making processes.

In this regard, let us define the total price, P , paid by all customers to the electricity retailer, as a function of $\mathbf{I} \triangleq \{\mathbf{I}_h\}_{h \in \mathcal{H}}$, where the strategy of user h is $\mathbf{I}_h \triangleq \{x_{ah,t}\}_{a \in \mathcal{A}_h}$:

$$P(\mathbf{I}) = \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} y_{ht} \cdot c_t(y_t) \quad (1)$$

where y_{ht} is a function of $x_{ah,t}$ and c_t is a function of y_t that represents the total power demand of users at time t .

The following Theorem holds:

Theorem 1. *The considered power scheduling game is a potential game if $c_t(y_t)$ is convex with respect to y_t , with $P(\mathbf{I})$ being the potential function. Specifically, if $c_t(y_t)$ is linear w.r.t. y_t , $P(\mathbf{I})$ is the exact potential function, i.e.,*

$$P(\mathbf{I}'_h, \mathbf{I}_{-h}) - P(\mathbf{I}_h, \mathbf{I}_{-h}) = P_h(\mathbf{I}'_h, \mathbf{I}_{-h}) - P_h(\mathbf{I}_h, \mathbf{I}_{-h}) \quad \forall h \in \mathcal{H}, \mathbf{I}_h' \quad (2)$$

where $P_h(\mathbf{I})$ is the price of customer h paid to the operator under the strategy profile \mathbf{I}_h . If $c_t(y_t)$ is strictly convex w.r.t. y_t , it holds that

$$P(\mathbf{I}'_h, \mathbf{I}_{-h}) \geq P(\mathbf{I}_h, \mathbf{I}_{-h}) \iff P_h(\mathbf{I}'_h, \mathbf{I}_{-h}) \geq P_h(\mathbf{I}_h, \mathbf{I}_{-h}) \quad \forall h \in \mathcal{H}, \mathbf{I}_h' \quad (3)$$

Due to the page limit, we do not detail the proof. The above theorem readily implies the following corollaries.

Corollary 1 (Efficiency of Equilibrium). *If $c_t(y_t)$ is convex w.r.t. y_t , then the equilibrium also minimizes the total price paid to the operator.*

Corollary 2 (Convergence to Equilibrium). *If $c_t(y_t)$ is convex w.r.t. y_t , then the game has the Finite Improvement Property (FIP). Any sequence of asynchronous improvement steps is finite and converges to a pure equilibrium. Particularly, the sequence of best response update converges to a pure equilibrium.*

Potential games have nice properties, such as existence of at least one pure Nash equilibrium, namely the strategy that minimizes $P(\mathbf{I})$. Furthermore, in such games, best response dynamics always converges to a Nash equilibrium.

Hereafter we describe a simple implementation of best response dynamics, which allows each householder to improve his cost function in the proposed power scheduling game. Such algorithm, detailed in the following, is the best response strategy for a user u minimizing objective function (4), $\sum_{t \in \mathcal{T}} (c_t \cdot y_{ht})$, assuming other users are not changing their strategies.

Specifically, each customer, in an iterative fashion, defines his optimal power scheduling strategy based on electricity tariffs (calculated according to other players' strategies) and broadcasts his energy plan (i.e., his daily power demand profile) to the group \mathcal{H} . At every iteration, energy prices are updated according to the last strategy profile and, as a consequence, other users can decide to modify their consumption scheduling by changing their strategy according to the new tariffs. The iterative process is repeated until convergence is reached. Once convergence is reached, householders' power scheduling and energy prices are fixed.

The best response mechanism is executed by solving, in an iterative way, an optimization model. Specifically, at every iteration and based on the energy demands of other users, this model is used to optimally decide the power plan of the user in charge of defining his energy demand at this step of the iterative process, with the goal of minimizing his bill. The optimization model that we have defined for the best response mechanism is described in the following section.

We will show in the Numerical Results section that our proposed algorithm converges, in few iterations, to a Nash equilibrium.

V. SINGLE USER OPTIMIZATION MODEL

The single-house problem is modeled as a Mixed Integer Nonlinear Programming (MINLP) model which determines, for each residential user u (here called *active user*), his energy plan, i.e., when to buy energy from the grid and when to start the shiftable appliances based on the energy plan of the other householders, with the final aim of minimizing his daily energy bill.

OBJECTIVE FUNCTION

The goal of the problem is to minimize the daily electricity bill of the *active householder* over a 24 hour time horizon divided into $|\mathcal{T}|$ time slots, \mathcal{T} being the set of such time slots. To this end, we define the continuous non-negative variables y_{ut} and c_t representing, respectively, the amount of electricity bought by *active user* u and the price of electricity in each time slot $t \in \mathcal{T}$. The objective function can be modeled as:

$$\min \sum_{t \in \mathcal{T}} (c_t \cdot y_{ut}) \quad (4)$$

CONSTRAINTS

Appliances scheduling: The house appliance activities to be executed by the user u are represented by the set \mathcal{A}_u . For each activity $a \in \mathcal{A}_u$, binary variables x_{aut} have to be defined, which are equal to 1 if the activity a starts in time slot t , and 0 otherwise. Such variables must satisfy two sets of constraints:

$$\sum_{t=ST_a}^{ET_a-d_a+1} x_{aut} = 1 \quad \forall a \in \mathcal{A}_u \quad (5)$$

$$p_{atf} = l_{auf} x_{au(t-f+1)} \quad \forall a \in \mathcal{A}_u, t \in \mathcal{T}, f \in \mathcal{F} : f \leq t \quad (6)$$

The first set of constraints guarantees that the activity a starts in exactly one time slot and it is carried out in the required interval (ST_{au}, ET_{au}) . The second set of constraints forces the power required by each appliance in each time slot, p_{atf} , to be equal to the load profile l_{auf} of the phase carried out in the considered time slot.

Power demand constraints: In every time slot $t \in \mathcal{T}$, the power demand y_{ut} of the *active householder* must meet the total power consumption of home appliances. For this reason we define the following constraints:

$$y_{ut} = \sum_{a \in \mathcal{A}_u} \sum_{f \in \mathcal{F}} p_{atf} \quad \forall t \in \mathcal{T} \quad (7)$$

Moreover, in every time slot $t \in \mathcal{T}$, the electricity bought from the grid cannot exceed the Peak Power Limit (PPL) defined by the retailer and denoted by π^{PPL} :

$$y_{ut} \leq \pi^{PPL} \quad \forall t \in \mathcal{T} \quad (8)$$

Electricity price constraints: The price of electricity c_t , in each time slot $t \in \mathcal{T}$, is a linear increasing function of the total power demand of the group of users \mathcal{H} . In particular, it is computed according to the following constraints:

$$c_t = c^{MIN} + s(y_{ut} + p_t) \quad \forall t \in \mathcal{T} \quad (9)$$

where p_t is the total power demand of other householders of the group \mathcal{H} , c^{MIN} is the minimum electricity price and s is the slope of the cost function [15].

VI. NUMERICAL RESULTS

This section presents the numerical results that illustrate the validity of our proposed distributed DSM system to reduce the peak demand of a group of residential neighbors.

We first describe the experimental methodology of our simulations, then we analyze and discuss the performance achieved by the proposed mechanism.

A. Experimental Methodology

In our simulations, we consider two typical residential scenarios composed of identical houses with four and five shiftable devices out of eleven realistically-modeled appliances², respectively. The basic domestic configuration and the load profile consumption of each appliance have been defined based on literature data relevant to the Italian standard user obtained in the project MICENE carried on at Politecnico di Milano [15]. We vary the number of houses in the range [1, 5] to assess the performance of the proposed DSM system when the competition increases.

In order to evaluate the effect of the scheduling flexibility on users bills and the efficiency of the electrical grid, we consider three different scenarios, where residential users have (1) no flexibility, (2) tight and (3) loose temporal constraints on the execution of shiftable devices. Specifically, for each appliance we fix different bounds both on the starting and ending time (i.e., ST_a and ET_a), thus modeling the interval during which the user is willing to use its shiftable devices. We consider a set \mathcal{T} of 24 time slots of 1 hour each.

Concerning the electricity prices, we fix the minimum electricity price $c^{MIN} = 50 \times 10^{-6}$ € and the slope of the cost function $s = 4 \times 10^{-8}$ €/kWh, according to the data gathered in the project [15].

In order to gauge the performance of the proposed DSM system, we measured the following performance metrics:

- *Social Welfare:* defined as the sum of users utility, $\sum_{h \in \mathcal{H}} u_h$. Note that this value represents the electricity bill of the group of houses.
- *Fairness:* we consider the *Jain's Fairness Index (JFI)* [16], defined according to Equation (10):

$$\text{Jain's Fairness Index} = \frac{(\sum_{h \in \mathcal{H}} u_h)^2}{|\mathcal{H}| \cdot \sum_{h \in \mathcal{H}} u_h^2} \quad (10)$$

The *Jain's Fairness Index* measures the spread of the price paid by users, and it varies from $1/|\mathcal{H}|$ (no fairness) to 1 (perfect fairness).

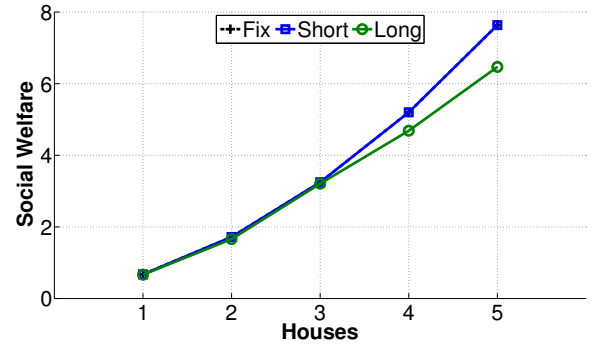
²Namely, Refrigerator, Purifier, Lights, Microwave Oven, Oven, TV, Iron, Washing machine, Dishwasher, Boiler, Vacuum Cleaner

B. Performance Evaluation

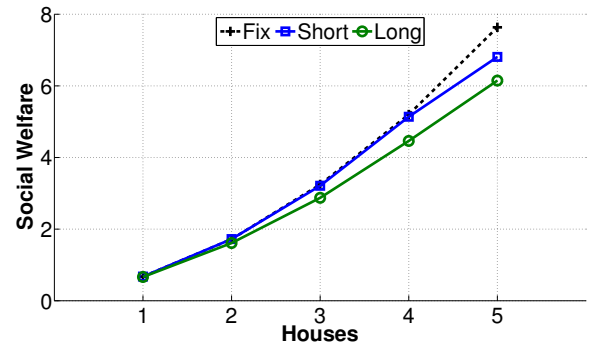
Figures 2(a) and 2(b) show the Social Welfare achieved by the proposed DSM system as a function of the number of houses participating in the power scheduling game. As illustrated in the figures, users always benefit from higher flexibility. Indeed, long scheduling intervals for shiftable appliances (curves identified by “Long” in the figures) result always in cheaper scheduling plans than those obtained with short and fixed intervals (curves identified by “Short” and “Fix”, respectively), since the distributed DSM system can explore a larger solution space.

It can be further observed that increasing the number of shiftable devices (from 4 to 5 out of 11) does not permit to reduce significantly the electricity bill when users have thight constraints on the starting and ending time of their appliances.

We underline that all houses pay an equal share of the electricity bill illustrated in Figures 2(a) and 2(b). Indeed, the Jain's Fairness Index, which we do not show for the sake of brevity, is always very close to 1. Finally, we observe that, throughout all our tests, convergence to the equilibrium is achieved within 5 iterations in the worst case, and in the large majority of the scenarios within only less than 2.



(a) 4 Appliances



(b) 5 Appliances

Figure 2: Social Welfare considering four (a) and five (b) shiftable appliances for each house.

In Figure 3, the overall electricity demand resulting from the proposed method, in the case of a five-house group, is compared to that of an unmanaged group of residential users. As illustrated in Figure 3(b), in addition to decreasing the

electricity bill, the proposed distributed DSM system reduces the energy demand during peak hours (i.e. high-price hours) up to 20% without any centralized coordination among users. Note, however, that a significant energy peak reduction can be obtained when users have loose temporal constraints on the execution of their devices (i.e., a high degree of flexibility). Indeed, as illustrated in Figure 3(a), when only 4 out of 11 devices are shiftable, the peak reduction is negligible (the curves *Short Flexibility* and *No Flexibility* are overlapped).

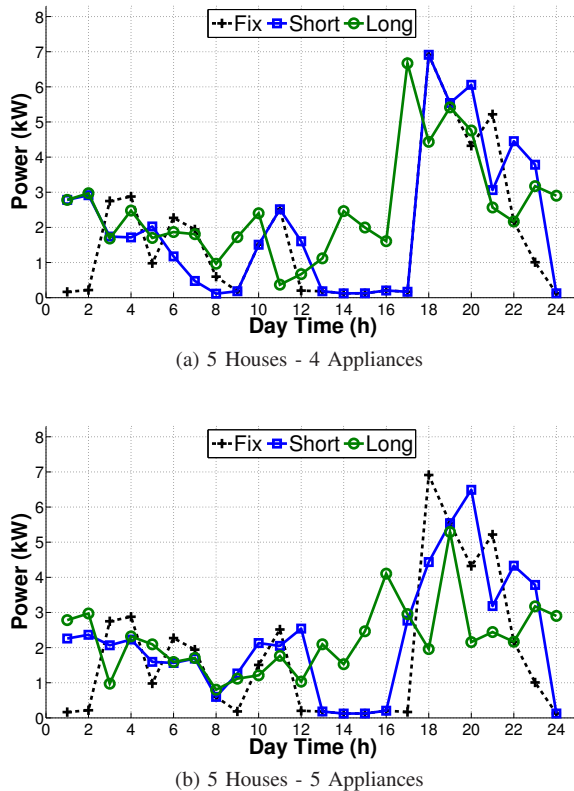


Figure 3: Electricity demand for five houses with four (a) and five (b) shiftable appliances, respectively.

VII. CONCLUSIONS

In this paper, we tackled a fundamental problem that has recently emerged in electric systems, namely the *peak absorption* in the power demand, which arises due to the high correlation among energy demands of residential customers. To solve this issue, we proposed a novel, fully distributed Demand Side Management (DSM) system aimed at reducing the peak demand of a group of residential users.

We modeled our system using a game theoretical approach, where the players are the end customers, the set of strategies is their daily energy demand, and the objective function the customers aim at minimizing is their daily electricity bill. We demonstrated that the proposed game is potential, and in particular exactly potential when the pricing scheme imposed by the energy retailer is linear in the total demand

of customers. For this reason, we proposed a best response dynamics mechanisms which converges in few steps to efficient Nash equilibrium solutions. Numerical results, obtained using realistic load profiles and appliance models demonstrate that residential users can minimize their electricity bill in a completely distributed fashion and reduce the peak absorption of the entire system.

ACKNOWLEDGMENT

This work was partially supported by Italian MIUR and French ANR in the framework of the PRIN Gatecom and ANR Green-Dyspan projects.

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