Design of Wireless Sensor Networks for Mobile Target Detection

Edoardo Amaldi, Antonio Capone, Matteo Cesana, and Ilario Filippini

Abstract—We consider surveillance applications through Wireless Sensor Networks (WSNs) where the areas to be monitored are fully accessible and the WSN topology can be planned a priori to maximize application efficiency. We propose an optimization framework for selecting the positions of wireless sensors to detect mobile targets traversing a given area. By leveraging the concept of path exposure as a measure of detection quality, we propose two problem versions: the minimization of the sensors installation cost while guaranteeing a minimum exposure, and the maximization of the exposure of the least exposed path subject to a budget on the sensors installation cost.

We present compact mixed-integer linear programming formulations for these problems that can be solved to optimality for reasonable-sized network instances. Moreover, we develop Tabu Search heuristics that are able to provide near-optimal solutions of the same instances in short computing time and also to tackle large size instances.

The basic versions are extended to account for constraints on the wireless connectivity as well as heterogeneous devices and non-uniform sensing. Finally, we analyze an enhanced exposure definition based on mobile target detection probability.

Index Terms—wireless sensor networks, mobile target detection, exposure, optimization

I. INTRODUCTION

Wireless Sensor Networks (WSNs) are generally composed of small sized battery-operated devices with embedded sensing, processing, and wireless communication capabilities. Due to their cost viability and flexibility, WSNs have emerged as effective solutions in a variety of civilian and military applications involving, among others, event detection, target tracking, environmental and industrial monitoring [1].

The ultimate goal of any deployed WSN is to provide measures of a given physical process, including the time evolution of physical parameters (air temperature, humidity level, light intensity, etc.) as well as the detection of specific events (invasion, mobile target, alarms, etc.). Therefore WSNs combine sensing/detecting functionalities to actually collect the measures, computation functionalities to process them (e.g., filter samples, or take decisions based on observations), and communication functionalities to distribute the collected data throughout the network, eventually reaching a sink device. Roughly speaking, the quality of a deployed WSN depends on the effectiveness of all the above functionalities.

Regardless of the application, the network topology plays a key role in determining the quality of WSNs, since it affects both the sensing capability and the wireless connectivity. Intuitively, sensors should be deployed in order to fully cover the area of interest, with a density able to capture characteristics and variations of the process to be measured (coverage). But sensors should also be able to set up a wireless communication backbone to exchange the collected measures, eventually taking cooperative decisions on them (connectivity).

A detailed and comprehensive evaluation of WSNs quality is a challenging task, due to the variety of applications and the difficulty to capture the realistic features of the sensing mechanism. Previous work in the field differs in the way coverage quality is measured. A common approach is based on a 0-1 coverage definition where sensor nodes are characterized by an on-off sensing range [2]–[23]. Enhanced performance measures, based on the concept of path exposure [25]–[32] have been recently proposed to quantify the network ability to detect mobile targets in surveillance applications [24]–[36]. This extends the concept of coverage range since, according to exposure definition, coverage quality depends on the distance from the sensor node and it smoothly decreases as the distance increases.

The performance measures proposed for quality evaluation can also be used to drive/improve network deployment. The type of deployment approach and the available degrees of freedom clearly depend on the application field and the accessibility of the specific environment. Whenever the environment is fully accessible by the network planner, sensors positions can be pre-determined and optimized. If the environment is hard to reach and/or hostile, or if node-by-node positioning is too expensive, random positioning may be the only alternative.

Considerable attention has been devoted to both scenarios. In the former one, the problem of sensors positioning is often tackled through mathematical optimization approaches to cover a set of target points [3], [4]. In the latter one, the main focus is on the evaluation of the minimum node density providing the required quality [36], or on energy saving strategies, often based on partitioning the set of installed sensors [11], [14], [19].

In this paper, we focus on fully accessible environments where sensors positions can be actually optimized. Previous work on this topic relies on the general 0-1 coverage model based on the sensing range. A common approach is to select a set of nodes satisfying some connectivity conditions and guaranteeing that each point of the area is within the range of k sensors (k-coverage). These models are appropriate for monitoring applications devoted to large scale measurements, or to the detection of localized and static target events.

In this work, instead, we propose an optimization framework for the problem of positioning sensors where the sensing quality depends on the distance from the sensor node. In particular,
the application scenario is mobile target detection and sensing quality is based on the concept of path exposure. Mobile targets detection is of utmost importance in several practical scenarios, including border protection/surveillance and critical areas surveillance (museums, factories, departments, etc.). To the best of our knowledge, this work is the first attempt to provide a wireless sensor network planning methodology where coverage is not just based on the sensing range (0-1 coverage model) but its quality depends on the distance from the sensor node (path exposure model).

Indeed, previous work on mobile target detection using the concept of exposure focuses on assessing the performance of randomly deployed WSNs [24], identifying the most/least exposed paths for given network topologies [27], computing the minimum node density that guarantees a given quality [30], [36], and optimizing multi-phase random deployment [32] (for a detailed review see Section VI).

The design of a WSN for mobile target detection requires a completely different deployment approach. In the classic coverage model, the entire area must be monitored to detect targets. Here, instead, the goal is to detect a target that crosses a protection border, moving from an external area to a sensible area. Therefore, the objective is not to detect an intruder at any point in the area, but to detect the crossing attempt. As we will see in the following sections, the sensor deployment pattern tends to be a “dense” barrier that the intruder is forced to go through in order to cross the area. The classic coverage model, instead, leads to more uniform deployment patterns, which usually require a higher number of installed sensors than those needed for mobile target detection.

In our approach we consider two main objectives: minimizing the number of nodes deployed (cost) and maximizing the exposure of the least exposed path. Given these contrasting objectives, we propose and investigate two WSN planning problems. In the first one, sensors must be positioned in order to maximize the exposure of the least exposed path, subject to a budget on the installation cost (number of sensors). In the second one, sensors have to be positioned so as to minimize the installation cost, provided that the exposure of the least-exposed path is above a given threshold.

Although the basic mathematical programming formulations of these problems are non-linear, we show that we can derive equivalent Mixed Integer Linear Programming (MILP) formulations. These formulations can be solved to optimality for reasonable-sized network instances with a state-of-the-art MILP solver. We also propose Tabu Search heuristics that provide near-optimal solutions in short computing time and that can be used to tackle large size instances. Furthermore, we extend the model to account for constraints on the wireless backbone (node capacity and multi-hop connectivity) as well as heterogeneous devices and non-uniform sensing. Finally, we analyze a different definition of the exposure based on mobile target detection probability.

In summary the main contributions of this paper are the following:
1) an optimization approach to the planning of sensor positions where coverage quality, defined according to the concept of path exposure, depends on the Euclidean distance from the intruder;
2) optimal solution of different planning problem versions based on MILP formulations;
3) efficient heuristic algorithms that can be used to solve large size instances in reasonable computing time;
4) several extensions to deal with common deployment issues such as wireless connectivity among sensors, heterogeneous devices and non-uniform sensing.

The paper is organized as follows. In Section II, we define the concept of exposure and introduce the planning objectives and constraints. The proposed mathematical programming formulations and the heuristic algorithms are described in Section III, along with the discussion on computational results obtained on randomly generated problem instances. Section IV outlines the extensions to deal with some deployment issues, while Section V analyzes a different definition of the exposure. A careful review of previous and related work is given in Section VI. Section VII contains our concluding remarks.

II. EXPOSURE AND PLANNING APPROACH

A. Exposure definitions

The concept of exposure [25] has been introduced to provide a quantitative measure of the quality of WSNs for the detection of mobile objects traversing areas of interest along a given path. Intuitively, the more exposed is a path, the better is the coverage provided by the WSN, and the higher is the probability to detect the mobile object moving along that path.

The formal definition of exposure obviously depends on the specific sensing model adopted, and the way in which sensed data are used for detection. A common sensing model assumes that the sensing mechanism is based on the energy of a signal received from the target (the signal can be either generated or just reflected by the target) [25]. Given a location \( p \) in the monitored area, the energy of the signal received by a sensor at position \( s \) from a target in \( p \) is:

\[
I_s(p) = \frac{\lambda}{[d(p, s)]^\gamma},
\]

where \( d(p, s) \) is the geometric distance between locations \( p \) and \( s \), \( \lambda \) is the energy emitted by the target, and \( \gamma \) is an energy decay factor, typically ranging from 2 to 5.

If \( l \) sensors located in \( s_1 \ldots s_l \) cooperate to detect the target in position \( p \), the total energy detected by the cooperating set is:

\[
I(p) = \sum_{i=1}^{l} I_{s_i}(p).
\]

A first definition of exposure along a path \( P \) is given in [25]:

\[
E_1(P) = \int_P I(p) dp = \sum_{i=1}^{l} \int_{P_i} I_{s_i}(p) dp,
\]

where \( \int_P \) represents the line integral along path \( P \).

This definition can be extended to the case where detection is corrupted by noise [32]. In this case, the energy received by a sensor in position \( s \) out of a target in position \( p \) becomes:

\[
I_s(p) = I_s(p) + N_s,
\]

where \( I_s(p) \) is defined in (1), and \( N_s \) accounts for additive noise on the measure. According to the value fusion detection model [32], one of the sensors detecting the target in \( p \) collects measurements of all the other sensors,
sums up the energy and decides that the target is actually in position \( p \) if the total received energy is above a given threshold \( \eta \). The probability of detection of a target in \( p \), is then given by:

\[
D(p) = Pr \left[ \sum_{i=1}^{l} I_{s_i}(p) \geq \eta \right].
\] (3)

A second definition of exposure [32] can be obtained by considering the probability of detecting a mobile target in at least one point of path \( P \), that is,

\[
E_2(P) = 1 - \prod_{p \in P} [1 - D(p)].
\] (4)

Since this is a detection problem in a noisy environment, the threshold value \( \eta \) must be such that the probability of a false intrusion detection (positive intrusion detection even though there is no intruder in the area) has a bounded value.

B. Optimization of the sensors positions

The concept of exposure and the two definitions (2) and (4) allow the estimation of the quality of WSNs for mobile target detection. So far, they have been used not only to analyze the quality of deployed WSNs, but also to define the minimum density of random networks that guarantees a given exposure level [36], and to optimize the cost of a heuristic-based multi-phase random deployment [32].

In this work, we use the exposure models to optimize the topology of a WSN assuming that sensor positions can be selected within a discrete set of candidate sites (CSs). These locations can be defined on the basis of a preliminary analysis/survey of the area to be protected, and they strongly depend on the specific application scenario. In many cases, sensors cannot be freely placed in the area, and possible positions depend on the availability of a support or infrastructure (e.g., points where sensors can be easily hidden, points where they cannot be damaged, etc...). When sensors can be located in continuous regions, a discrete approximation (e.g., grids) can be considered. In general, the goal of the network planning is to select a subset of CSs where to install sensors.

The exposure does not only depend on the sensors positions but also on the path followed by the mobile target traversing the area. As in previous work [31]–[35], instead of considering all possible continuous trajectories across a given area, we assume that the paths are constrained on a grid and that they all start from (end to) some virtual points on the left (right) of the grid. The accuracy of this approximation obviously depends on the grid density.

Since the exposure value of the least exposed path is a measure of the vulnerability level of the network, maximizing this value is one of the objectives of the network planning problem. Clearly, another objective is to minimize the installation cost, which is directly related to the number of sensors deployed. Thus we address two versions of the problem: maximizing the exposure of the least-exposed path subject to a budget constraint on the cost, and minimizing the installation cost while guaranteeing that all paths have an exposure value above a given threshold.

By adding constraints to these two basic versions, we can easily ensure wireless communications among the sensors. Given a sensor layout (a set of sensors located in a subset of CSs) and their transmission ranges, consider the communication graph where sensors are connected by a link if they are in range. An additional node of the graph represents the sink device where all data must be delivered. To guarantee communication, a sensor layout must correspond to a connected communication graph. To account for resiliency to failures, we may require \( k \)-connectivity. Based on the same graph, we can also include capacity constraints on the node traffic, corresponding to limited transmission rates on the radio channel and/or to energy consumption limitations.

Moreover, practical surveillance systems may integrate multiple types of sensing devices running different sensing technologies, with different capability/accuracy, as well as different installation costs. To this extent, we show how the basic optimization formulation can be extended to plan such heterogenous systems, capturing further degrees of freedom.

In the next section, we formalize the two basic versions of the WSN planning problem based on the exposure \( E_1 \), and derive compact MILP formulations by applying linear programming duality. These two basic versions are then extended to account for wireless connectivity and limited node capacity as well as heterogeneous devices and sensing ability. We also address a problem version based on the exposure definition \( E_2 \) instead of \( E_1 \). Although no compact MILP formulation is available in this case due to the intrinsic non-linearity of the detection probability, we show the heuristic we propose can be easily adapted to deal with exposure \( E_2 \).

III. INTRUSION DETECTION BASED ON \( E_1 \)

A. Problem formulations

We first consider the two basic versions of the problem with exposure definition \( E_1 \) in (2) where only coverage is considered. The variants capturing additional features such as connectivity and node capacity, and the problem version based on the exposure definition \( E_2 \) in (4) are discussed in the following subsections.

Given the area of interest, let \( S \) denote the set of all CSs where sensors can be installed. The area is approximated by a grid on which targets can move. The grid graph \( G = (V, A) \) is defined as follows. The vertex set \( V \) includes one vertex for each grid intersection point, and two distinguished vertices \( o \) and \( t \) that represent the virtual origin and, respectively, destination of any path. The arc set \( A \) includes two arcs \( (i,j) \) and \( (j,i) \) for each grid edge \( \{i,j\} \), the incoming/outgoing arcs connecting \( o \) to the leftmost column vertices of the grid, and the rightmost column vertices to \( t \), respectively ¹. Note that CSs are not forced to lie on the grid. They can be placed in any position of the area surrounding the grid, according to the available installation sites.

Without loss of generality, we assume that \( G \) is a square grid of size \( n \times n \).

¹The general case with multiple origins and destinations can be easily reduced to this single origin and destination case.
Given an instance defined by \((S, V, A)\) with two special vertices \(o\) and \(t\), and a budget value \(B\), the first basic version of the problem consists in deciding where to install the sensors so as to maximize the exposure of the least exposed path from \(o\) to \(t\) while guaranteeing a total installation cost of at most \(B\).

To derive a mathematical model for this problem version, we consider the decision variables: \(y_s\), which is equal to 1 if a sensor is installed in candidate site \(s\) and 0 otherwise, \(x_{ij}\), which is equal to 1 if arc \((i, j)\) belongs to the least exposed path and 0 otherwise, and the non-negative continuous variable \(z\) that expresses the exposure of the least exposed path. These variables lead to the following formulation:

Maximize \(z\)

subject to

\[
\sum_{s \in S} c_s y_s \leq B \tag{5}
\]

\[
\min \sum_{(i,j) \in A} \left( \sum_{s \in S_{ij}} e_{ij}^s y_s \right) x_{ij} \geq z \tag{7}
\]

\[
\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = \begin{cases} 1 & i = o \\ -1 & i = t \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in V \tag{8}
\]

\[
x_{ij} \geq 0 \quad \forall (i, j) \in A \tag{9}
\]

\[
y_s \in \{0, 1\} \quad \forall s \in S \tag{10}
\]

where \(c_s\) is the cost of installing a sensor in \(CS\) \(s\), and the parameter \(e_{ij}^s\) is the exposure estimated according to (2). The set \(S_{ij}\), which denotes the set of all \(CS\) that cover/affect the arc \((i, j)\), clearly depends on the devices’ sensing range. A \(CS\) belongs to \(S_{ij}\) if its exposure over arc \((i, j)\) is above a certain threshold. Constraint (6) enforces the limitation on the installation budget. Since \(z\) is maximized, constraint (7) ensures that \(z\) is equal to the exposure of the least exposed path. As expected, each installed sensor affects the weights (exposure) of all the arcs in \(S_{ij}\), and the weight (exposure) of edge \((i, j)\) is given by \(\sum_{s \in S_{ij}} e_{ij}^s y_s\). Constraints (8) are the flow balance equations that define a path from the origin \(o\) to the destination \(t\).

The formulation (5)-(10) is a so-called bi-level mathematical program, since a minimization subproblem (appearing in constraints (7), (8) and (9)) is plugged into the overall maximization problem. Although this formulation is nonlinear due to constraint (7), it can be linearized.

Proposition 1: The formulation (5)-(10) is equivalent to the mixed integer-linear formulation (F1):

Maximize \(z\)

subject to

\[
\sum_{s \in S} c_s y_s \leq B \tag{11}
\]

\[
\pi_t - \pi_o \geq z \tag{12}
\]

\[
\pi_j - \pi_i \leq \sum_{s \in S_{ij}} e_{ij}^s y_s \quad \forall (i, j) \in A \tag{13}
\]

\[
\pi_o = 0 \tag{14}
\]

Proof: If the \(y_s\) variables are considered as parameters, the minimization subproblem reduces to a shortest path problem in which the left-hand side of constraint (7) is minimized subject to constraints (8) and (9). Then its dual (SP-dual) is

\[
\max \quad \pi_t - \pi_o \tag{16}
\]

subject to

\[
\pi_j - \pi_i \leq \sum_{s \in S_{ij}} e_{ij}^s y_s \quad \forall (i, j) \in A \tag{17}
\]

\[
\pi_o = 0, \tag{18}
\]

where \(\pi_i\) is the dual variable associated with the \(i\)-th constraint (8).

Since all arc weights of \(G\) are non-negative, this shortest path subproblem always admits a finite optimal solution and, according to the linear programming strong duality theorem, the optimal values of the primal and dual objective functions coincide. By replacing the primal subproblem in (7)-(9) with the dual one in (16)-(18), we obtain the equivalent mixed integer-linear formulation (11)-(15).

Note that for each arc of \(G\) there is a single linear constraint (14) involving the corresponding dual \(\pi\) variables.

Let us now turn to the second basic problem version, where we minimize the total installation cost while guaranteeing that the exposure of any path traversing the detection area is above a certain real threshold \(\tau > 0\).

Proposition 2: For any \(\tau > 0\), the second basic problem version admits the following MILP formulation (F2):

Minimize \(\sum_{s \in S} c_s y_s\)

subject to

\[
\pi_t - \pi_o \geq \tau \tag{20}
\]

\[
\pi_j - \pi_i \leq \sum_{s \in S_{ij}} e_{ij}^s y_s \quad \forall (i, j) \in A \tag{21}
\]

\[
\pi_o = 0. \tag{22}
\]

Proof: Consider again the dual (SP-dual) of the shortest-path subproblem in (7)-(9). According to weak duality, the objective function value \(\pi_t - \pi_o\) of any feasible solution of (SP-dual) is not larger than that of \(\sum_{(i,j) \in A} \left( \sum_{s \in S_{ij}} e_{ij}^s y_s \right) x_{ij}\) of any feasible solution of the (primal) shortest-path subproblem. Therefore, by imposing that the variable vector \(\pi\) satisfies constraints (20)-(22) of (F2), we make sure that the total exposure of any path from \(o\) to \(t\) (in particular that of any shortest path) is at least \(\tau\). Note that here, given the constraint (20) and the objective function (19), the vector \(\pi^*\) of an optimal solution of (F2) does not need to be an optimal solution of (SP-dual).

Proposition 3: Both basic problems (F1) and (F2) are NP-hard even when the grid is a single path and each \(CS\) can detect the target across a single arc.

Proof: We consider the decision versions associated to the optimization problems of formulations (F1) and (F2) where, given a threshold value \(K\), one has to decide whether there exists a feasible solution with an objective function value greater or equal to \(K\). To verify that these optimization problems are NP-hard, we exhibit a simple polynomial-time reduction from the well-known knapsack problem to the corresponding decision versions. Consider an arbitrary instance \(I\) of the knapsack problem: a finite set of items \(S\), a cost \(c(s) \in \mathbb{Z}^+\) and a value \(v(s) \in \mathbb{Z}^+\) for each \(s \in S\), and two thresholds...
B ∈ Z⁺ and K ∈ Z⁺, is there a subset S’ ⊆ S such that \(\sum_{s \in S'} c(s) \leq B\) and \(\sum_{s \in S'} v(s) \geq K\)? We construct the special instance \(I'\) of the decision version of problem (F₁) whose grid consists of a single path with |S| arcs (one for each element \(s \in S\)) and with a CS in the middle of each arc which does only affect that arc (\(|S_{ij}| = 1\) for each arc \((i,j)\)). Note that if \(c_a = c(s)\) and \(e_{ij} = v(s)\) for each \(s \in S\), and sensors are installed in a subset \(S' \subseteq S\) of the CSs, the only available path has a total cost \(\sum_{s \in S'} c_a\) and a total exposure \(\sum_{s \in S'} e_{ij}\). Clearly, the instance \(I'\) of the decision version of (F₁) has a positive answer (in particular there exists a binary vector \(y\) corresponding to a subset \(S' \subseteq S\) such that \(\sum_{s \in S'} c_a \leq B\) and a total exposure \(\sum_{s \in S'} e_{ij} \geq K\)) if and only if the corresponding knapsack instance \(I\) has a positive answer. The same reduction also holds for the problem (F₂).

### B. Heuristic algorithms

To tackle large-size instances of both formulations (F₁) and (F₂), we have developed Tabu Search (TS) algorithms, see e.g. [41]. TS is a metaheuristic that guides a local search procedure to explore the solution space of optimization problems beyond local optima, which has been successfully adapted to a number of other challenging network design problems. Starting from an initial feasible solution \(y₀\), a set of neighboring solutions \(N(y₀)\) are generated by applying a set of possible “moves” to \(y₀\). Then the best solution in the “neighborhood” \(N(y₀)\) is selected as the next iterate \(y₁\), even if it does not strictly improve the value of the objective function. The process is iterated to generate a sequence of solutions \(\{y_k\}\). The rationale is to try to improve an initial solution by iteratively installing and removing sensors selected on the basis of their exposure contribution to a least exposed path of the current solution

1) **Formulation (F₁):** The objective is to minimize the minimum path exposure subject to budget constraint on installation costs. Let \(y\) be a random initial solution satisfying the budget constraint. We consider swap moves in which a sensor in the current feasible solution \(y\) is deleted while a new sensor is installed in an empty CS (a CS \(i\) with \(y_i = 0\)). At each iteration, given a shortest path \(P_k\) corresponding to \(y\), all the CSs are ordered according to their contribution \(E_1(P_k)\) to the path exposure. Consider the \(l\) empty CSs with the largest such values and the \(l\) CSs where a sensor is located with the lowest such values. Then the next iterate \(y^{k+1}\) is obtained by performing, among all the above \(l^2\) possible swaps, the swap leading to the best objective function value.

In order to prevent cycling (considering feasible solutions that have already been generated) and try to escape from local maxima, a list of “tabu moves” is maintained. If \(L\) denotes the length of the list and \(M\) the move carried out at a given iteration, the opposite of \(M\) is forbidden (tabu) for the next \(L\) iterations, until it is extracted from the list. According to the “aspiration criteria”, tabu moves can be clearly made if they lead to a best found solution. The best solution encountered during the search process is stored and returned after a maximum number of iterations \(\text{max}_{\text{it}}\).

The tabu list is implemented in a simple way: sensors that are installed (deleted) cannot be deleted (reinstalled) during \(L\) iterations. To favor search diversification, if the best solution found in the neighborhood does not improve the current solution during \(R\) moves, a random swap is carried out: a randomly selected sensor is deleted while a sensor is installed in an empty CS selected at random. Algorithm 1 reports steps to solve the problem of Formulation (F₁).

The computational results reported in the next section have been obtained with the following parameter settings: \(L = 15, L = 4, R = 2, \text{max}_{\text{it}} = 250\).

2) **Formulation (F₂):** The objective is to minimize the total installation costs while guaranteeing a minimum path exposure \(\bar{z}\). We start from the initial solution \(y₀\) where all sensors are installed. Each iteration consists of two steps: 1) \(m\) sensors are deleted from the current solution \(y_k\) to obtain \(\tilde{y}^k\); 2) the “neighborhood” \(N(\tilde{y}^k)\) is explored as in the previous algorithm looking for a better solution.

In the first step, given a shortest path \(P_k\) corresponding to \(y_k\), all the CSs are ordered according to their contribution \(E_1(P_k)\) to the path exposure. The algorithm randomly selects \(m\) among the first \(h\) CSs and removes them from the solution (setting \(y_i = 0\) for corresponding \(i\)), the result is \(\tilde{y}^k\). The second step consists of swap moves, similarly to the algorithm for formulation (F₁). At each second-step iteration \(e\), given a shortest path \(P_{e}\) corresponding to \(\tilde{y}^e\), all the CSs are ordered according to their contribution \(E_1(P_{e})\) to the path exposure.
Consider the $l$ empty CSs with the largest such values and the $l$ CSs where a sensor is located with the lowest such values. Then the next iterate $y^{k+1}$ is obtained by performing, among all the above $l^2$ possible swaps, the swap leading to the best objective function value. The second step ends after $\max_e$ iterations giving the next $y^{k+1}$.

The same tabu list of the previous algorithm is used. Sensors that are installed (deleted) in the first or the second step cannot be deleted (reinstalled) during the next $L$ iterations of the same type.

The whole algorithm execution is characterized by two phases. A first phase, named Steepest Descent, where $m = 5$ among $h$ CSs are removed in the first step and $l = 5$ CSs are considered in the second step, at each iteration $k$. The Steepest Descent phase continues until it reaches iteration $k'$ where the best solution found during its second step has a path with exposure smaller than the desired $\bar{e}$. Sensors removed during the first step of iteration $k'$ are reinstalled and the second phase, named Slow Descent, begins at iteration $k' + 1$. During the Slow Descent phase $m = 1$ over $h$ CSs is removed in the first step and $l = 10$ CSs are considered in the second step of every iteration $k$. Slow Descent phase stops when the second step of the current iteration ends providing a solution which does not guarantee the desired minimum exposure $\bar{e}$. The algorithm stops when the Slow Descent phase ends, the final solution is the solution $y^{k'}$ of the penultimate iteration. This twofold behavior arises from the idea of having a first rough phase to quickly get close to a good solution and then a refined and wide search for the best solution. The entire procedure is summarized in Algorithm 2.

Finally, we have included a multi-start strategy starting from the end of the Steep Descent phase. At each multi-start run, the first set of $h$ CSs candidate to be removed is randomly created. After $P$ scheduled runs, the solution of the run which ended with the best results (smallest number of installed sensors) is provided as final solution.

Computational results reported in the next section have been obtained with the following parameter settings: $L = 4$, $h = 20$, $\max_e = 10$, the above mentioned $m$ and $l$ values, $P = 3$.

### C. Numerical results

We now report and discuss some numerical results obtained on sample network instances to provide some insights on the characteristics of the proposed models, and to evaluate the performance of the exact and heuristic solution approaches.
Instances have been generated by considering an 80m × 80m square area and by randomly selecting the positions of 80 CSs. The sensing range is set to 10m. The exposure coefficient λ in (2) is set to 1, while the decay factor γ to 3.

Unless otherwise specified, results are averages on 20 randomly generated instances. Computational tests have been run on a 3.0GHz PC with 1GB of RAM under Linux. To solve MILP formulations to optimality, we used the commercial solver CPLEX 10.0 [43]. Our Tabu Search heuristics have been implemented in C++ using Boost libraries [42].

Network planning versus random deployment. First we compare the quality of randomly deployed networks with that of optimized layouts, to assess the potential gain when the area of interest is fully accessible. Figure 1 plots the exposure $E_1(P)$ of a least exposed path versus the number of deployed sensors, for random and optimized layouts. The optimal layouts have been obtained by solving the formulation ($F_1$) with 80 CS to optimality, whereas, in random deployment, sensors are randomly drawn in the area.

As expected, given a number of deployed sensors, the difference in the exposure value of a least exposed path is remarkable. Random deployment requires many more sensors to achieve a given protection level (minimum exposure value). For example, for a minimum exposure value of 0.20, 20 nodes are sufficient in optimal sensors layouts, while more than 50 are required in random deployments. Figure 2 illustrates two network topologies, almost equivalent from the minimum exposure point of view, with random deployment ($B = 50$) and optimized sensors positions ($B = 20$), respectively. Each figure indicates sensors positions, their sensing range and one of the least exposed paths. It is worth noting that, in general, optimized layouts admit many equivalent least exposed paths. For example, in the case of Figure 2, the optimal deployment has hundreds of equivalent least exposed paths. This is due to the specific max-min structure of the problem. Indeed, the objective function only depends on the exposure of a least exposed path, regardless of the exposure distribution of the other paths. Intuitively, maximizing the exposure of the least exposed path tends to balance the exposure of all the paths, given the cost budget.

Comparison between exact and heuristic solutions. In Table I, we compare the performance of the optimal solutions of formulation ($F_1$) with those provided by the TS algorithm. Different classes of instances with two values of the grid density (25 × 25 and 50 × 50) and three different values of the budget $B$ on the number of installed sensors (30, 40, and 50) are considered. The three pairs of columns from 3 to 8 refer to the optimal layouts, and indicate the average and standard deviation values of computing time (in seconds), the minimum exposure, and the actual number on installed sensors, respectively. The two pairs of columns from 9 to 12 report the same information for the TS algorithm (the number of sensors is omitted since it is always equal to the budget). The last column shows exposure gaps between optimal solutions and ones obtained with the TS algorithm.

First, note that CPLEX allows to solve to optimality medium-to-large sized instances within 4 hours of CPU time. The TS algorithm performs very well on all instances, since it provides solutions that are within 5% of the optimum in at most 3 minutes.

The analysis of the results indicates some interesting characteristics of the problem. As expected, the minimum exposure increases as the budget increases, as more sensors can be used to detect targets. For a given budget, the minimum exposure slightly decreases as the grid density increases, since with higher-density grids the formulation can select the least exposed path in a more accurate way. Finally, it is worth noting that the number of deployed sensors is sometimes lower than the budget. Again, this is due to the objective function that only depends on least exposed paths. In fact, if none of the remaining CSs (those where sensors are not installed) can improve the exposure of all these paths, due to the limited sensing range, deploying further sensors is useless, since it would only improve the exposure of some non-minimal paths.

The heuristic algorithm proposed can also be adopted to plan much larger instances in reasonable time. Figure 3 reports an example network topology spanning an area of 400m × 400m with 1500 CSs, a budget $B = 1000$ and a grid $200 \times 200$. In this case, the heuristic algorithm required approximately 24 minutes of computation time.

The results obtained with the minimum cost formulation ($F_2$) are shown in Table II. Even if formulation ($F_2$) is not too different from ($F_1$), its computing time requirements turn out to be much higher. With a CPU time limit of 4 hours, we are able to solve only instances with a limit on the minimum exposure up to 0.35, with a 25 × 25 grid, and up to 0.20, with a 50 × 50 grid. The TS algorithm performs very well also for this formulation, it provides solutions that are within 5% of the optimum in no more than 5 minutes. Here again, for the same reason as in Table I, we can see that with higher-density grids more sensors must be installed to guarantee the same minimum exposure.
TABLE II
COMPARISON BETWEEN EXACT AND HEURISTIC SOLUTIONS FOR THE MINIMUM COST FORMULATION ($F_2$).

<table>
<thead>
<tr>
<th>Grid</th>
<th>Exposure</th>
<th>Time [s]</th>
<th># Installed sensors</th>
<th>Time [s]</th>
<th># Installed sensors</th>
<th># Gap [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>average</td>
<td>std dev</td>
<td>average</td>
<td>std dev</td>
<td></td>
</tr>
<tr>
<td>50x50</td>
<td>0.20</td>
<td>9110.96</td>
<td>5983.07</td>
<td>15.40</td>
<td>1.51</td>
<td>272.96</td>
</tr>
<tr>
<td>50x50</td>
<td>0.15</td>
<td>8396.64</td>
<td>6383.17</td>
<td>12.60</td>
<td>0.84</td>
<td>225.81</td>
</tr>
<tr>
<td>50x50</td>
<td>0.10</td>
<td>7298.60</td>
<td>5950.95</td>
<td>9.90</td>
<td>0.57</td>
<td>199.94</td>
</tr>
<tr>
<td>50x50</td>
<td>0.05</td>
<td>3279.45</td>
<td>2651.75</td>
<td>7.30</td>
<td>0.48</td>
<td>96.69</td>
</tr>
<tr>
<td>25x25</td>
<td>0.30</td>
<td>3597.90</td>
<td>2619.76</td>
<td>19.90</td>
<td>2.18</td>
<td>72.24</td>
</tr>
<tr>
<td>25x25</td>
<td>0.25</td>
<td>3474.35</td>
<td>2690.67</td>
<td>17.00</td>
<td>2.26</td>
<td>64.96</td>
</tr>
<tr>
<td>25x25</td>
<td>0.20</td>
<td>3480.92</td>
<td>2627.70</td>
<td>14.50</td>
<td>2.01</td>
<td>52.70</td>
</tr>
<tr>
<td>25x25</td>
<td>0.15</td>
<td>2746.77</td>
<td>4443.32</td>
<td>12.10</td>
<td>0.74</td>
<td>44.27</td>
</tr>
<tr>
<td>25x25</td>
<td>0.10</td>
<td>1431.76</td>
<td>1558.14</td>
<td>9.80</td>
<td>0.42</td>
<td>35.18</td>
</tr>
<tr>
<td>25x25</td>
<td>0.05</td>
<td>965.86</td>
<td>601.28</td>
<td>7.20</td>
<td>0.42</td>
<td>18.06</td>
</tr>
</tbody>
</table>

Fig. 3. Example of a large size network planned using the heuristic algorithm.

Thus our compact MILP formulations yield optimal solutions for medium-size instances, and our efficient TS heuristics provide near-optimal solutions in short computing time and can be used to tackle large-size instances.

Fig. 4. Example of circular sensible area.

Finally, we want to remark that the application of our approach is not limited to square grid areas, but it can be adopted in different geometries that are relevant for mobile target detection applications. In Fig. 4 we show an example scenario where we have applied our algorithms to a circular crossing area (white) that protects a sensible core area (black).

The optimal placement of sensor nodes with their coverage areas is shown along with the least exposed path. Note that the deployment is concentrated in circumferences with smaller radii, a configuration which allows a better exposure over breach paths.

Minimum exposure versus average exposure. The planning problem considered so far assumes the worst possible intruder, that is, an intruder which is able to find the minimum exposure path through the region to be protected. In practical surveillance systems, such assumption on the "worst intruder" may be over pessimistic, as the intruder may only have partial knowledge on the topology of the surveillance system itself. To this extent, it is worth studying the case when the network planner is interested in designing the network to counteract the behavior of an "average" intruder(s). To do that, we compare the results obtained with the previous proposed approaches with results obtained planning the network using an average exposure measure over a set of randomly selected crossing paths that uniformly cover the sensible area. With such a comparison we can quantify the difference in sensor nodes requirement and intrusion detection quality between the two planning approaches.

In more detail, we have applied the technique based on average exposure to the same instances as in Tables I and II. We have generated a set of $N_P = 100,500$ random paths starting from the leftmost column of the grid and ending to the rightmost one. The set is populated with disjoint (as much as possible in the grid) paths that uniformly cover the area. The node deployment that maximizes the average exposure of the paths subject to a budget constraint is computed solving a simple ILP model:

$$
\max \sum_{P_h \in \mathcal{P}} \sum_{(i,j) \in P_h} \left( \sum_{s \in S_{ij}} e_{ij}^s y_s \right) = (23)
$$

subject to

$$
\sum_{s \in S} e_s y_s \leq B = (24)
$$

$$
y_s \in \{0, 1\} \quad \forall s \in S, = (25)
$$

where $\mathcal{P}$ is the set of considered paths. Similarly to formulation ($F_2$), the ILP model has been modified to deal also with the problem of minimizing the number of installed nodes with
TABLE III
RESULT FOR THE AVERAGE EXPOSURE MAXIMIZATION OVER A SET OF RANDOM PATHS WITH AN INSTALLATION BUDGET CONSTRAINT.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Budget</th>
<th>100 paths</th>
<th>500 paths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ave. Exp. on paths</td>
<td>Min Exposure</td>
</tr>
<tr>
<td>25x25</td>
<td>30</td>
<td>1.80</td>
<td>0.07</td>
</tr>
<tr>
<td>25x25</td>
<td>40</td>
<td>1.98</td>
<td>0.12</td>
</tr>
<tr>
<td>50x50</td>
<td>30</td>
<td>1.93</td>
<td>0.09</td>
</tr>
<tr>
<td>50x50</td>
<td>40</td>
<td>2.13</td>
<td>0.16</td>
</tr>
<tr>
<td>50x50</td>
<td>50</td>
<td>2.14</td>
<td>0.18</td>
</tr>
</tbody>
</table>

TABLE IV
RESULT FOR THE MINIMIZATION OF THE NUMBER OF INSTALLED SENSORS WITH A CONSTRAINT ON THE AVERAGE EXPOSURE OVER A SET OF RANDOM PATHS.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Exposure</th>
<th>100 paths</th>
<th>500 paths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ave. Exp. on paths</td>
<td>Min Exposure</td>
</tr>
<tr>
<td>50x30</td>
<td>0.200</td>
<td>0.2102</td>
<td>0.0028</td>
</tr>
<tr>
<td>50x30</td>
<td>0.150</td>
<td>0.1587</td>
<td>0.0056</td>
</tr>
<tr>
<td>50x30</td>
<td>0.100</td>
<td>0.1089</td>
<td>0.0035</td>
</tr>
<tr>
<td>50x30</td>
<td>0.050</td>
<td>0.0541</td>
<td>0.0020</td>
</tr>
<tr>
<td>25x25</td>
<td>0.350</td>
<td>0.3698</td>
<td>0.0045</td>
</tr>
<tr>
<td>25x25</td>
<td>0.300</td>
<td>0.3143</td>
<td>0.0024</td>
</tr>
<tr>
<td>25x25</td>
<td>0.250</td>
<td>0.2684</td>
<td>0.0103</td>
</tr>
<tr>
<td>25x25</td>
<td>0.200</td>
<td>0.2053</td>
<td>0.0177</td>
</tr>
<tr>
<td>25x25</td>
<td>0.150</td>
<td>0.1563</td>
<td>0.0129</td>
</tr>
<tr>
<td>25x25</td>
<td>0.100</td>
<td>0.1080</td>
<td>0.0133</td>
</tr>
<tr>
<td>25x25</td>
<td>0.050</td>
<td>0.0530</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

A constraint on the minimum average path exposure:

\[
\min \sum_{s \in S} c_s y_s \quad (26)
\]

\[
s.t. \quad \frac{1}{|P|} \sum_{P_k \in P} \sum_{(i,j) \in P_k} \left( \sum_{s \in S_{i,j}} c_{i,j}^s y_s \right) \geq \tau \quad (27)
\]

\[y_s \in \{0, 1\} \quad \forall s \in S. \quad (28)\]

Results are shown in Tables III and IV, which report data for the average exposure maximization problem and the installed nodes minimization problem, respectively. Reported results are organized in two groups, for 100 and 500 random paths, each group is composed of four columns.

Table IV shows again a huge gap between the minimum exposure computed on random paths and the one of the real breach path. Note that the average technique allows to save no more than 40% of installed sensor nodes, but the difference in terms of breach path exposure is close to a factor of 10 (with 500 paths).

IV. DEALING WITH CONNECTIVITY AND NON-UNIFORM SENSING

In this section we want to extend the formulations of the Section III-A to deal with two important deployment issues. The first one arises from the need of analyzing data collected by sensor nodes in order to establish whether or not an intrusion has occurred. Since nodes must be able to exchange data among themselves and with a processing sink node through multi-hop wireless transmissions, node deployment must optimize the intrusion detection quality while inducing a connected wireless network among sensor nodes and sink.

The second issue concerns the specific sensor devices and detection technologies nodes are equipped with (passive infrared, acoustic, light, etc.). The network planner may have the chance of choosing among different sensor nodes with different detection technologies and intrusion detection capabilities. Moreover, different detection technologies may have different fields of view, or, in general, they may not have an omnidirectional exposure contribution on the directions of possible intruders. The optimization process must consider these aspects when computing the optimal setting.

A. Network connectivity

We consider the communication graph given by the sensor layout, where sensors can communicate only if they are in range. We define the communication range as the maximum distance at which a transmission from a sensor can be correctly
decoded at a receiver. The sink node acts as a centralized server that collects events detected by sensors, and estimates whether an intruder is present.

Consider a new directed graph $G' = (S, A')$ with one vertex per CS and an arc set $A' \subseteq S \times S$ containing one arc $(i, j)$ for each pair of CSs that are within communication range. A special node $\sigma$ and its relative incoming connections are added to $S$ and $A'$ to represent the sink. To guarantee connectivity among the sensors and the sink, a unit of traffic is required from each installed sensor to the sink. If a continuous non-negative variable $f_{ij}$ represents the traffic flow on arc $(i, j) \in A'$, the extension of $(F_1)$ accounting for connectivity, referred to as $(F_{1C})$, is:

$$\text{max } z$$
$$\text{s.t. } \sum_{s \in \sigma} y_s \leq B$$
$$\pi_i - \pi_o \geq z$$
$$\pi_j - \pi_i \leq \sum_{s \in \sigma} e_{ij} y_s \quad \forall (i, j) \in A$$
$$\pi_o = 0$$
$$\sum_{(s,r) \in A'} f_{sr} - \sum_{(r,s) \in A'} f_{rs} = \begin{cases} -y_r \sum_{q \in S} y_q & r \neq \sigma \\ \sum_{q \in S} y_q & r = \sigma \end{cases} \quad \forall r \in S. \quad (34)$$

The objective function (29) and constraints (30)-(33) are as in $(F_1)$. Constraints (34) are flow balance equations for the graph $G'$, ensuring that one unit of flow is sent from each installed sensor to the sink $\sigma$.

Connectivity can thus be imposed by adding constraints that do not modify the shortest-path subproblem and only place additional conditions on sensor installation. Similarly, model $(F_2)$ can be extended to account for connectivity by adding constraints (34). For the sake of brevity, we do not report the full formulation.

Sensor’s RF equipment has usually limited bandwidth and energy resources. In general, we want to avoid overloading a sensor with communication relay traffic, thus leading it to early depletion or load congestion. To this end, we introduce a node capacity $u_s$ that limits the maximum sum of flows entering and exiting each node $s$, and we add the following constraints:

$$\sum_{(r,s) \in A'} f_{rs} + \sum_{(s,r) \in A'} f_{sr} \leq u_s y_s \quad \forall s \in S: s \neq \sigma. \quad (35)$$

For failure tolerance purposes, one or more backup paths should be selected so that the sink is reached even in case sensors fail. This is achieved by imposing $k$-connectivity on the subgraph of graph $G'$ defined by the installed sensors. The variables $f_{ij}$ are substituted with the new binary variables:

$$x_{qr}^s = \begin{cases} 1 & \text{if path from sensor } s \text{ includes arc } (q, r) \in A' \\ 0 & \text{otherwise.} \end{cases}$$

To enforce $k$-connectivity, we add to the basic formulations the following set of constraints:

$$\sum_{(q,r) \in A'} x_{qr}^s - \sum_{(r,q) \in A'} x_{rq}^s = \begin{cases} -k y_s & r = s \\ k y_s & r = \sigma \\ 0 & \text{otherwise} \end{cases} \quad \forall s \in S, r \in S: r \neq \sigma \quad (36)$$

$$\sum_{(r,q) \in A'} x_{rq}^s \leq y_q \quad \forall s, q \in S: s \neq \sigma \quad (37)$$

$$\sum_{(q,r) \in A'} x_{qr}^s + \sum_{(r,q) \in A'} x_{rq}^s \leq u_r y_r \quad \forall r \in S: r \neq \sigma, \quad (38)$$

where parameter $k$ is the desired number of node-disjoint paths from each sensor to the sink. Constraints (36) are flow balance equations. Since the $x_{qr}^s$ variables are binary, they make sure that $k$ links leave the source sensor $s$ and $k$ links enter the sink $\sigma$ of each $s$-$\sigma$ path. They also enforce the number of entering and leaving links at each intermediate sensor to be equal. Since constraints (37) impose that each installed intermediate sensor has at most 1 outgoing link, constraints (37), together with constraints (36), enforce one entering link and one leaving link at each intermediate sensor of the $k$ $s$-$\sigma$ paths, thus, ensuring node-disjointness. Constraints (38) are capacity constraints, similar to (35). Notice that this set of constraints can be added to both $(F_1)$ and $(F_2)$ formulations, as the simple connectivity constraints (34).

Finally, note that the feasibility of connected, or $k$-connected, WSN designs strongly depends on the set $S$ of CSs and on the sensor communication range. If the CSs are too sparse and/or the communication range is too short, no solution exists even if there is enough capacity. In those cases, the network planner is forced to find additional possible locations for sensors or to choose sensor nodes with different technology (longer range, higher capacity, etc.)

Numerical results reported in Table V have been obtained with the following parameter settings. A communication range of 15m is considered for the problem formulations including connectivity constraints, and a sink node with a communication range of 20m has also been randomly added. In the formulations with sensor capacity, the maximum amount of traffic per node is set to the total communication load generated by 15 sensors.

When wireless connectivity and node capacity constraints are included, both the number of feasible solutions and their quality decrease. Table V shows the results obtained for problem formulation $(F_1)$ with additional constraints (36)–(38) with $k = 2$, accounting for communication graph 2-connectivity and capacity. It is interesting to note that Algorithm 1 can be easily extended to deal with connectivity by restricting attention to the swap moves that lead to connected WSNs.

Since the solution space shrinks, the exposure values are smaller than the corresponding ones in Table I. The price to pay in terms of budget in order to provide connectivity and resiliency to the wireless backbone and to limit per-node traffic is not negligible. For example, to achieve almost the same
exposure level corresponding to \( B = 30 \) in Table I, the number of sensors to be installed is now \( B = 50 \).

### TABLE V

EXACT SOLUTIONS WITH 2-CONNECTIVITY AND CAPACITY CONSTRAINTS
FOR FORMULATION \((F_1)\).

<table>
<thead>
<tr>
<th>Grid</th>
<th>8</th>
<th>Exposure ave. [s]</th>
<th>Exposure std dev</th>
<th># Installed Sensors ave.</th>
<th># Installed Sensors std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>50x50</td>
<td>60</td>
<td>4014.59</td>
<td>8378.80</td>
<td>0.32</td>
<td>0.46</td>
</tr>
<tr>
<td>50x50</td>
<td>50</td>
<td>9214.14</td>
<td>8214.14</td>
<td>0.32</td>
<td>0.46</td>
</tr>
<tr>
<td>50x50</td>
<td>40</td>
<td>13686.04</td>
<td>9773.73</td>
<td>0.32</td>
<td>0.46</td>
</tr>
<tr>
<td>50x50</td>
<td>30</td>
<td>15116.85</td>
<td>8414.59</td>
<td>0.32</td>
<td>0.46</td>
</tr>
</tbody>
</table>

#### B. Non-uniform sensing

Formulations \((F_1)\) and \((F_2)\) can be easily extended to include aspects related to cost, sensing ability and sensing field of each sensor node. In particular, parameters \( c_s \) and \( e_{ij}^s \) can be used to express different types of nodes. Beside setting different costs for sensor nodes of different quality by properly setting \( c_s \), the different sensing ability can be quantified by including a parameter \( \eta_s \) into \((1)\). The new definition is

\[
I_s(p) = \eta_s \frac{\lambda}{d(p,s)}^{1/\gamma},
\]

where \( \eta_s \) indicates the sensing gain achieved by sensor \( s \) with respect to a reference sensor. The value of \( I_s(p) \) is then used to compute the exposure contribution \( e_{ij}^s \) of sensor \( s \) to grid edge \( i,j \) according to \((2)\). A typical scenario is depicted in Fig. 5(a) where two types of sensor nodes are used to monitor the area. One costs three times as much as the other, but with a sensing gain \( \eta_s \) equal to 3, instead of 1.

We consider sensors with a field of view that consists in a circular sector of width \( \alpha \). This implies that during the deployment phase, in addition to the location, the orientation of each sensor must be optimized. Therefore, we extend the decision variables \( y_s \) to include such information. The new binary variables are \( y_{ij}^a \), which are equal to 1 if a sensor is installed in candidate site \( s \) with orientation \( a \). The orientation can be chosen among a set of predefined angles, \( \mathcal{A} \). Similarly, parameter \( e_{ij}^s \) is extended to \( e_{ij}^{s,a} \), which indicates the exposure contribution of a sensor installed in candidate site \( s \) to grid edge \((i,j)\) when it is oriented to direction \( a \). Finally, models must be modified replacing each sum \( \sum_{s \in S_{ij}} e_{ij}^s y_s \) with \( \sum_{s \in S_{ij}} \sum_{a \in \mathcal{A}} e_{ij}^{s,a} y_{ij}^a \) and including the following set of constraints that force a single orientation to be chosen for each sensor node

\[
\sum_{a \in \mathcal{A}} y_{ij}^a \leq 1 \quad \forall s \in \mathcal{S}
\]

An example of planned network where \( \mathcal{A} = \{ \pi/4, 3\pi/4, 5\pi/4, 7\pi/4 \} \) and \( \alpha = \pi/2 \) is reported in Fig. 5(b).

#### V. EXPOSURE AND DETECTION PROBABILITY \((E_2)\)

We now consider the version of the intrusion detection problem with the objective of maximizing the detection probability of a mobile target that traverses the protected area (as defined in \((4)\)), subject to a budget constraint on the total sensors’ installation cost. More precisely, we want to maximize the detection probability of any \( o-t \) path \( P \) with the lowest \( E_2(P) \) in the underlying grid \( G \).

Finding a path with the lowest detection probability for a given sensor deployment is equivalent to finding a path with maximum probability of non-detection which, according to \((4)\), is given by

\[
1 - E_2(P) = \prod_{p \in P} [1 - D(p)],
\]

where \( D(p) \) is the probability of detection in point \( p \). By taking the logarithm on both sides, we have

\[
\log(1 - E_2(P)) = \sum_{p \in P} \log(1 - D(p)) \quad \text{(41)}
\]

Since the probability of non-detection tends to 1 when \( \log(1 - E_2(P)) \to 0 \), we must determine a path \( P \) that minimizes

\[
|\log(1 - E_2(P))| = \sum_{p \in P} |\log(1 - D(p))|. \quad \text{(42)}
\]

Now, common sensor devices survey the area around them at a given rate. If the target moving along an arc \((i,j)\) is sampled at several points \( p_1, \ldots, p_r \), the above problem reduces to finding a shortest path from \( o \) to \( t \) in \( G \) with respect to the arc weights \( w_{ij} = \sum_{l=1}^r |\log(1 - D(p_l))| \) [35].

In order to maximize the detection probability, we have to maximize the total weight of the above shortest path. A formulation for the basic problem version with the detection
probability $E_2$ can thus be obtained by replacing the right-hand side of (14) in formulation ($F_1$) with
\[ \sum_{i=1}^r \left| \log \left( 1 - D(p_{ij}^l | \mathbf{y}) \right) \right|, \] (43)
where $p_{ij}^l$ denotes the $l$-th sampling point on the arc $(i, j)$, and $\mathbf{y}$ the 0–1 incidence vector of the installed sensors. It is worth pointing out that the resulting formulation is non-linear because the computation of the above probabilities cannot be expressed as the sum of contributions related to single installed sensors. Similar formulations can be derived for variants with connectivity requirements and capacities.

The non-linearity prevents from solving the problem to the optimality with classical MILP methods. However, we can obtain sub-optimal solutions using the Tabu Search (TS) algorithms described in Sec. III-B. Indeed, the rationale of trying to improve an initial solution by iteratively installing and removing sensors selected on the basis of their exposure contribution can be applied to this version as well. Note that TS algorithms rely on the exposure definition only when CSs/sensors are ordered according to the contribution to the path $P_k$ exposure, $E_1(P_k)$, and when the minimum exposure of a solution is evaluated. Therefore, we can run the same algorithms replacing the way exposure contributions and minimum path exposure are computed, i.e., using (3) and (4) instead of (1) and (2).

We now compare the solutions obtained with the two exposure definitions $E_1$ and $E_2$. For the exposure definition based on $E_2$ in (4), the variance of AWGN noise is set to 0.01 and the detection threshold to 0.5.

Table VI reports the probability of non-detection (defined as in $E_2$) of the solution obtained by running the TS for $E_1$ and then computing $E_2$ on the output, and the one obtained with TS directly for $E_2$. Results are averaged over 20 instances of the same size with a budget of $B = 30, 40, 50$. Note that the probability of non-detection drastically drops (from $10^{-4}$ to $10^{-16}$) while the budget raises from 40 to 50 sensors. On the other hand, imposing a budget of $B = 30$ leads to scarcely protected networks (probability of non-detection around 0.45).

It is worth pointing out that the particular structure of the non-linear formulation may lead to difficulties in designing effective solution strategies. Indeed, the logarithm that allowed us to turn the product of edge probabilities along a path into a sum of arc weights may cause numerical problems. This occurs in particular when the non-detection probabilities approach zero, which of course is the interesting range from the application point of view.

Thus, the limited average discrepancy between the non-detection probability values deriving from the two alternative exposure definitions indicates that $E_1$ not only leads to compact MILP formulations, but also gives a reasonable quality approximation of the much "nastier" (from the modelling point of view) exposure definition $E_2$.

VI. RELATED WORK

The study of WSNs coverage capabilities has recently attracted much attention in the research community. The work appeared in the field generally differentiate on the basis of the specific metric adopted to assess the coverage quality. Two coverage models are commonly adopted: a 0-1 discrete general model based on coverage ranges, and a continuous model based on the concept of exposure.

Works resorting the 0-1 coverage model address problems of geographical coverage of continuous areas or of discrete sets of points. Within this field, [3]–[6] study the optimal placement of sensors with the goal of minimizing the number of installed devices, while ensuring coverage of target points, and wireless connectivity among sensors. A more complex version of the area coverage problem is the k-coverage problem, in which each target point must be covered by k sensors, at least [2]. A probabilistic coverage has been proposed as well. Each covered point has a detection/coverage probability which depends on the type of sensing model, as in [7]. Unlike in these works, our optimal strategy is based on the concept of path exposure rather than on the 0-1 coverage model.

While the aforementioned papers aims at optimizing network topology, many other researchers have taken a different approach more oriented to the performance evaluation of predefined network topologies. Liu and Townsley [11] evaluate coverage quality when sensors are distributed according to a 2-D Poisson point process, while Kumar et al. [8] analyze the case of k-coverage considering deterministic and random deployment strategies. In [36], sensor density thresholds to ensure full coverage are derived, whereas, in [12], information theory is used to assess connectivity features of deterministic WSN deployments. Joint coverage and connectivity properties of regular deployment pattern are studied in [16, 17, and 18].

Wang et al. [23] and Zhang and Hou [10] study the conditions to achieve simultaneously coverage and connectivity in randomly drawn WSNs, and Huang et al. [9] extend the previous work studying the cases of k-coverage and k-connectivity, and k-coverage and 1-connectivity. Recently, Lazos and Poovendran have provided a comprehensive study on stochastic coverage in heterogeneous sensor network in [13], while bounds on the coverage of WSN with mobile nodes are obtained in [14]. Finally, Wettergren and Costa analyze in [15] the problem of finding the optimal sensor density distribution to maximize the network’s detection probability.

Since energy efficiency is central in any WSN, many papers [19–21, 23] have focused on extending the aforementioned pieces of work on coverage and connectivity to account for energy efficiency. The common approach here is to define the largest possible number of disjoint subsets of sensors, each of which fulfilling coverage and connectivity requirements, to be
activated in different time intervals.

Different from the previous references, [24]–[26] introduce and study the problem of designing WSNs for the detection of mobile targets traversing a sensitive region along specific paths. The work in [24], later on extended in [26], provides geometrical methods to find, in known network topologies, the maximal breach path (MBP) and the maximal support path (MSP) of an object that moves through the network. The MBP (MSP) corresponds to the worst (best) case of coverage, in the sense that maximizes (minimizes) the distance of the object to the closest sensor. The case of MSP is further studied in [28]. In [29] the deployment pattern that maximizes the number of intersections of a path with sensor coverage areas is studied.

A recent approach to the intrusion detection problem comes from the definition of barrier coverage. A sensor network, usually deployed over a boundary strip region, is said to provide \( k \)-barrier coverage if every path which completely crosses the width of the strip is covered by at least \( k \) distinct sensors, that is, it intersects at least \( k \) distinct coverage areas [38]. This is an interesting approach that brings together the 0-1 coverage (it counts the number of intersections) and the path coverage (it is not required to detect an object at every point in the area, but just at some points along its trajectory). Despite the similarities with the methodology used in this paper, the \( k \)-barrier coverage differentiates from the definition of exposure as it does not consider the length of the intersection with coverage areas and the distance between the path and the covering sensors. Similarly to classical approaches, there are works on quality assessment [37], optimal regular deployment patterns [38], sensor density thresholds to achieve \( k \)-barrier coverage in randomly deployed sensor networks [39], and energy-saving strategies [40].

The concept of exposure, extensively used throughout this paper, is first proposed in [25]. Besides defining exposure, authors provide an algorithm to calculate the minimal exposure path in a grid-based sensor network. The problem of calculating the minimal exposure path is also considered in [27].

Differently from us, these references assume given sensors positions, whereas we jointly optimize positions and minimum exposure.

Clouqueur et al. [31] extend the concept of exposure introducing AWGN noise in the sensing process, defining the detection probability along given paths. In [32] and [35], this new measure is used to find the optimal number of sensors to be randomly deployed to maximize the detection probability along paths traversing a region of interest. The same sensing/detection models is used in following papers [33], [34], where authors considers the case of regions of interest with obstacles.

Finally, in [44] the so-called shortest-path network interdiction problem is considered in the completely different field of transportation. The problem consists in using a budget in order to increase the length of the arcs of the network so as to maximize a shortest path from a source to a destination. In this case the weight of each arc has to be independently selected, while in (\( F \)) the weight (exposure) of each arc is affected by all the sensors installed in closeby CSs.

VII. CONCLUDING REMARKS

We have proposed an optimization framework to position the sensors of a WSN in order to achieve high detection quality along paths traversing the area of interest. Unlike in previous work based on coverage quality, the capability of the network to detect a mobile object moving along a given path depends on the distance from sensors and is measured by the path exposure.

We have derived compact mixed-integer linear programming formulations of different problem versions aiming at maximizing exposure or minimizing cost. The constraints on wireless connectivity among sensors and sensor nodes capacity have been taken into account as well as heterogeneous sensors with different sensing abilities. Moreover, we have introduced a non-linear variant based on a definition of path exposure that explicitly considers the detection probability.

We have shown that reasonable-sized instances of the linear formulations can be solved to optimality with a state-of-the-art MILP solver. To tackle large size instances of the linear and non-linear problems, we have also proposed heuristic algorithms. We have shown that our Tabu Search heuristics provide near-optimal solutions, within 5% of the optimum, and that they are able to deal with large size instances, with up to 1500 CSs, in reasonable computing time.

REFERENCES


