Channel Assignment Problem in Cellular Systems: A New Model and a Tabu Search Algorithm

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***Abstract*—The channel assignment in cellular systems has the task of planning the reuse of available frequencies in a spectrum efficient way. A classical approach to frequency assignment problems, when applied to the frequency planning of cellular networks, does not enable this task to be performed in an efficient way, since it does not consider the cumulative effect of interferers. In the paper, we propose a new model for the channel assignment problem in narrow-band cellular networks, which accounts for the cumulative effect of interferers. In this model, the service area is partitioned into regions and the propagation characteristics are assigned by means of the levels received in each region by the considered base stations (BS's). The objective is to maximize the sum of traffic loads offered by regions in which the ratio between the received power and the sum of powers received from interfering transmissions is above a threshold value. In the paper, we also present an algorithm, based on tabu search (TS) techniques, to solve this problem. This algorithm has been tested on some instances obtained by using a simple radio channel model and on a real world instance.**

***Index Terms*—Channel assignment problem, tabu search.**

I. Introduction

MOBILE cellular systems provide telecommunications services by means of a network of base stations (BS's) which can handle connections with mobile stations (MS's) within their service area (cell) using radio resources. Unfortunately, radio channels constitute a limited resource and therefore must be reused in the system according to some strategy [1]. The transmission in one cell can be successfully detected if the power of interfering signals due to transmissions in other cells is small enough.

The ability of the receiver to detect a signal in the presence of interference noise is known as *capture effect.* A common capture model assumes that a receiver can correctly detect a packet of information if the ratio between its received power Γ_0 and the sum of received powers from interfering transmissions Γ_i is greater than the *capture ratio* b [2]

$$\frac{C}{I} = \frac{\Gamma_0}{\sum_{i=1}^{n} \Gamma_i} > b. \tag{1}$$

In narrow-band cellular system with fixed channel assignment, C/I is kept above the capture ratio by means of a frequency planning strategy which assigns channels to BS's and must guarantee the following.

- C/I constraints are satisfied in all cells.
- The number of channels assigned to each cell guarantees the required grade of service (GoS).

Of course, the channel assignment policy is a critical factor on which the capacity of a cellular system depends [3]. For this reason, many optimization techniques have been proposed in order to build efficient channel assignment algorithms.

The solution to the channel assignment problem (CAP) has been given for regular networks of hexagonal cells [4]. In such a case, the optimal assignment is obtained by defining a *cluster* of cells to which all channels are assigned, and repeating this assignment pattern in the regular cell grid in a periodic fashion. This method cannot provide suitable solutions to the frequency planning problems arising in real-world systems [5]. Therefore, a more general approach has been developed which transforms the CAP into a graph coloring problem (see [6]). The model adopted by this approach [7] defines a compatibility matrix $C = \{c_{ij}\}$, which is an $n \times n$ symmetric matrix where n is the number of cells in the mobile network. Frequencies are assumed to be evenly spaced in the available bandwidth, so they can be identified with positive integers. The generic element of the compatibility matrix c_{ij} is the minimum frequency separation between the frequencies assigned to cells i and j. The number of channels needed for each cell is reported in the requirement vector $M = \{m_i\}$ where $i = 1, \cdots, n$. A frequency assignment is represented by an n-vector $F = \{F_i\}$ such that each F_i is a finite subset of the positive integers which defines the frequencies assigned to cell i. F is called admissible frequency assignment if

$$|F_i| = m_i, \qquad \forall i \tag{2}$$

$$|f - f'| \geq c_{ij}, \qquad \forall i, j: f \in F_i, f' \in F_j. \tag{3}$$

Constraints (2) require that the number of assigned frequencies to each cell be equal to the number needed to satisfy GoS requirements, while constraints (3) define electromagnetic compatibility between each couple of cells. The *span* of a frequency assignment is the largest integer contained in it, i.e., the number of frequencies used to satisfy all constraints. The solution of the channel assignment problem is the frequency assignment F with the minimum span. This problem is known to be NP hard [8].

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The CAP has already been investigated by many authors. The proposed algorithms are usually heuristic methods, even though it is possible to use partially enumerative algorithms [9] to achieve the exact solution. In [7], some lower bounds for the channel assignment problem are provided and in [10] a heuristic algorithm which use these bounds is presented. Algorithms derived from the graph coloring problems can be found in [4] and [8].

Another class of algorithms uses some heuristics of general applicability. Interesting applications of neural networks are presented in [11]–[13]. An evolutionary optimization approach has been used in [14]–[16] where genetics algorithms are proposed for the channel assignment problem, while in [17] a performance comparison of a genetic algorithm and a neural network is presented. Last, in [18] the problem is solved using simulated annealing.

The main drawback of the classical modeling approach is that it does not correctly mirror the interference situation in cellular radio systems for two principal reasons.

- With the compatibility matrix, the accumulated influence of several interferers is ignored.
- Cellular network is a multiple access system, thus, the interference compatibility must be guaranteed for all possible radio links between an MS and its BS in the service area where propagation conditions may be quite different from point to point.

For these reasons, the classical model can be viewed as a first approximation to the problem of finding a compatible and efficient frequency plan in mobile cellular networks, as already recognized by Gamst in [10]. Indeed, these model approximations may cause a misuse of system resources and the reduction of the capacity when the derived assignments are evaluated by using the actual C/I in the service area. Moreover, the definition of compatibility constraints starting from the information on the received signal levels in the service area may vary according to the *planning tool* used, and the criteria adopted to tune these tools are not reported in the literature (see [11]–[16]). A different model addressing the cumulative interference is presented in [19]: the objective of the optimization process is to maximize the minimum C/I in the network. Unfortunately, no details on the algorithmic solution are given in that paper.

In this paper, we propose a new model for the CAP in narrow-band cellular networks. The aim of the model is to overcome the limits of the classical approach, by addressing the objective of keeping the long term C/I above the capture ratio throughout the service area of the wireless network. In addition, we present a heuristic algorithm based on tabu search (TS) techniques [21]. The new model is presented in Section II. In Section III, we make a comparison of the characteristics and the efficiency of the classical and the new modeling approaches. In Section IV, a TS algorithm for the CAP is proposed. In Section V, some instances of the problem are defined in order to test the TS algorithm and to tune parameters. Moreover, a computational analysis is performed on a real-world instance with 43 cells and 24 frequencies. Finally, the paper is summarized and conclusions are given in Section VI.

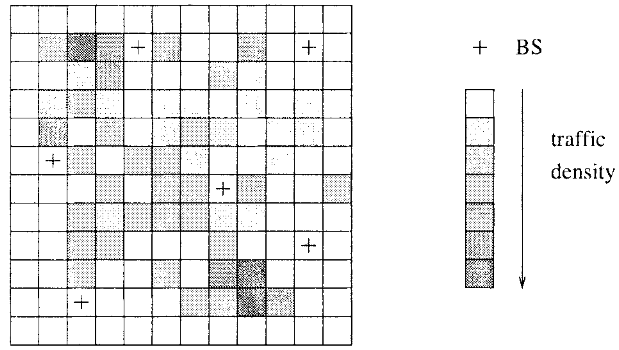

Fig. 1. Network service area.

II. The Model

The proposed model describes the network service area by means of a grid of regions characterized by a traffic density (Fig. 1). BS's are placed in the area and each of them serves a subset of regions.

The propagation characteristics are assigned by means of a $r \times n$ matrix of received levels $A = \{a_{ij}\}$, where r is the number of regions in the grid, n is the number of BS's, and a_{ij} ($a_{ij} > 0 \ \forall i, j$) is the long-term signal level which can be received in region i if a signal is transmitted by BS j. These values can be obtained with standard tools for the coverage prediction.

The coverage area of each BS is obtained by associating each region to the BS from which the highest level is received

$$r_i \in R_k, \qquad \text{if } \max_j a_{ij} = a_{ik} \tag{4}$$

where r_i is the ith region of the grid and R_k is the coverage area of BS k.

We associate an offered traffic η_i with each region; thus, the traffic load of a BS can be calculated as the sum of traffics offered by the regions of its coverage area. The n-vector $M = \{m_i\}$ defines the number of frequencies required by BS's. These values are obtained on the basis of the required GoS, the offered traffic load and the number of channels per carrier. With this information, the CAP can be represented with the following mathematical programming model.

A. Admissible Assignments

A frequency assignment is represented by an $n \times f$ matrix $X = \{x_{ij}\}$, with f being the number of frequencies available in the system and where the decision variables x_{ij} are defined as

$$x_{ij} = \begin{cases} 1, & \text{if frequency } j \text{ is assigned to base } i \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The GoS constraints are expressed as

$$\sum_{j=1}^{f} x_{ij} = m_i, \qquad \forall i = 1, \cdots, n. \tag{6}$$

The system may also require a minimum separation between frequencies assigned to the same BS k, i.e., if a frequency i is assigned to BS k, all frequencies in the range $[i - c_k, i + c_k]$

cannot be assigned to k. These requirements, called cosite constraints, are expressed as

$$\sum_{i=1}^{c_k} x_{k,(h+i)} \leq 1, \qquad h = 0, \cdots, f - c_k \tag{7}$$

where c_k is the minimum required separation.

The matrix X is an *admissible frequency assignment* if both GoS and cosite constraints are satisfied.

B. Objective Function

For a given admissible frequency assignment, we can evaluate the long term interference to carrier ratio I/C in each region: let r_i be a generic region belonging to the coverage area of BS k and h a frequency assigned to base k ($x_{kh} = 1$); if only cochannel interference is considered

$$\left(\frac{I}{C}\right)_{ih} = \frac{\sum_{j=1, j\neq k}^{n} a_{ij} x_{jh}}{a_{ik}}, \qquad i = 1, \cdots, r \quad h = 1, \cdots, f. \tag{8}$$

For all frequencies not assigned to base k it is assumed that $(I/C)_{ih} = \epsilon$, where ϵ is a small value. In order to take into account the adjacent channel interference, we define the net filter discriminator (NFD), which is the filter reduction constant for adjacent frequencies. The I/C expression now becomes

$$\left(\frac{I}{C}\right)_{ih} = \sum_{j=1, j\neq k}^{n} \frac{1}{a_{ik}} \left(a_{ij} x_{jh} + \frac{a_{ij}}{\text{NFD}} x_{j,h+1} + \frac{a_{ij}}{\text{NFD}} x_{j,h-1}\right). \tag{9}$$

Let b be the capture ratio, and $Q = \{q_i\}$ an r vector called *coverage vector* which defines the set of regions whose (C/I) is over the capture ratio

$$q_i = \begin{cases} 1, & \text{if } \left(\frac{I}{C}\right)_{ih} \leq \frac{1}{b} \quad \forall h = 1, \cdots, f \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

The assignment problem can, then, be formulated as the problem of maximizing the capacity of the system measured as the total served traffic, i.e.,

$$\max f(X) = \sum_{i=1}^{r} \eta_i q_i \tag{11}$$

where X varies among all the admissible frequency assignments and η_i denotes the offered traffic of region i.

This problem is NP hard since its decision version, asking if an admissible assignment exists such that in all regions it is possible to capture the signal, is NP complete (see Appendix).

III. Comparisons

The model presented in the previous section overcomes the limits of the classical model since it considers the cumulative effect of interferers and each region of the system serving area is analyzed. On the other hand, the model complexity is higher than that of the classical model, because, besides the number of available frequencies, the problem dimension is defined by the number of regions (i.e., grid definition) rather than the number of BS's. Of course, we can accept an higher complexity if it is possible to prove that the new model enables channels to be assigned in a more efficient way. To this end, we compare, from a theoretical point of view, the solutions which can be obtained by applying the two models. As a first step, in order to evaluate the efficiency of the classical approach we need to define the procedure which translates the C/I constraints into compatibility constraints. Commercial tools (see, for example, [20]) usually analyze a couple of cells at a time. For each region of one cell (e.g., cell i), the C/I is computed by considering only the interference from the base of the other cell (e.g., cell j), and the minimum frequency separation c_{ij} is obtained by means of an algorithm which summarizes all the constraints of the cell i in a merit figure. As a consequence, it is clearly possible for this method to consider as admissible also assignments which do not satisfy all the C/I constraints since the cumulative effect of interferers has been neglected when defining the compatibility matrix C. To see that, suffice it to consider the following example. A simple system has five BS's and a single region for each base. Let $M = [1, 2, 2, 2, 2]$, $f = 3$, and $b = 9$. The values of a_{ij} for $i, j = 1, 2, 3, 4, 5$ are reported in the following matrix A (note that $a_{ij} = a_{ji}$) together with a compatibility matrix C which can be derived by applying the rules above

$$A = \begin{bmatrix} 100 & 20 & 4 & 4 & 4 \\ 20 & 100 & 5 & 5 & 4 \\ 4 & 5 & 100 & 4 & 4 \\ 4 & 5 & 4 & 100 & 4 \\ 4 & 4 & 4 & 4 & 100 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Given the assignment $F_1 = \{1\}$ and $F_2 = \{2, 3\}$ for the first two BS's, there are 3^3 different assignments for the other BS's all respecting the compatibility constraints. But among them only two ($F_3 = \{1, 3\}$, $F_4 = \{1, 2\}$, $F_5 = \{2, 3\}$, and $F_3 = \{1, 2\}$, $F_4 = \{1, 3\}$, $F_5 = \{2, 3\}$) also satisfy the capture ratio constraints. As a consequence, with the classical model even infeasible assignments can be considered as feasible (and such a situation is usually approached with a *trial and error* tuning).

On the other hand, if we could apply some exhaustive search to define a set of compatibility constraints such that the derived assignments always satisfied the C/I constraints, then a different kind of drawback would arise. Let us consider, for example, a simple system with four BS's and a single region for each base. The values of a_{ij} for $i, j = 1, 2, 3, 4$ are reported in the following matrix A:

$$A = \begin{bmatrix} 100 & 5 & 20 & 5 \\ 5 & 100 & 5 & 5 \\ 20 & 5 & 100 & 5 \\ 5 & 5 & 5 & 100 \end{bmatrix}.$$

Setting the capture ratio $b = 9$, it is easy to show that, according to the C/I constraints for region 1, base 1 can use the same frequencies as bases 2 and 4, whereas it must use frequencies different from that of base 3. Therefore, the row C_1 of the compatibility matrix is $[1, 0, 1, 0]$. In the same way, we can obtain $C_3 = [1, 0, 1, 0]$. On the contrary, according to the C/I constraints for region 2, base 2 can use the same frequencies as two other bases arbitrarily chosen. The row C_2 can be $[0, 1, 0, 1]$ or $[0, 1, 1, 0]$ or $[1, 1, 0, 0]$. In the same way, we can have $C_4 = [0, 0, 1, 1]$ or $C_4 = [0, 1, 0, 1]$ or $C_4 = [1, 0, 0, 1]$. As a consequence, nine different compatibility matrices can be defined. One of these is

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

If $f = 2$ and $M = [1, 2, 1, 2]$, it is easy to verify that no assignment satisfies all compatibility constraints. Moreover, the use of all the other eight compatibility matrices produces the same result. With this model, the only way to satisfy all compatibility constraints is to modify the traffic requirements by setting $M = [1, 1, 1, 1]$.

Nevertheless, if the new model is adopted the assignment

$$X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

enables the C/I constraints to be satisfied for all regions, with a coverage vector $Q = [1, 1, 1, 1]$, while respecting the original GoS requirements.

As a consequence, the optimal solution of the classical model can be characterized by a capacity (number of channels per cell) lower than that of the optimal solution of the proposed model.

A final point has to be considered. Both the classical and the new model define NP-hard problems, but in the new model a higher number of variables and constraints are required. However, we believe that it is worthwhile to try to define good algorithms for a problem formulation which will enable a higher reuse degree to be achieved. Indeed, we believe that good algorithms for the new CAP will produce more efficient assignments than that produced by adopting the classical CAP. The next section presents the proposed algorithm.

IV. Tabu Search Approach

Tabu search is a *metaheuristic* that guides a local heuristic search procedure to explore the solution space beyond local optimality. It is based on the use of prohibition techniques and "intelligent" schemes which exploit the past history of the search in order to influence its future steps. The modern TS paradigm derives from work by Glover [21], [22] with seminal ideas and contributions from various sources.

The basis for TS may be described as follows. Let S denote the set of the feasible solutions of a problem. With each $s \in S$, we associate a subset of S, called *neighborhood* of s and denoted with $N(s)$. The neighborhood of s contains all those solutions which can be obtained with a (simple) modification of s, called *move*. Given a starting feasible solution s, ordinary local search iterates the following steps: 1) compute $N(s)$; 2) select within $N(s)$ the solution s_N with the best objective function value; and 3) if such value is better than that of s then replace s with s_N and go to step 1), otherwise stop and return s. The final solution is clearly a local optimum with respect to the defined neighborhood. To go beyond local optimality TS continues the search by performing even nonimproving moves and adopting a memory structure to avoid a cyclic behavior. Given a starting feasible solution s, at each iteration TS computes $N(s)$, but selects the solution with the best objective function value within a *subset* of $N(s)$. The heuristics forbids the selection of a solution in $N(s)$ if it recognizes that solution as one selected in a previous iteration. We denote with $TN(s)$ the subset of $N(s)$ containing the forbidden (*tabu*) solutions. To identify a solution, at each iteration TS records in a memory structure, called *tabu list*, some information, called *attributes*, of the selected solution. Such information is not the complete structure of that solution, since otherwise we could not manage the tabu list after a few iterations. It is, instead, those aspects which are modified by the application of the move, i.e., the aspects by which the current solution differs from the selected one. However, since a few attributes are not enough to completely identify a solution, i.e., a recorded set of attributes is usually shared both by already visited solutions and by unexplored ones, the length T of the tabu list, called *tabu tenure*, is limited and the tabu list is managed with a *first in first out* policy. The tabu status of a solution is not an absolute, but can be overruled if certain conditions, called *aspiration criteria*, are verified. For example, a commonly adopted aspiration criterion accepts a tabu solution $s_T \in TN(s)$ if its objective function value is strictly better than that of all the solutions already explored. Such a condition easily guarantees that s_T has not already been selected. To stop the search the commonly adopted criterion is based on the total elapsed time or on the total number of iterations. A pseudoalgorithm for TS is shown in Table I.

In order to apply TS to the channel assignment problem presented in Section II, we need to define the basic local search procedure and an efficient initialization procedure.

A. Local Search

In our approach, the set $N(X)$ contains all the solutions which can be obtained from a feasible assignment X by swapping in only one BS an assigned frequency with an unused one. More formally, we define the distance between two vectors $d(x_1, x_2)$ as the minimum number of components variations able to change x_1 in x_2 and vice versa. Let $\tilde{x}_i$ be the ith row of an admissible solution $\tilde{X}$ of the problem (11),

TABLE I
PSEUDOALGORITHM FOR TS

```
Procedure Tabu-Search(x)

begin
x_opt := x;
f_opt := f(x);
while(termination criteria not satisfied) do
      begin
      evaluate f(.) for all x ∈ N(x);
      if (∃ x' | f(x') > (<)f_opt) then
         x_opt := x';
      else
         select x' best point in N(x) but not in TN(x)
      update tabu list;
      x := x';
      end
end
```

then the neighborhood $N(\tilde{X})$ can be denoted as

$$X \in N(\tilde{X}), \qquad \text{if} \qquad \sum_{i=1}^{n} d(x_i, \tilde{x}_i) \leq 2 \tag{12}$$

$$\text{and} \qquad \sum_{j=1}^{f} x_{ij} = m_i \quad \forall\, i = 1, \cdots, n. \tag{13}$$

With this definition, the cardinality of the neighborhood is $O(nf^2)$, and, in consequence, the evaluation of the objective function for all points of $N(X)$ may be heavy in terms of computation time. For this reason, we have defined a *reduction procedure* capable of reducing the number of points to be explored by excluding all sets of points for which the objective function is bounded by its value in $\tilde{X}$. For this purpose, the objective function is evaluated for a not admissible solution obtained from $\tilde{X}$ by removing a frequency from the set of frequencies assigned to one BS. The obtained value will be less or equal to the value in $\tilde{X}$. In particular, if this relation is verified with the sign of equality (which means that the set of regions over the capture ratio is constant), all the solutions obtained from $\tilde{X}$ by swapping that frequency with an unused one cannot have a better value of the objective function and do not need to be explored.

For the numerical examples presented in the next section, we have verified that this procedure is able to reduce the computation times considerably.

B. Initialization

The initialization module is used to generate a starting admissible solution. This starting point may be a random frequency assignment, but if this point is too far from the region where the best solutions are, we shall need many steps of the exploration phase merely to approach that region. For this reason, we have defined an initialization procedure capable of building up a good starting point in a short time.

For each BS k, the algorithm selects m_k frequencies. In the first phase of the procedure, each available frequency is assigned to a BS; therefore, at the end of this phase a subset of BS's have all the required frequencies, one BS may have only some of the required frequencies, and a subset of BS's have no frequencies. In the second phase, we apply the following steps.

1) Choose a BS k which has less than m_k frequencies assigned; if all BS's have completed their assignment, then *stop*.
2) Compute for each frequency h the sum of interference levels received in the service area

$$I_{kh} = \sum_{i|r_i \in R_k} \sum_{j \neq k} a_{ij} \cdot x_{jh}. \tag{14}$$

3) Put the frequencies in a sorted list in nondecreasing order of I_{kh} and assign the first frequencies of the list to BS k until the required number m_k is reached.
4) Go to step 1).

The aim of the procedure is to minimize the total interference level in the network. The admissible assignments obtained with this procedure are usually characterized by high values of the objective function. While assignments with a high level of total interference are usually characterized by low values of the objective function.

C. Tabu Search Algorithm

The optimization algorithm proposed for the channel assignment problem applies the TS techniques and is based on the local search and initialization procedures presented above.

The basic moves are defined as swaps of a used frequency with an unused one for one BS. At each iteration we associate with the current solution the following attributes: the BS and the couple of frequencies involved. For example, if the applied move sets as used an unused frequence $f1$ and as unused a used frequence $f2$ in BS $b1$, we record the attributes "$f1$, $f2$, $b1$." For a number T of iterations all the selected moves which propose the contemporary modification of $f1$ *and* $f2$ in the BS $b1$ are forbidden.

During the local phase, the above-described neighborhood *reduction procedure* is applied. This approach is appropriate only when improving solutions are reachable by means of admissible moves. If this is not the case, the whole neighborhood must be explored.

In order to keep computation times acceptably low, we have defined an algorithm which performs a partial sampling of the neighborhood. This *sampling procedure* is based on the compilation of the list of the *hot* frequencies, evaluating a modified objective function which considers one frequency at a time

$$\hat{q}_{ih} = \begin{cases} 1, & \text{if } \left(\dfrac{C}{I}\right)_{ih} \geq b \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

$$f_h(X) = \sum_{i=1}^{r} \hat{q}_{ih}(X)\eta_i. \tag{16}$$

The evaluation of $f_h(X)$ for all frequencies can be easily done in the same time required for the evaluation of $f(X)$. Frequencies are ordered in the list with nondecreasing values

TABLE II
TABU SEARCH ALGORITHM

```
Procedure CAP-TABUSEARCH(n, f, M, A);
X :=INIZ(n, f, M, A);
X_opt := X
FO_opt := eval-fo(X, A);
counter := 0;
while counter < MAXITER do
    begin
    S := 0; FO_best := 0;
    RedN := Reduce-Neigh(X, A);
    for X_i ∈ RedN do
        begin
        FO_temp :=eval-fo(X_i, A);
        if FO_temp > FO_opt then
          begin
          FO_opt := FO_temp; X_opt := X_i;
          FO_best := FO_temp; X_best := X_i; S := 1;
          end
        else if FO_temp > FO_best AND tabu(X, X_i) then
              begin
              FO_best := FO_temp; X_best := X_i;
              end
        end
    if S=0 then
        begin
        SamN = Sample-Neigh(X, A);
        for(X_i ∈ SamN) AND NOT(X_i ∈ RedN)
              AND tabu(X, X_i)) do
              begin
              FO_temp :=eval-fo(X, A);
              if (FO_temp > FO_best) then
                 begin
                 FO_best := FO_temp; X_best := X_i;
                 end
              end
        end
    update-tabu(X, X_best);
    X := X_best;
    end
return X_opt;
end;
```

of $f_h(X)$. The reduction of the neighborhood is obtained by taking into account only the solutions obtained with acceptable moves which swap one of the first K frequencies of the list with one of the subsequent $f-K$ frequencies. We have to observe that the sampling procedure and the reduction procedure are applied together in such a way that the algorithm is always able to reach local optima even if it does not explore all neighbors. The final TS algorithm is shown in Table II. Function *tabu*(X_1, X_2) returns one if the move from X_1 to X_2 is admissible and zero otherwise, while procedure *update-tabu*(X_1, X_2) updates the tabu list after a move, and procedure $INIZ(n, f, M, A)$ performs the initialization procedure described above. Function *Sample-Neigh* performs the *sampling procedure*, while *Reduce-Neigh* performs the *reduction procedure.*

In order to evaluate the performance of the sampling procedure, in the next section, some numerical results are presented which make a comparison between TS algorithms with a complete neighborhood and with a reduced one.

The main search parameter of the algorithm is the tabu list size T. In a first step, we used a fixed TS approach where T is assigned only at the beginning of the search and kept constant. In a subsequent step, we adopted an adaptive TS approach where T varies during the search according to system conditions. After each iteration, which improves the current objective function value, we decrease the tabu tenure T by one unit (obviously not allowing negative values). In all the other cases, we increase the tabu tenure by one unit. The aim of decreasing T is that of locally intensifying the search within those regions which are more promising, while the aim of increasing T is that of speeding up the departure from the neighborhood of already visited local minima.

V. COMPUTATIONAL ANALYSIS

In this section, we present a numerical analysis of the proposed algorithm. A direct comparison between the results obtained with our algorithm and those reported in the literature cannot be made. That is because, as already explained in Section III, since the translation from the C/I constraints to the compatibility constraints is not unique, the solution depends on the choice of the compatibility matrix. Moreover, in the literature [11]–[16] the A matrices corresponding to the used C matrices are not reported, therefore, the C/I values cannot be computed. However, since our problem is characterized by an upper bound, which is the total traffic offered to the system, we can always make a comparison with this bound.

In order to analyze the performance characteristics of the proposed algorithm and to validate our choices, we have defined three instances of the channel assignment problem. The system considered is described by a grid of 30 $\times$ 30 regions served by 10, 15, or 20 BS's. The received levels matrix has been calculated by adopting a simple radio channel model where only the deterministic path loss effect is taken into account. In particular, the channel attenuation due to path loss is expressed by d^η, where d is the distance between a BS and the center of a region and $\eta = 4$ is the *propagation loss exponent* [23]. In addition, we have assumed that all regions offer the same traffic, so the value of the objective function is uniquely determined by the number of regions (NR) whose C/I is over the capture ratio, set to 9 dB, for all frequencies. Each BS requires 4 of the 16 available frequencies; the *reuse factor* (ratio between the number of available frequencies and the number of frequencies used in each cell) being equal to four, the defined instances thus refer to a case in which the theory used for regular networks of hexagonal cells is not able to provide a solution that guarantees a value of C/I higher than 9 dB throughout the service area. The adopted algorithms have been developed using C++ language and SUN sparc 10 workstations have been used to obtain numerical results.

Table III presents the numerical results for the three instances defined above, for which the upper bound is 900, obtained by using fixed TS with different values of the parameter T, with 500 iterations and $K = 3$. As we can see, the value of the objective function is not sensitive to the value of T in the instance with ten BS's, while the values obtained for the instance with 20 BS's vary with the value of T in a significant way. The computation times are about 323 s for the

TABLE III
FIXED TS ALGORITHM RESULTS

T	NR - 10-BSs instance	NR - 15-BSs instance	NR - 20-BSs instance
50	849	888	833
75	849	890	835
100	849	892	848
125	849	890	865
150	849	892	861
175	849	890	862
200	849	887	854
225	849	887	850
250	849	887	846

TABLE IV
ADAPTIVE TS ALGORITHM RESULTS

instance	NR
10-BSs	849
15-BSs	892
20-BSs	861

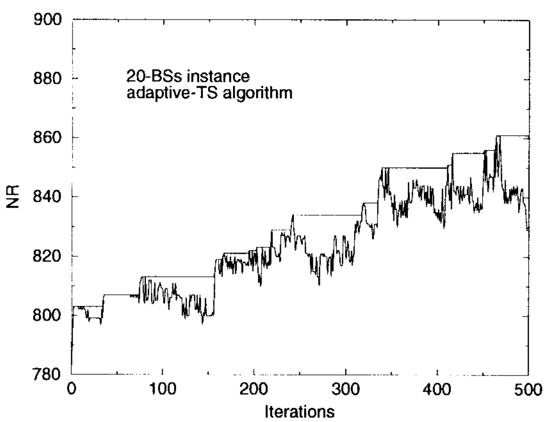

Fig. 2. NR versus iteration number: 20-BS's instance with adaptive-TS algorithm.

ten BS's instance, 508 s for the 15 BS's instance, and 1017 s for the 20 BS's instance.

These results show that the tuning of T may be a key issue for the efficiency of the algorithm. Therefore, we have also analyzed the adaptive TS approach, described in the previous section, which does not need the tuning of T. The results obtained, shown in Table IV, are equal or very close to the maximum values achievable with fixed TS.

For a more complete analysis of the characteristics of our TS algorithm, in Fig. 2 the NR values versus the iteration number are shown. These curves point out how the algorithm enables exploration of state-space regions where the admissible solutions have increasing value of the objective function.

In order to verify the efficiency of the partial sampling procedure based on the *hot* frequencies list, the results for the algorithm that perform a complete exploration of the neighborhoods have been calculated with the same number of iterations as that adopted in the previous case. As shown in Table V, the values obtained are somewhat lower than that of Table IV, while computation times are, of course, higher.

The final validation of the presented algorithm has been performed using a real world instance.[1] The considered area

[1] The coverage data were provided by Italtel SpA.

TABLE V
ADAPTIVE TS ALGORITHM WITH COMPLETE NEIGHBORHOOD EXPLORATION

instance	NR	comp. time (s)
10-BSs	841	888
15-BSs	877	1672
20-BSs	841	3090

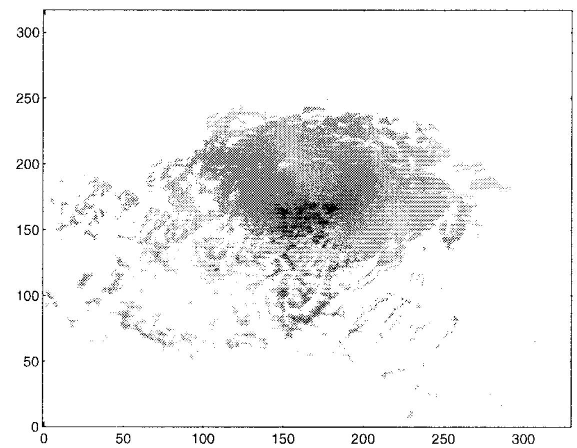

Fig. 3. Grid of received levels in a gray scale.

TABLE VI
REAL WORLD INSTANCE RESULTS

freq. per BS	NR
2	6647
3	6387
2/3	6571

is described by a 316 × 329 grid and is served by 43 BS's. All power levels lower than −140 dBm are considered undetectable by receivers and represented by zeros in the received levels matrix. Fig. 3 depicts the levels received in each region by the strongest BS. Only the 6649 regions which receive a level higher than −100 dBm from their BS's belong to the considered coverage area and, therefore, 6649 is also the upper bound of this instance.

The number of frequencies available for global system for mobile communications (GSM) systems in the real case considered is equal to 24, and the number of frequencies usually assigned to each BS is two or three. The results obtained using the adaptive TS algorithm with 2000 iterations are reported in Table VI in the cases in which all BS's require two or three frequencies and in the case in which the 18 BS's with the largest service area require three frequencies, while the other BS's require two frequencies.

For the last case, the NR values versus the iteration number are shown in Fig. 4. For all these cases, the computation times are about 5 h.

VI. CONCLUSION

In this paper, we have considered channel assignment in narrow-band cellular systems. The classical definition of the problem, borrowed from the problem of frequency assignment to a set of radio links, is based on the compatibility constraints which determine the minimum frequency separation between

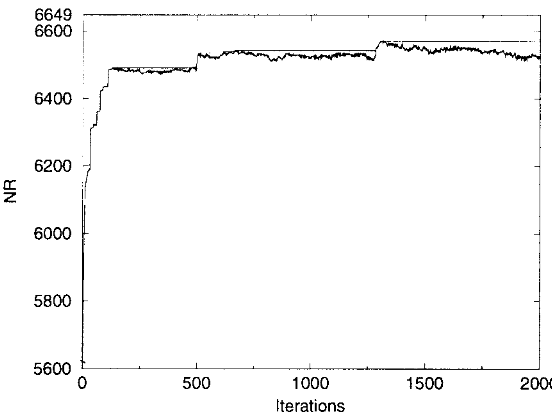

Fig. 4. NR versus the iteration number for the real world instance with adaptive-TS algorithm.

two channels assigned to a couple of BS's. The main drawback of this approach is that possible interferers are singly considered, while the cumulative effect of all possible interferers is neglected. This has driven our research to define a new model of the problem which considers all interferers by evaluating the carrier-to-interference ratio in the whole service area. Based on the capture effect, the objective function is expressed by the sum of the traffic loads offered by the regions of the service area whose C/I is over the capture ratio.

The dimensions of the new problem are bigger than that of the classical approach, since they are mainly defined by the number of regions considered in the service area rather than only by the number of BS's and the number of available frequencies. Nevertheless, only the new definition of the problem enables the maximization of the system's capacity to be defined as the objective of channel assignment.

The algorithm, proposed in the paper, is based on TS since these techniques perform an aggressive exploration of solution space which enables computation times to be reduced in comparison to techniques such as simulated annealing, when the problem is characterized by very large neighborhoods. We have tested the algorithm on some instances obtained by using a simple radio channel model and on a real world instance; it has revealed a good performance, since the obtained values are very close to the upper bound of the problem.

APPENDIX
NP COMPLETENESS OF THE PROBLEM

We can prove the NP completeness of the decision version of the problem by reducing the *three-dimensional matching* (3DM), known to be NP complete [24], to this problem.

The problem 3DM can defined as follows: given a finite set of elements $S = \{e_1, \cdots, e_n\}$ and a family $F = \{F_i\}$ of subsets of S such as $|F_i| = 3$ for all i, say if

$$\exists P \subseteq F\colon |P| = l = n/3,\, P_i \cap P_j = 0 \,\forall j \neq i,\, \bigcup_{i=1}^{l} P_i = S.$$

We consider a simplified version of the problem where the number n of BS's is equal to the number r of regions, the capture ratio is equal to one, each BS requires only one channel and the received level in a region from its own BS is equal to one.

Let the number f of available frequencies equal to l and the elements e_i be the regions of the simplified problem; the matrix A can be defined as

$$a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1/2, & \text{if } \exists F_k\colon e_i,\, e_j \in F_k \text{ and } i \neq j \\ T > 1, & \text{otherwise} \end{cases}$$

where elements a_{ii} identify the received level in region i from its own BS.

The correspondence between solutions of the simplified problem of channel assignment and the 3DM is assured by the fact that a solution of the simplified problem, if it exists, can only consist of sets of three BS's (and regions) which use the same frequency.

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