

On the Capacity of a Cellular CDMA System

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Abstract—The use of spread spectrum or code division techniques for multiple access (CDMA) has long been debated. Certain advantages, such as multipath mitigation and interference suppression are generally accepted, but past comparisons of capacity with other multiple access techniques were not as favorable. This paper shows that, particularly for terrestrial cellular telephony, the interference suppression feature of CDMA can result in a many-fold increase in capacity over analog and even over competing digital techniques.

I. INTRODUCTION

SPREAD-SPECTRUM techniques, long established for antijam and multipath rejection applications as well as for accurate ranging and tracking, have also been proposed for code division multiple access (CDMA) to support simultaneous digital communication among a large community of relatively uncoordinated users. Yet, as recently as 1985 a straightforward comparison [1] of the capacity of CDMA to that of conventional time division multiple access (TDMA) and frequency division multiple access (FDMA) for satellite applications suggested a reasonable edge in capacity for the latter two more conventional techniques. This edge was shown to be illusory shortly thereafter [2] when it was recognized that since CDMA capacity is only interference limited (unlike FDMA and TDMA capacities which are primarily bandwidth limited), any reduction in interference converts directly and linearly into an increase in capacity. Thus, since voice signals are intermittent with a duty factor of approximately 3/8 [3], capacity can be increased by an amount inversely proportional to this factor by suppressing (or squelching) transmission during the quiet periods of each speaker. Similarly, any spatial isolation through use of multibeamed or multisector antennas, which reduces interference, also provides a proportional increase in capacity. These two factors, voice activity and spatial isolation, were shown to be sufficient to render CDMA capacity at least double that of FDMA and TDMA under similar assumptions for a mobile satellite application [2].

While previous comparisons primarily applied to satellite systems, CDMA exhibits its greatest advantage over TDMA and FDMA in terrestrial digital cellular systems, for here isolation among cells is provided by path loss, which in terrestrial UHF propagation typically increases with the fourth power of the distance. Consequently, while conventional techniques must provide for different frequency allocation for

contiguous cells (only reusing the same channel in one of every 7 cells in present systems), CDMA can reuse the same (entire) spectrum for all cells, thereby increasing capacity by a large percentage of the normal frequency reuse factor. The net improvement in capacity, due to all the above features, of CDMA over digital TDMA or FDMA is on the order of 4 to 6 and over current analog FM/FDMA it is nearly a factor of 20.

The next section deals with a single cell system, such as a hubbed satellite network, and develops the basic expression for capacity. The subsequent two sections derive the corresponding expressions for a multiple cell system and determine the distribution on the number of users supportable per cell. The last section presents conclusions and system comparisons.¹

II. SINGLE CELL CDMA CAPACITY

The network to be considered throughout consists of numerous mobile (or personal) subscribers communicating with one or multiple cell sites (or base stations) which are interconnected with a mobile telephony switching office (MTSO), which also serves as a gateway to the public switched telephone network. We begin by considering a single cell system, which can also serve as a model for a satellite system whose "cell site" is a single hub.

Each user of a CDMA system occupies the entire allocated spectrum, employing a direct sequence spread spectrum waveform. Without elaborating on the modulation and spreading waveform, we assume generic CDMA modems at both subscriber units and the cell site with digital baseband processing units as shown in Fig. 1 for the transmitter sides of each. These consist of (digital) forward-error correction (FEC), modulation and (direct sequence) spreading functions, preceding the (analog) amplification and transmission functions. Each of the digital functions can be performed using binary sequences in the subscriber modulator.

At the cell-site transmitter, the spread signals directed to the individual subscribers are added linearly and phase randomness is assured by modulating each signal with independent pseudorandom sequences on each of the two quadrature phases. The weighting factors $\phi_1, \phi_2, \dots, \phi_N$ can be taken to be equal for the time being, but for the multiple cell case they will provide power control based on considerations to be

Manuscript received May 1, 1990; revised September 14, 1990. This paper was presented at the 1990 IEEE GLOBECOM Conference, San Diego, CA.

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IEEE Log Number 9144470.

¹It should be noted that our purpose is not to evaluate or optimize modem performance for the channels under consideration. Rather, assuming an efficient modulation and FEC code for the given channels, we shall establish conditions under which the modems will achieve an acceptable level of performance, particularly in terms of the maximum number of users supportable per cell.

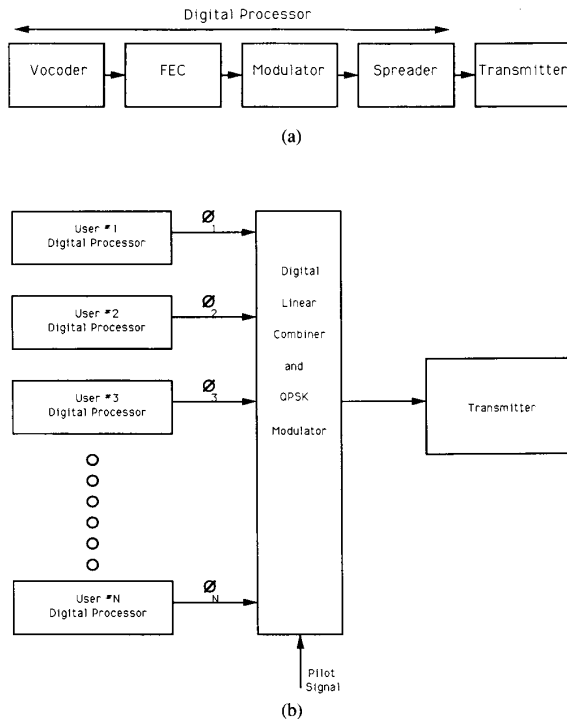


Fig. 1. Cellular system simplified block diagram. (a) Reverse link subscriber processor/transmitter. (b) Forward link cell-site processor/transmitter.

described later. The receiver processors in both subscriber and cell-site receivers provide the inverse baseband functions, which are of course considerably more complex than the transmitter baseband functions.

One other key feature of the cell-site transmitter is the inclusion of a pilot signal in the forward (cell-site-to-subscriber) direction. This provides for acquisition by the mobile terminals, including initial power control by the mobile, which adjusts its output power inversely to the total signal power it receives. Power control is a basic requirement in CDMA and will be expanded on in a later section.

We note also that the pilot signal is used by the subscriber demodulator to provide a coherent reference which is effective even in a fading environment since the desired signal and the pilot fade together. In the subscriber-to-cell-site (reverse) direction, no pilot is used for power efficiency considerations, since unlike the forward case, an independent pilot would be needed for each signal. A modulation consistent with, and relatively efficient for, noncoherent reception is, therefore, used for the reverse direction.

Without elaborating further on the system implementation details, we note that for a single cell site with power control, all reverse link signals (subscribers-to-cell site) are received at the same power level. For N users, each cell-site demodulator processes a composite received waveform containing the desired signal having power S and $(N - 1)$ interfering signals each also of power S . Thus the signal-to-noise (inter-

ference) power is

$$\text{SNR} = \frac{S}{(N - 1)S} = \frac{1}{N - 1}.$$

Of greater importance for reliable system operation is the bit energy-to-noise density ratio, whose numerator is obtained by dividing the desired signal power by the information bit rate, R , and dividing the noise (or interference) by the total bandwidth, W . This results in

$$E_b/N_0 = \frac{S/R}{(N - 1)S/W} = \frac{W/R}{N - 1}. \quad (1)$$

This paper does not explicitly address modulation techniques and their performance. Rather, an E_b/N_0 level is assumed which ensures operation at the level of bit error performance required for digital voice transmission. Among the factors to be considered in establishing the modulation and the resulting required E_b/N_0 level are phase coherence, amplitude fading characteristics and power control techniques and their effectiveness, particularly for the reverse link. One of the lesser considerations, albeit one of the most cited, is the probability distribution of the interfering signals. While Gaussian noise is often assumed, this is not strictly necessary to establish the E_b/N_0 requirements. Nonetheless, the assumption is quite reasonable when powerful forward error-correcting codes are employed, particularly at low code rates, because in such cases decisions are based on long code sequence lengths over which the interfering signal sequence contributions are effectively the sums of a large number of binomial variables, which closely approximate Gaussian random variables.

Equation (1) ignores background noise, η , due to spurious interference as well as thermal noise contained in the total spread bandwidth, W . Including this additive term in the denominator of (1) results in a required

$$E_b/N_0 = \frac{W/R}{(N - 1) + (\eta/S)}. \quad (2)$$

This implies that the capacity in terms of number of users supported is

$$N = 1 + \frac{W/R}{E_b/N_0} - \frac{\eta}{S} \quad (3)$$

where W/R is generally referred to as the "processing gain" and E_b/N_0 is the value required for adequate performance of the modem and decoder, which for digital voice transmission implies a BER of 10^{-3} or better. In words, the number of users is reduced by the inverse of the per user signal-to-noise ratio (SNR) in the total system spread bandwidth, W . In a terrestrial system, the per user SNR is limited only by the transmitter's power level. As will be justified below, we shall assume SNR just below unity corresponding to a reduction in capacity equivalent to removing one user. The background noise, therefore establishes the required received signal power at the cell site, which in turn fixes the subscriber's power or the cell radius for a given maximum transmitter power.

For the reverse (subscriber-to-cell-site) direction, noncoherent reception and independent fading of all users is assumed. With dual antenna diversity, the required $E_b/N_0 = 7$ dB for a relatively powerful (constraint length 9, rate 1/3) convolutional code. Since the forward link employs coherent demodulation by the pilot carrier which is being tracked, and since its multiple transmitted signals are synchronously combined, its performance in a single cell system will be much superior to that of the reverse link. For a multiple cell system however, other cell interference will tend to equalize performance in the two directions, as will be described below.

All this leaves us at the point of our previous conclusions [1], only worse because of the Rayleigh fading encountered in terrestrial mobile applications. In the next section we begin to remedy the situation.

III. AUGMENTED PERFORMANCE THROUGH SECTORIZATION AND VOICE-ACTIVITY MONITORING

Short of reducing E_b/N_0 through improved coding or possibly modulation, which rapidly reaches the point of diminishing returns for increasing complexity (and ultimately the unsurmountable Shannon limit), we can only increase capacity by reducing other user interference and hence the denominator of (1) or (2). This can be achieved in two ways.

The first is the common technique of sectorization, which refers to using directional antennas at the cell site both for receiving and transmitting. For example, with three antennas per cell site, each having 120° effective beamwidths, the interference sources seen by any antenna are approximately one-third of those seen by an omnidirectional antenna. This reduces the $(N - 1)$ term in the denominator of (2) by a factor of 3 and consequently, in (3) N is increased by nearly this factor. Henceforth, we shall take N_s to be the number of users per sector and the interference to be that received by one sector's antenna. Using three sectors, the number of users per cell $N = 3N_s$.

Secondly, voice activity can be monitored, a function which virtually already exists in most digital vocoders, and transmission can be suppressed for that user when no voice is present. Extensive studies show that either speaker is active only 35 % to 40 % of the time [3]. We shall assume for this the "voice activity factor," $\alpha = 3/8$ throughout. On the average, this reduces the interference term (in the denominator of (2)) from $(N - 1)$ to $(N - 1)\alpha$. Below, we will find through a more careful analysis that the net improvement in capacity due to voice activity is reduced from 8/3 to about 2 due to the fact that with a limited number of calls per sector, there is a nonnegligible probability that an above average number of users are talking at once. We ignore this in this preliminary discussion but include it in the results described below. Thus with sectorization and voice activity monitoring, the average $\overline{E_b/N_0}$, is increased relative to (2) to become²

$$\frac{\overline{E_b}}{N_0} = \frac{W/R}{(N_s - 1)\alpha + (\eta/S)}. \quad (4)$$

² These arguments leading to (4) were first advanced by Cooper and Nettleton [4].

This suggests that the average number of users per cell is increased by almost a factor of 8. In fact, because of variability in E_b/N_0 this increase will need to be backed off to a factor of 5 or 6. We shall return to this variability issue and other more precise results after we consider multiple cell interference in the next section. For now, note from (3) and (4) that this is enough to bring the number of users/cell up to the processing gain, $N \approx W/R$ users/cell which makes CDMA at least competitive with other multiple access techniques (FDMA or TDMA) on a single cell basis. As we will presently show, in multiple-cell systems additional advantages accrue through frequency reuse of the same spectrum in all cells. To assess this advantage, we must first consider the power control techniques and their effect on multicell interference.

IV. REVERSE LINK POWER CONTROL IN MULTIPLE-CELL SYSTEMS

As should be clear by now, power control is the single most important system requirement for CDMA, since only by control of the power of each user accessing a cell can resources be shared equitably among users and capacity maximized. In a single cell system, the principle is straightforward, though the implementation may not be. Prior to any transmission, each of the subscribers monitors the total received signal power from the cell site. According to the power level it detects, it transmits at an initial level which is as much below (above) a nominal level in decibels as the received pilot power level is above (below) its nominal level. Experience has shown that this may require a dynamic range of control on the order of 80 dB. Further refinements in power level in each subscriber can be commanded by the cell site depending on the power level it receives from the subscriber.

The relatively fast variations associated with Rayleigh fading may at times be too rapid to be tracked by the closed-loop power control but variations in relative path losses and shadowing effects, which are modelled as an attenuation with log-normal distribution, will generally be slow enough to be controlled. Also, while Rayleigh fading may not be the same for forward and reverse links, log-normal shadowing normally will exhibit reciprocity. For the forward link, no power control is required in a single cell system, since for each subscriber any interference caused by other subscriber signals remains at the same level relative to the desired signal; inasmuch as all signals are transmitted together and hence vary together, there are no resulting degradations due to fading assuming the background noise may be neglected.

In multiple-cell CDMA systems, the situation becomes more complicated in both directions. First, for the reverse link, subscribers are power controlled by the base station of their own cell. Even the question of cell membership is not simple. For it is not minimum distance which determines which base station (cell site) the subscriber joins, but rather the maximum pilot power among the cell sites the subscriber receives. In any case the interference level from subscribers in the other cells varies not only according to the attenuation in the path to the subscriber's cell site, but also inversely to the attenuation from the interfering user to his own cell site,

which through power control by that cell site may increase, or decrease, the interference to the desired cell site. These issues will be treated in the next section.

As for the forward link for a multiple cell system, interference from neighboring cell sites fade independently of the given cell site and thereby degrade performance for any level of interference. This becomes a particularly serious problem in the region where two or even three cell transmissions are received at nearly equal strengths. Techniques for mitigating this condition are treated in Section VI.

V. REVERSE LINK CAPACITY FOR MULTIPLE CELL CDMA

Recalling that power control to a given mobile is exercised by the cell whose pilot signal power is maximum to that mobile, it follows that if the path loss were only a function of distance from the cell site, then the mobile would be power controlled by the nearest cell site, which is situated at the center of the hexagon in which it lies, as shown in Fig. 2(a) for an idealized placement of cell sites. In fact, the loss is proportional to other effects as well, the most significant being shadowing. The generally accepted model is an attenuation which is the product of the fourth power of the distance and a log-normal random variable whose standard deviation is 8 dB. That is, the path loss between the subscriber and the cell site is proportional to $10^{(\xi/10)}r^{-4}$ where r is distance from subscriber to cell site and ξ is a Gaussian random variable with standard deviation $\sigma = 8$ and zero mean. Fast fading (due largely to multipath) is assumed not to affect the (average) power level.

We note that other propagation exponents can be found in different environments. In fact, within a single cell the propagation may vary from inverse square law very close to the cell antenna to as great as the inverse 5.5 power far from the cell in a very dense urban environment such as Manhattan. The present analysis is primarily concerned with interference from neighboring and distant cells so the assumption of inverse fourth law propagation is a reasonable one.

The interference from transmitter within the given subscriber's cell is treated as before; that is, since each user is power controlled by the same cell site, it arrives with the same power S , when active. Thus given N subscribers per cell, the total interference is never greater than $(N - 1)S$, but on the average it is reduced by the voice activity factor, α . Subscribers in other cells, however, are power controlled by other cell sites (Fig. 2(a)). Consequently, if the interfering subscriber is in another cell and at a distance r_m from its cell site and r_0 from the cell site of the desired user, the other user, when active, produces an interference in the desired user's cell site equal to

$$\begin{aligned} \frac{I(r_0, r_m)}{S} &= \left(\frac{10^{(\xi_0/10)}}{r_0^4} \right) \left(\frac{r_m^4}{10^{(\xi_m/10)}} \right) \\ &= \left(\frac{r_m}{r_0} \right)^4 10^{(\xi_0 - \xi_m)/10} \leq 1 \end{aligned} \quad (5)$$

where the first term is due to the attenuation caused by

distance and blockage to the given cell site, while the second term is the effect of power control to compensate for the corresponding attenuation to the cell site of the out-of-cell interferer.³ Of course ξ_0 and ξ_m are independent so that the difference has zero mean and variance $2\sigma^2$. For all values of the above parameters, the expression is less than unity, for otherwise the subscriber would switch to the cell site which makes it less than unity (i.e., for which the attenuation is minimized).

Then, assuming a uniform density of subscribers, and normalizing the hexagonal cell radius to unity, and since the average number of subscribers/cell is $N = 3N_s$, the density of users is

$$\rho = \frac{2N}{3\sqrt{3}} = \frac{2N_s}{\sqrt{3}} \text{ per unit area.} \quad (6)$$

Consequently, the total other-cell user interference-to-signal ratio is

$$I/S = \iint \psi \left(\frac{r_m}{r_0} \right)^4 \{ 10^{(\xi_0 - \xi_m)/10} \} \cdot \mathcal{O}(\xi_0 - \xi_m, r_0/r_m) \rho dA \quad (7)$$

where m is the cell-site index for which

$$r_m^4 10^{-\xi_m} = \min_{k \neq 0} r_k^4 10^{-\xi_k} \quad (8)$$

and

$$\mathcal{O}(\xi_0 - \xi_m, r_0/r_m) = \begin{cases} 1, & \text{if } (r_m/r_0)^4 10^{(\xi_0 - \xi_m)/10} \leq 1 \\ & \text{or } \xi_0 - \xi_m \leq 40 \log_{10}(r_0/r_m) \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

and ψ is the voice activity variable, which equals 1 with probability α and 0 with probability $(1 - \alpha)$. To determine the moment statistics of the random variable I , the calculation is much simplified and the results only slightly increased if for m we use the smallest distance rather than the smallest attenuation. Thus (7), with (9), holds as an upper bound if in place of (8) we use that value of m for which

$$r_m = \min_{k \neq 0} r_k. \quad (8')$$

In Appendix I, it is shown that the mean or first moment, of the random variable I/S is upper bounded (using (8') rather than (8) for m) by the expression

$$E(I/S) = \alpha \iint \frac{r_m^4}{r_0} f\left(\frac{r_m}{r_0}\right) \rho dA$$

where

$$f\left(\frac{r_m}{r_0}\right) = \exp\left[(\sigma \ln 10/10)^2\right] \left\{ 1 - Q\left[\frac{40}{\sqrt{2}\sigma^2} \cdot \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2}\sigma^2 \frac{\ln 10}{10}\right] \right\} \quad (10)$$

³Cooper and Nettleton [4] employed similar geometric arguments to compute interference, but did not consider log-normal statistical variations due to blockage.

and

$$Q(x) = \int_x^\infty e^{-y^2/2} dy / \sqrt{2\pi}.$$

This integral is over the two-dimensional area comprising the totality of all sites in the sector (Fig. 2(a)). The integration, which needs to be evaluated numerically, involves finding for each point in the space the value of r_0 , the distance to the desired cell site and r_m , which according to (8), is the distance to the closest cell site, prior to evaluating at the given point the function (10). The result for $\sigma = 8$ dB is

$$E(I/S) \leq 0.247N_s.$$

Calculation of the second moment, $\text{var}(I/S)$ of the random variable requires an additional assumption on the second-order statistics of ξ_0 and ξ_m . While it is clear that the relative attenuations are independent of each other, and that both are identically distributed (i.e., have constant first-order distributions) over the areas, their second-order statistics (spatial correlation functions) are also needed to compute $\text{var}(I)$. Based on experimental evidence that blockage statistics vary quite rapidly with spatial displacement in any direction, we shall take the spatial autocorrelation functions of ξ_0 and ξ_m to be extremely narrow in all directions, the two-dimensional spatial equivalent of white noise. With this assumption, we obtain in Appendix I that

$$\text{var}(I/S) \leq \iint \left(\frac{r_m}{r_0}\right)^8 \left[\alpha g\left(\frac{r_m}{r_0}\right) - \alpha^2 f\left(\frac{r_m}{r_0}\right) \right] \rho dA$$

where

$$g\left(\frac{r_m}{r_0}\right) = \exp \left[\left(\frac{\sigma \ln 10}{5} \right)^2 \right] \left\{ 1 - Q \left[\frac{40}{\sqrt{2\sigma^2}} \right. \right. \\ \left. \left. \cdot \log_{10} \left(\frac{r_0}{r_m} \right) - \sqrt{2\sigma^2} \left(\frac{\ln 10}{5} \right) \right] \right\}. \quad (11)$$

This integral is also evaluated numerically over the area of Fig. 2(a), with r_m defined at any given point by condition (8). The result for $\sigma = 8$ dB is $\text{var}(I/S) \leq 0.078N_s$. The above argument also suggests that I , as defined by (7), being a linear functional on a two-dimensional white random process, is well modelled as a Gaussian random variable.⁴

We may now proceed to obtain a distribution on the total interference, both from other users in the given cell, and from other-cell users on the desired user's reverse link transmission. With sectorization, variable voice activity and the other-cell interference statistics just determined, the received E_b/N_0 on the reverse link of any desired user becomes the random variable

$$E_b/N_0 = \frac{W/R}{\sum_{i=1}^{N_s-1} \chi_i + (I/S) + (\eta/S)} \quad (12)$$

where N_s is the users/sector and I is the total interference from users outside the desired user's cell. This follows easily

⁴Of course, it can never be negative, but since the ratio of mean-to-standard deviation is approximately $\sqrt{N_s}$, with typical values of $N_s > 30$ the approximating Gaussian distribution is nearly zero for negative values.

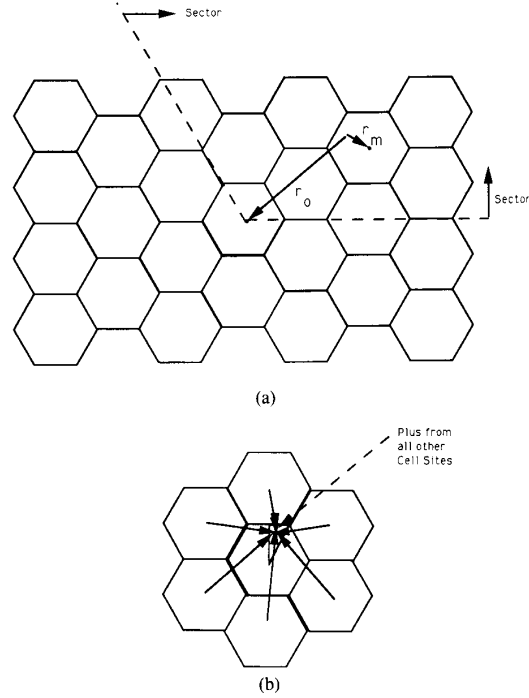


Fig. 2. Capacity calculation geometrics. (a) Reverse link geometry. (b) Forward link allocation geometry.

from (2) with the recognition that the $N_s - 1$ same sector normalized power users, instead of being unity all the time, now are random variables χ_i with distribution

$$\chi_i = \begin{cases} 1, & \text{with probability } \alpha \\ 0, & \text{with probability } 1 - \alpha. \end{cases} \quad (13)$$

The additional term I represents the other (multiple) cell user interference for which we have evaluated mean and variance,

$$E(I/S) \leq 0.247N_s \quad \text{and} \quad \text{var}(I/S) \leq 0.078N_s \quad (14)$$

and have justified taking it to be a Gaussian random variable. The remaining terms in (12), W/R and S/η , are constants.

As previously stated, with an efficient modem and a powerful convolutional code and two-antenna diversity, adequate performance ($\text{BER} < 10^{-3}$) is achievable on the reverse link with $E_b/N_0 \geq 5$ (7 dB). Consequently, the required performance is achieved with probability $P = \Pr(\text{BER} < 10^{-3}) = \Pr(E_b/N_0 \geq 5)$. We may lower bound the probability of achieving this level of performance for any desired fraction of users at any given time (e.g., $P = 0.99$) by obtaining an upper bound on its complement, which according to (12), depends on the distribution of χ_i and I , as follows

$$1 - P = \Pr(\text{BER} > 10^{-3}) = \Pr \left(\sum_{i=1}^{N_s} \chi_i + I/S > \delta \right) \quad (15)$$

where

$$\delta = \frac{W/R}{E_b/N_0} - \frac{\eta}{S}, \quad E_b/N_0 = 5.$$

Since the random variable χ_i has the binomial distribution

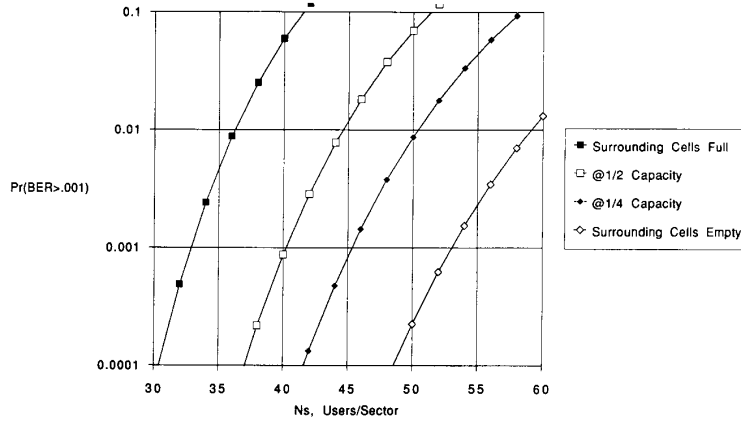


Fig. 3. Reverse link capacity/sector. ($W = 1.25$ MHz, $R = 8$ kb/s, voice activity = $3/8$).

given by (13) and I/S is a Gaussian variable with mean and variance given by (14) and all variables are mutually independent, (15) is easily calculated to be

$$1 - P = \sum_{k=0}^{N_s-1} \Pr(I/S > \delta - k | \sum x_i = k) \Pr(\sum x_i = k) \\ = \sum_{k=0}^{N_s-1} \binom{N_s-1}{k} \alpha^k (1 - \alpha)^{N_s-1-k} \\ \cdot Q\left(\frac{\delta - k - 0.247N_s}{\sqrt{0.078N_s}}\right). \quad (16)$$

This expression is plotted for $\delta = 30$ (a value chosen as discussed in the conclusion) and $\alpha = 3/8$, as the leftmost curve of Fig. 3. The rightmost curve applies to a single cell without other cell interference ($I = 0$), while the other intermediate curves assume that all cells other than the desired user's cells are on the average loaded less heavily (with averages of $1/2$ and $1/4$ of the desired user's cell).

We shall discuss these results further in the concluding section, and now concern ourselves with forward link performance.

VI. MULTIPLE-CELL FORWARD LINK CAPACITY WITH POWER ALLOCATION

As noted earlier, although with a single cell no power control is required, with multiple cells it becomes important, because near the boundaries of cells considerable interference can be received from other cell-site transmitters fading independently.

For the forward link, power control takes the form of power allocation at the cell-site transmitter according to the needs of individual subscribers in the given cell. This requires measurement by the mobile of its relative SNR, defined as the ratio of the power from its own cell site transmitter to the total power received. Practically, this is done by acquiring (correlating to) the highest power pilot and measuring its energy, and also measuring the total energy received by the mobile's omnidirectional antenna from all cell site transmitters. Both measurements can be transmitted to the

selected (largest power) cell site when the mobile starts to transmit. Suppose then that based on these two measurements, the cell site has reasonably accurate estimates of S_{T_1} and $\sum_{i=1}^K S_{T_i}$, where

$$S_{T_1} > S_{T_2} > \dots > S_{T_K} > 0 \quad (17)$$

are the powers received by the given mobile from the cell site sector facing it, assuming all but K (total) received powers are negligible. (We shall assume hereafter that all sites beyond the second ring around a cell contribute negligible received power, so that $K \leq 19$.) Note that the ranking indicated in (17) is not required of the mobile—just the determination of which cell site is largest and hence which is to be designated T_1 .

The i th subscriber served by a particular cell site will receive a fraction of S_{T_1} the total power transmitted by its cell site, which by choice and definition (17) is the greatest of all the cell site powers it receives, and all the remainder of S_{T_1} as well as the other cell site powers are received as noise. Thus its received E_b/N_0 can be lower bounded by

$$\left(\frac{E_b}{N_0}\right)_i \geq \frac{\beta \varnothing_i S_{T_1} / R}{\left[\left(\sum_{j=1}^K S_{T_j}\right)_i + \eta\right] / W} \quad (18)$$

where S_{T_j} is defined in (17), β is the fraction of the total cell site power devoted to subscribers ($1 - \beta$ is devoted to the pilot) and \varnothing_i is the fraction of this devoted to subscriber i . Because of the importance of the pilot in acquisition and tracking, we shall take $\beta = 0.8$. It is clear that the greater the sum of other cell-site powers relative to S_{T_1} , the larger the fraction \varnothing_i which must be allocated to the i th subscriber to achieve its required E_b/N_0 . In fact, from (18) we obtain

$$\varnothing_i \leq \frac{(E_b/N_0)_i}{\beta W/R} \left[1 + \left(\frac{\sum_{j=2}^K S_{T_j}}{S_{T_1}}\right)_i + \frac{\eta}{(S_{T_1})_i} \right] \quad (19)$$

where

$$\sum_{i=1}^{N_s} \phi_i \leq 1 \quad (20)$$

since βS_{T_i} is the maximum total power allocated to the sector containing the given subscriber and N_s is the total number of subscribers in the sector. If we define the relative received cell-site power measurements as

$$f_i \triangleq \left(1 + \sum_{j=2}^K S_{T_j} / S_{T_1} \right)_i, \quad i = 1, \dots, N_s \quad (21)$$

then from (19) and (20) it follows that their sum over all subscribers of the given cell site sector is constrained by

$$\sum_{i=1}^{N_s} f_i \leq \frac{\beta W / R}{E_b / N_0} - \sum_{i=1}^{N_s} \frac{\eta}{S_{T_{1i}}} \triangleq \delta'. \quad (22)$$

Generally, the background noise is well below the total largest received cell site signal power, so the second sum is almost negligible. Note the similarity to δ in (15) for the reverse link. We shall take $\beta = 0.8$ as noted above to provide 20 % of the transmitted power in the sector to the pilot signal, and the required $E_b / N_0 = 5$ dB to ensure $\text{BER} \leq 10^{-3}$. This reduction of 2-dB relative to the reverse link is justified by the coherent reception using the pilot as reference, as compared to the noncoherent modem in the reverse link. Note that this is partly offset by the 1-dB loss of power due to the pilot.

Since the desired performance ($\text{BER} \leq 10^{-3}$) can be achieved with N_s subscribers per sector provided (22) is satisfied with $E_b / N_0 = 5$ dB, capacity is again a random variable whose distribution is obtained from the distribution of variable f_i . That is, the BER can not be achieved for all N_s users/sector if the N_s subscribers combined exceed the total allocation constraint of (22). Then following (15),

$$1 - P = \Pr(\text{BER} > 10^{-3}) = \Pr\left(\sum_{i=1}^{N_s} f_i > \delta'\right). \quad (23)$$

But unlike the reverse link, the distribution of the f_i , which depends on the sum of ratios of ranked log-normal random variables, does not lend itself to analysis. Thus we resorted to Monte Carlo simulation, as follows.

For each of a set of points equally spaced on the triangle shown in Fig. 2(b), the attenuation relative to its own cell center and the 18 other cell centers comprising the first three neighboring rings was simulated. This consisted of the product of the fourth power of the distance and the log-normally distributed attenuation

$$10^{(\epsilon_k/10)} r_k^{-4}, \quad k = 0, 1, 2, \dots, 18.$$

Note that by symmetry, the relative position of users and cell sites is the same throughout as for the triangle of Fig. 2(b). For each sample, the 19 values were ranked to determine the maximum (S_{T_i}), after which the ratio of the sum of all other 18 values to the maximum was computed to obtain $f_i - 1$. This was repeated 10 000 times per point for each of 65

equally spaced points on the triangle of Fig. 2(b). From this, the histogram of $f_i - 1$ was constructed, as shown in Fig. 4.

From this histogram the Chernoff upper bound on (23) is obtained as

$$\begin{aligned} 1 - P &= \min_{s>0} E \exp \left[s \sum_{i=1}^{N_s} f_i - s\delta' \right] \\ &= \min_{s>0} \left[(1 - \alpha) + \alpha \sum_k P_k \exp(s f_k) \right]^{N_s} e^{-s\delta'} \quad (24) \end{aligned}$$

where P_k is the probability (histogram value) that f_i falls in the k th interval. The result of the minimization over s based on the histogram of Fig. 4, is shown in Fig. 5.

VII. CONCLUSIONS AND COMPARISONS

Figs. 3 and 5 summarize performance of reverse and forward links. Both are theoretically pessimistic (upper bounds on probability). Practically, both models assume only moderately accurate power control.

The parameters for both links were chosen for the following reasons. The allocated total spread bandwidth $W = 1.25$ MHz represents ten percent of the total spectral allocation, 12.5 MHz, for cellular telephone service of each service provider, which as will be discussed below, is a reasonable fraction of the band to devote initially to CDMA and also for a gradual incremental transition from analog FM/FDMA to digital CDMA. The bit rate $R = 8$ kb/s is that of an acceptable nearly toll quality vocoder. The voice activity factor, 3/8, and the standard sectorization factor of 3 are used. For the reverse channel, the received SNR per user $S/\eta = -1$ dB reflects a reasonable subscriber transmitter power level. In the forward link, 20 % of each site's power is devoted to the pilot signal for a reduction of 1 dB ($\beta = 0.8$) in the effective processing gain. This ensures each pilot signal (per sector) is at least 5 dB above the maximum subscriber signal power. The role of the pilot, as noted above, is critical to acquisition, power control in both directions and phase tracking as well as for power allocation in the forward link. Hence, the investment of 20 % of total cell site power is well justified. These choices of parameters imply the choices $\delta = 30$ and $\delta' \approx 38$ in (16) and (24) for reverse and forward links, respectively.

With these parameters, according to Fig. 3, the reverse link can support over 36 users/sector or 108 users/cell, with 10^{-3} bit error rates better than 99 % of the time. This number becomes 44 users/sector or 132 users/cell if the neighboring cells are kept to half of this loading. The forward link according to Fig. 5, can do the same or better for 38 users/sector or 114 users/cell.

Clearly, if the entire cellular allocation is devoted to CDMA, these numbers are increased tenfold. Similarly, if a lower bit rate vocoder algorithm is developed, or if narrower sectors are employed, the number of users may be increased further.

Remaining with the parameters assumed, interesting comparisons can be drawn to existing analog FM/FDMA cellular systems as well as other proposed digital systems. First, the former employs 30-kHz channel allocation, and assuming 3

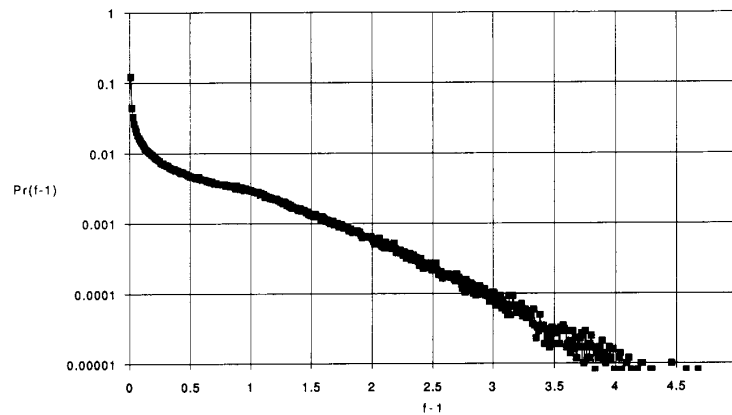
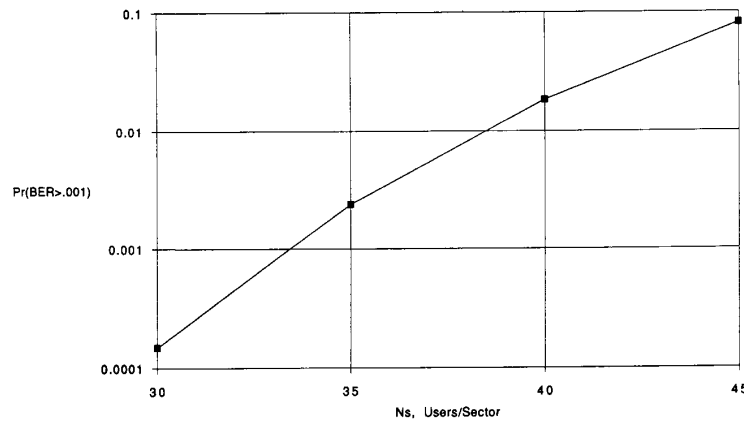


Fig. 4. Histogram of forward power allocation.

Fig. 5. Forward link capacity/sector. ($W = 1.25$ MHz, $R = 8$ kb/s, voice activity = $3/8$, pilot power = 20 %).

sectors/cell, requires each of the six contiguous cells in the first ring about a given cell to use a different frequency band. This results in a "frequency reuse factor" of $1/7$. Hence, given the above parameters, the number of channels in a 1.25-MHz band is slightly less than 42, and with a frequency reuse factor of $1/7$, this results in slightly less than 6 users/cell for a 1.25-MHz band. Thus, CDMA offers at least an eighteenfold increase in capacity. Note further that use of CDMA over just ten percent of the band supports over 108 users/cell whereas analog FM/FDMA supports only 60 users/cell using the entire 12.5 MHz band. Thus by converting *only 10 % of the band* from analog FDMA to digital CDMA, overall capacity is increased *almost threefold*.

Comparisons of CDMA with other digital systems are more speculative. However, straightforward approaches such as narrower frequency channelization with FDMA or multiple time slotting with TDMA can be readily compared to the analog system. The proposed TDMA standard for the U.S. is based on the current 30-kHz channelization but with sharing of channels by three users each of whom is provided one of three TDMA slots. Obviously, this triples the analog capacity but falls over a factor of 6 short of CDMA capacity.

In summary, properly augmented and power-controlled multiple-cell CDMA promises a quantum increase in current cellular capacity. No other proposed scheme appears to even approach this performance. Other advantages of CDMA not treated here include inherent privacy, flexibility in supporting multiple services and multiple voice and data rates, lower average transmit power requirements and soft limit on capacity, since if the bit error rate requirement is relaxed more users can be supported. With all these inherent advantages, CDMA appears to be the logical choice henceforth for all cellular telephone applications.

APPENDIX I REVERSE LINK OUTER-CELL INTERFERENCE

Outer-cell normalized interference, I/S , is a random variable defined by (7), (8), and (9), and upper bounded by replacing (8) by (8'). Then the upper bound on its first moment, taking into account also the voice activity factor of the outer-cell subscribers, α , becomes

$$E(I/S) \leq \int \int_{\text{sector}} \left(\frac{r_m}{r_0} \right)^4 E(\psi \cdot 10^{\chi/10} \mathcal{O}(\chi, r_0/r_m)) \rho dA$$

where r_m is defined by (8) for every point in the sector, $\psi = 1$ with probability α and 0 with probability $(1 - \alpha)$, and $\chi = \xi_0 - \xi_m$ is a Gaussian random variable of zero mean and variance $2\sigma^2$ with $\mathcal{O}(\chi, r_0/r_m)$ defined by (9),

$$\mathcal{O}(\chi, r_0/r_m) = \begin{cases} 1, & \text{if } \chi \leq 40 \log(r_0/r_m) \\ 0, & \text{otherwise.} \end{cases}$$

The expectation is readily evaluated as

$$\begin{aligned} \alpha f\left(\frac{r_m}{r_0}\right) &\triangleq E(\psi) E(e^{\chi \ln 10/10} \mathcal{O}(\chi, r_0/r_m)) \\ &= \alpha \int_{-\infty}^{40 \log r_0/r_m} e^{\chi \ln 10/10} \frac{e^{-\chi^2/4\sigma^2}}{\sqrt{4\pi\sigma^2}} d\chi \\ &= \alpha e^{(\sigma \ln 10/10)^2} \int_{-\infty}^{40 \log(r_0/r_m)} \frac{\exp\left[-\frac{1}{2}\left(\frac{\chi}{\sqrt{2\sigma^2}} - \sqrt{2\sigma^2} \ln 10/10\right)^2\right]}{\sqrt{2\pi(2\sigma^2)}} d\chi \\ &= \alpha e^{(\sigma \ln 10/10)^2} \left\{ 1 - Q\left[\frac{40 \log(r_0/r_m)}{\sqrt{2\sigma^2}} - \sqrt{2\sigma^2} \frac{\ln 10}{10}\right] \right\} \end{aligned}$$

which yields (10).

To evaluate $\text{var}(I/S)$, assuming the "spatial whiteness" of the blockage variable, we have

$$\text{var}(I/S) \leq \iint_{\text{sector}} \left(\frac{r_m}{r_0}\right)^8 \cdot \text{var}(\psi \cdot 10^{\chi/10} \mathcal{O}(\chi, r_0/r_m)) \rho dA.$$

Rewriting the variance in the integral as

$$\begin{aligned} &E(\psi^2) E[10^{2\chi/10} \mathcal{O}^2(\chi, r_0/r_m)] \\ &- \{E(\psi) E[10^{\chi/10} \mathcal{O}(\chi, r_0/r_m)]\}^2 \\ &= \alpha g\left(\frac{r_m}{r_0}\right) - \alpha^2 f^2\left(\frac{r_m}{r_0}\right) \end{aligned}$$

where $f(r_m/r_0)$ was derived above and

$$\begin{aligned} \alpha g\left(\frac{r_m}{r_0}\right) &= E(\psi^2) E[e^{\chi \ln 10/5} \mathcal{O}^2(\chi, r_0/r_m)] \\ &= \alpha e^{(\sigma \ln 10/5)^2} \left\{ 1 - Q\left[\frac{40 \log(r_0/r_m)}{\sqrt{2\sigma^2}} - \frac{\sqrt{2\sigma^2} \ln 10}{5}\right] \right\} \end{aligned}$$

which yields (11).

ACKNOWLEDGMENT

The authors gratefully acknowledge the contribution of Dr. Audrey Viterbi and Dr. Jack Wolf.

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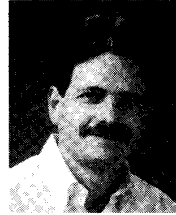
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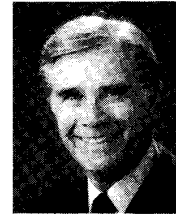
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