31 OPTIMIZATION PROBLEMS AND MODELS FOR PLANNING CELLULAR NETWORKS

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Abstract: During the last decade the tremendous success of mobile phone systems has triggered considerable technological advances as well as the investigation of mathematical models and optimization algorithms to support planning and management decisions. In this chapter, we give an overview of some of the most significant optimization problems arising in planning second and third generation cellular networks, we describe the main corresponding mathematical models, and we briefly mention some of the computational approaches that have been devised to tackle them. For second generation systems (GSM), the planning problem can be subdivided into two distinct subproblems: coverage planning, in which the antennas are located so as to maximize service coverage, and capacity planning, in which frequencies are assigned to the antennas so as to maximize a measure of the overall quality of the received signals. For third generation systems (UMTS) network planning is even more challenging, since, due to the peculiarities of the radio interface, coverage and capacity issues must be simultaneously addressed.
Keywords: Wireless communications, cellular networks, coverage, capacity, location problems, frequency assignment.

31.1 INTRODUCTION

Wireless communications, and in particular mobile phone systems, have rapidly pervaded everyday life. This success has triggered considerable technological advances as well as the investigation of mathematical models and optimization algorithms to support planning and management decisions. The main contribution of optimization in this field is to improve the way the scarce resources (e.g., transmission band, antennas) are used, and to enhance the service quality (e.g., bandwidth, transmission delay).

When planning and managing a cellular system, a number of aspects must be considered, including traffic estimation, signal propagation, antenna positioning, capacity allocation, transmission scheduling, power and interference control. Most of these aspects gives rise to interesting and challenging optimization problems which must account for the peculiarities of the specific network technology.

In this chapter, we summarize the most significant optimization problems arising in planning a cellular network, we describe the related mathematical models and mention some of the computational approaches that have been devised to tackle them. After recalling the main technological features and the most relevant telecommunication aspects (Section 31.2), we focus on second generation cellular systems (Sections 31.3 and 31.4). Since the corresponding planning problem is very challenging computationally, it is usually decomposed into two distinct phases: coverage planning (e.g., antennas location and transmission power selection) and capacity planning. Interference clearly plays a crucial role in the latter phase, in which frequencies have to be assigned to transmitters, and a wide class of mathematical models and of elegant solution methods have been proposed. Due to the peculiarities of air interface of third generation cellular systems, a two-phase approach is no longer appropriate: coverage planning and capacity planning have to be simultaneously addressed (Section 31.5). Different optimization models and algorithms are thus under investigation.

Although the management of cellular networks, in particular third generation ones, gives also rise to interesting optimization problems such as code assignment and packet scheduling, we do not consider this class of problems here and we refer for instance to Minn and Siu (2000), Agnetis et al. (2003), and Dell'Amico et al. (2004), and the references therein.

31.2 CELLULAR TECHNOLOGIES

Mobile phone systems provide telecommunication services by means of a set of base stations (BSs) which can handle radio connections with mobile stations (MSs) within their service area (Walke, 2001). Such an area, called *cell*, is the set of points in which the intensity of the signal received from the BS under consideration is higher than that received from the other BSs. The received power level depends on the transmitted power, on the attenuation effects of signal propagation from source to destination (path loss due to distance, multi-path effect, shadowing due to obstacles, etc.) as well as on the antenna characteristics and configuration parameters such as maximum emission power, height, orientation and diagram (Parsons, 1996). As a result, cells can have different shapes and sizes depending on BSs location and configuration parameters as well as on propagation.

When users move in the service area crossing cell boundaries, service continuity is guaranteed by handover procedures. During handovers, a connection is usually switched from a BS to a new one (hard-handover). In some cases, simultaneous connections with two or more BSs can be used to improve efficiency.

In order to allow many simultaneous connections between BSs and MSs, the radio band available for transmissions is divided into radio channels by means of a multiple access technique. In most of second generation systems (such as GSM and DAMPS) the radio band is first divided into carriers at different frequencies using FDMA (Frequency Division Multiple Access) and then on each carrier a few radio channels are created using TDMA (Time Division Multiple Access) (Walke, 2001). With bidirectional connections, a pair of channels on different carriers is used for transmissions from BS to MS (downlink) and from MS to BS (uplink), according to the FDD (Frequency Division Duplexing) scheme. BSs can use multiple frequencies by means of a set of transceivers (TRX).

Unfortunately, the number of radio channels obtained in this way (several hundreds in second generation systems) are not enough to serve the large population of mobile service users. In order to increase the capacity of the system the radio channels must be reused in different cells. This generates interference that can affect the quality of the received signals. However, due to the capture effect, if the ratio between the received power and the interference (sum of received powers from interfering transmissions), referred to as SIR (Signal-to-Interference Ratio), is greater than a capture threshold, SIR_{min}, the signal can be correctly decoded.

In order to guarantee that such a condition is satisfied during all system operations the assignment of radio channels to BSs must be carefully planned. Obviously, the denser the channel reuse, the higher the number of channels available per cell. Therefore, the channel assignment determines system capacity. Since usually BSs of second generation systems are not synchronized, radio channels within the same carrier cannot be assigned independently and only carriers are considered by the reuse scheme. For these reasons the process of assigning channels to cells is usually referred to as capacity planning or frequency planning. Although the main source of interference derives from transmissions on the same frequency (carrier), transmissions on adjacent frequencies may also cause interference due to partial spectrum overlap and should be taken into account.

Planning a mobile system involves selecting the locations in which to install the BSs, setting their configuration parameters (emission power, antenna height, tilt, azimuth, etc.), and assigning frequencies so as to cover the service area and to guarantee enough capacity to each cell. Due to the problem complexity, a two-phase approach is commonly adopted for second generation systems. First coverage is planned so as to guarantee that a sufficient signal level is received in the whole service area from at least one BS. Then available frequencies are assigned to BSs considering SIR constraints and capacity requirements.

Second generation cellular systems were devised mainly for the phone and low rate data services. With third generation systems new multimedia and high speed data services have been introduced. These systems, as UMTS (Holma and Toskala, 2000) and CDMA2000 (Karim et al., 2002), are based on W-CDMA (Wideband Code Division Multiple Access) and prior to transmission, signals are spread over a wide band by using special codes. Spreading codes used for signals transmitted by the same station (e.g., a BS in the downlink) are mutually orthogonal, while those used for signals emitted by different stations (base or mobile) can be considered as pseudo-random. In an ideal environment, the de-spreading process performed at the receiving end can completely avoid the interference of orthogonal signals and reduce that of the others by the spreading factor (*SF*), which is the ratio between the spread signal rate and the user rate. In wireless environments, due to multipath propagation, the interference among orthogonal signals cannot be completely avoided and the *SIR* is given by:

$$SIR = SF\,\frac{P_{received}}{\alpha I_{in} + I_{out} + \eta}, \tag{31.1}$$

where $P_{received}$ is the received power of the signal, I_{in} is the total interference due to the signals transmitted by the same BS (intra-cell interference), I_{out} that due to signals of the other BSs (inter-cell interference), α is the orthogonality loss factor $(0 \leq \alpha \leq 1)$, and η the thermal noise power. In the uplink case, no orthogonality must be accounted for and $\alpha = 1$, while in the downlink usually $\alpha \ll 1$.

The *SIR* level of each connection depends on the received powers of the relevant signal and of the interfering signals. These, in turn, depend on the emitted powers and the attenuation of the radio links between the sources and destinations. A power control (PC) mechanism is in charge of dynamically adjusting the emitted power according to the propagation conditions so as to reduce interference and guarantee quality. With a *SIR*-based PC mechanism each emitted power is adjusted through a closed-loop control procedure so that the *SIR* of the corresponding connection is equal to a target value SIR_{tar}, with $SIR_{tar} \geq SIR_{min}$ (Grandhi et al., 1995).

For third generation systems, a two-phase planning approach is not appropriate because in CDMA systems the bandwidth is shared by all transmissions and no frequency assignment is strictly required. The network capacity depends on the actual interference levels which determine the achievable *SIR* values. As these values depend in turn on traffic distribution, as well as on BSs location and configuration, coverage and capacity must be jointly planned.

31.3 COVERAGE PLANNING

The general *Coverage Problem* can be described as follows. Given an area where the service has to be guaranteed, determine where to locate the BSs and select their configurations so that each point (or each user) in the service area receives a sufficiently high signal. Since the cost associated to each BS may depend on its location and configuration, a typical goal is that of minimizing the total antenna installation cost while guaranteeing service coverage.

In the literature, the coverage problem has been addressed according to two main types of approaches. In the first one, the problem is considered from a continuous optimization point of view. A specified number of k BSs can be installed in any location of the space to be covered, possibly avoiding some forbidden areas, and antenna coordinates are the continuous variables of the problem. Sometimes also other parameters, such as transmission powers and antenna orientations, can be considered as variables. The crucial element of this type of approach is the propagation prediction model used to estimate the signal intensity in each point of the coverage area. The coverage area is usually subdivided into a grid of pixels, and for each pixel the amount of traffic is assumed to be known. The signal path loss from transmitter j to the center of pixel i is estimated according to a function $g_i(x_j, y_j, z_j)$ that depends on the transmitter coordinates x_j, y_j, z_j, the distance and the obstacles between the transmitter and the pixel. In the literature, many prediction models have been proposed, from the simple Okumura-Hata formulas (Hata, 1980) to the more sophisticated ray tracing techniques (Parsons, 1996). The objective function of the coverage problem is usually a combination of average and maximum-minimum signal intensity in each pixel or other measures of Quality of Service. If this objective function is denoted by $f(\mathbf{x},\mathbf{y},\mathbf{z})$, where $\mathbf{x},\mathbf{y},\mathbf{z}$ are the vector coordinates of the k BSs, the coverage problem is simply stated as follows:

$$\begin{array}{ll} \max & f(\mathbf{x},\mathbf{y},\mathbf{z}) \\ & 0 \le x_j \le h_1, 0 \le y_j \le h_2, 0 \le z_j \le h_3 \quad j = 1,\ldots,k, \end{array}$$

where the coverage area is the hyper-rectangle with sides h_1, h_2 and h_3.

Although these problems have simple box constraints, the very involved path loss functions, which cannot always be defined analytically, make them beyond the reach of classical location theory methods (Francis et al., 1992). Global optimization techniques were thus adapted to tackle them, as for example in Sherali et al. (1996) where an indoor optimal location problem is considered.

The alternative approach to the coverage problem is based on discrete mathematical programming models. A set of *test points* (TPs) representing the users are identified in the service area. Each TP can be considered as a traffic centroid where a given amount of traffic (usually expressed in Erlang) is requested (Tutschku et al., 1996). Instead of allowing the location of BSs in any position, a set of *candidate sites* (CSs) where BSs can be installed is identified. Even though parameters such as maximum emission power, antenna height, tilt and azimuth are inherently continuous, the antenna configurations can be discretized by only considering a subset of possible values. Since we can evaluate (or even measure in the field) the signal propagation between any pair of TP and CS for a BS with any given antenna configuration, the subset of TPs covered by a sufficiently strong signal is assumed to be known for a BS installed in any CS and with any possible configuration. The coverage problem then amounts to an extension of the classical minimum cost set covering problem, as discussed for instance in Mathar and Niessen (2000).

Let $S = \{1,\ldots,m\}$ denote the set of CSs. For each $j \in S$, let the set K_j index all the possible configurations of the BS that can be installed in CS j. Since the installation cost may vary with the BS configuration (e.g., its maximum emission power, or the antenna diagram), an installation cost c_{jk} is associated with each pair of CS j and BS configuration k, $j \in S$, $k \in K_j$. Let $I = \{1,\ldots,n\}$ denote the set of test points.

The propagation information is summarized in the attenuation matrix G. Let g_{ijk}, $0 < g_{ijk} \le 1$, be the attenuation factor of the radio link between test point i, $i \in I$, and a BS installed in j, $j \in S$, with configuration $k \in K_j$.

From the attenuation matrix G, we can derive a 0-1 incidence matrix containing the coverage information that is needed to describe the BS location and configuration problem. The coefficients for each triple TP i, BS j and configuration k are defined as follows:

$$a_{ijk} = \begin{cases} 1 & \text{if the signal of a BS installed in CS } j \text{ with configuration } k \text{ is} \\ & \text{sufficient to cover TP } i \\ 0 & \text{otherwise.} \end{cases}$$

Introducing the following binary variable for every pair of candidate site j and BS configuration k:

$$y_{jk} = \begin{cases} 1 & \text{if a BS with configuration } k \text{ is installed in CS } j \\ 0 & \text{otherwise,} \end{cases}$$

the problem of covering all the test points at minimum cost can be formulated as:

$$\min \quad \sum_{j \in S} \sum_{k \in K_j} c_{jk} y_{jk}$$

$$\sum_{j \in S} \sum_{k \in K_j} a_{ijk} y_{jk} \geq 1 \quad \forall i \in I \tag{31.2}$$

$$\sum_{k \in K_j} y_{jk} \leq 1 \quad \forall j \in S \tag{31.3}$$

$$y_{jk} \in \{0,1\} \quad \forall j \in S, \forall k \in K_j. \tag{31.4}$$

Constraints (31.2) ensure that all TPs are within the service range of at least one BS, and constraints (31.3) state that in each CS at most one configuration is selected for the base station.

This problem can be solved by adapting the algorithms for set covering, see Ceria et al. (1997). In practice, however, the covering requirement is often a “soft constraint” and the problem actually involves a trade-off between coverage and installation cost. In this case, constraints (31.2) are modified by introducing for each $i \in I$, an explicit variable z_i which is equal to 1 if TP i is covered and 0 otherwise. The resulting model, which falls within the class of maximum coverage problems, is then:

$$\max \quad \lambda \sum_{i \in I} z_i - \sum_{j \in S} \sum_{k \in K_j} c_{jk} y_{jk}$$

$$\sum_{j \in S} \sum_{k \in K_j} a_{ijk} y_{jk} \geq z_i \quad \forall i \in I \tag{31.5}$$

$$\sum_{k \in K_j} y_{jk} \leq 1 \quad \forall j \in S \tag{31.6}$$

$$y_{jk} \in \{0,1\} \quad \forall j \in S, \forall k \in K_j \tag{31.7}$$

$$z_i \in \{0,1\} \quad \forall i \in I, \tag{31.8}$$

where $\lambda > 0$ is a suitable trade-off parameter which allows to express both objectives in homogeneous economic terms. This problem can be efficiently solved by using, for instance, GRASP heuristics (Resende, 1998).

Note that these two discrete models do not account for the interference between cells or the overlaps between them, which are very important to deal with handover, i.e., the possibility for a moving user to remain connected to the network while moving from one cell to another. In Amaldi et al. (2005b), for instance, the classical set covering and maximum coverage problems are extended to consider overlaps in the case of Wireless Local Area Network (WLAN) design by introducing suitable non linear objective functions.

The influence of BS locations on the "shape" of the cells can be captured by introducing variables that explicitly assign test points to base stations. These binary variables are defined for every pair of TP i and CS j such that there exist at least one configuration of the BS in CS j that allows them to communicate:

$$x_{ij} = \begin{cases} 1 & \text{if TP } i \text{ is assigned to BS } j \\ 0 & \text{otherwise.} \end{cases}$$

If $K(i,j)$ denotes the set of the available configurations for the BS in CS j that allow the connection with TP i, the formulation of the full coverage problem becomes:

$$\min \quad \sum_{j \in S} \sum_{k \in K_j} c_{jk} y_{jk}$$

$$\sum_{j \in S} x_{ij} = 1 \quad \forall i \in I \tag{31.9}$$

$$\sum_{k \in K_j} y_{jk} \leq 1 \quad \forall j \in S \tag{31.10}$$

$$x_{ij} \leq \sum_{k \in K(i,j)} y_{jk} \quad \forall i \in I, \forall j \in S : K(i,j) \neq \emptyset \tag{31.11}$$

$$y_{jk} \in \{0,1\} \quad \forall j \in S, \forall k \in K_j \tag{31.12}$$

$$x_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in S : K(i,j) \neq \emptyset. \tag{31.13}$$

The crucial constraints of the above model are (31.11) stating that a TP can be assigned to a BS only if the configuration of this BS allows that connection. In order to account for the maximum coverage variant, the equality constraints (31.9) expressing full coverage can be transformed into inequalities ($\leq$) and a suitable term proportional to the number of connected TPs can be added to the objective function. Note that, in this case, a cell is defined by the set of TPs assigned to it and hence is not predefined by the incidence matrix, as in the models based on set covering.

This basic model can be amended by adding constraints related with the actual "shape" of the resulting cells. Some authors proposed a quality measure of a cell C given by

$$\frac{\sqrt{area(C)}}{boundary(C)}$$

(Zimmerman et al., 2000). Since this quantity, which is maximized when the cell is circular, is difficult to deal with in a mathematical model and in a solution algorithm, other rules based on connectivity are usually preferred. For instance, each TP is assigned to the "closest" (in terms of signal strength) activated BS. One way to express this constraint for a given TP i is to consider all the pairs of BSs and configurations that would allow connection with i and sort them in decreasing order of signal strength. Let $\{(j_1,k_1),(j_2,k_2),\ldots,(j_L,k_L)\}$ be the ordered set of BS-configuration pairs, the constraints enforcing the assignment of TP i to the closest activated BS are:

$$y_{j_\ell k_\ell} + \sum_{h=\ell+1}^{L} x_{i j_h}, \leq 1 \quad \ell = 1,\ldots,L-1. \tag{31.14}$$

According to the above constraints, if a BS is activated in configuration ℓ, then TP i cannot be connected to a less convenient BS. In some settings, including second generation systems, capacity constraints can also be introduced so as to limit the number of TPs assigned to the same BS (Mathar and Niessen, 2000).

These location-allocation models can be solved efficiently with known exact and heuristic methods. See, for instance, Ghosh and Harche (1993).

31.4 CAPACITY PLANNING

In second generation systems, after the coverage planning phase, available carriers (frequencies) must be assigned to BSs in order to provide them with enough capacity to serve traffic demands. Frequencies are identified by integers (denoting their relative position in the spectrum) in the set $F = \{1,2,\ldots,f_{max}\}$. To efficiently exploit available radio spectrum, frequencies are reused in the network. However, frequency reuse may deteriorate the received signal quality. The level of such a deterioration depends on the *SIR* and can be somehow controlled by a suitable assignment of transmission frequencies.

The *Frequency Assignment Problem* (FAP) is the problem of assigning a frequency to each transmitter of a wireless network so that (a measure of) the quality of the received signals is maximized. Depending on spectrum size, objectives and specific technological constraints, the FAP may assume very different forms.

It is worth noting that the FAP is probably the telecommunication application which has attracted the largest attention in the Operations Research literature, both for its practical relevance and for its immediate relation to classical combinatorial optimization problems. This wide production has been analyzed and organized in a number of surveys and books (Aardal et al., 2003; Eisenblätter et al., 2002; Jaumard et al., 1999; Leese and Hurley, 2002; Murphey et al., 1999; Roberts, 1991). In this section, we give an overview of the most significant contributions to the models and algorithms for the FAP and provide a historical perspective.

In the 1970s, frequencies were licensed by governments in units; since operators had to pay for each single frequency, they tried to minimize the total number of frequencies required by non-interfering configurations. It was soon understood (Metzger, 1970) that this corresponds to solving a suitable version of the well-known graph coloring problem, or some generalization of it. This immediate correspondence is obtained by associating a graph $G = (V,E)$ with network R, defining V to be the set of antennas (TRXs) of R, and by letting $\{i,j\} \in E$ if and only if TRX i and TRX j interfere. Any coloring of the vertices of G (i.e., assignment of colors such that adjacent vertices have different colors) is then an assignment of frequencies to R such that no mutual interfering TRXs receive the same frequency. A minimum cardinality coloring of G is a minimum cardinality non-interfering frequency assignment of R. Early solution approaches to the graph coloring model of the FAP were proposed in Metzger (1970) and Zoellner and Beall (1977): both papers discuss simple greedy heuristics.

The graph coloring model assumes that distinct frequencies do not interfere: this is not always the case. In general, a frequency h interferes with all frequencies $g \in [h-\delta, h+\delta]$ where, δ depends upon channel bandwidth, type of transmission and power of signals. To overcome this drawback, in the early 80's a number of generalizations of the graph coloring problem were proposed (Gamst and Rave, 1982; Hale, 1980).

In the new offspring of works an instance of FAP is represented by a complete, undirected, weighted graph $G = (V, E, \delta)$, where $\delta \in Z_+^{|E|}$ is the *distance vector*, and δ_{uv} is the (minimum) admissible distance (in channel units) between a frequency f_u assigned to u and a frequency f_v assigned to v. The problem of defining a free-interference plan becomes now that of finding an assignment f such that $|f_v - f_u| \geq \delta_{uv}$ for all $\{u, v\} \in E$ and the difference between the largest and the smallest frequency, denoted by $Span(f)$, is minimized. The Span of a minimum Span assignment of $G = (V, E, \delta)$ is a graph invariant denoted by $Span(G)$. Clearly, when $\delta_{uv} \in \{0, 1\}$, then $Span(G) = \chi(G)$ (the minimum cardinality of a coloring of G). This version of FAP, called MS-FAP, has been widely addressed in the literature; most of solution approaches are heuristic methods, ranging from the simple generalizations of classical graph coloring heuristics (Costa, 1993; Borndörfer et al., 1998) like DSATUR (Brélaz, 1979) to specific implementations of local search such as, for instance, simulated annealing (Costa, 1993), genetic algorithm (Valenzuela et al., 1998), and tabu search (Hao and Perrier, 1999).

It was soon remarked (Box, 1978) that, given an assignment f, one can build one (or more) total ordering $\sigma(V)$ on the vertices V by letting $\sigma(u) < \sigma(v)$ whenever $f_u < f_v$ (ties are broken arbitrarily). Similarly, given an ordering $\sigma(V) = (u_1, \ldots, u_n)$, one can immediately associate an assignment $f \in \{1, \ldots, f_{max}\}^{|V|}$ by letting $f_{u_1} = 1$ and

$$f_{u_j} = max_{i<j} \ f_{u_i} + \delta_{u_i u_j}$$

for $j = 2, \ldots, n$ (with f_{max} large enough). This observation led to the definition of a number of models and algorithms based on the correspondence between orderings of V and acyclic orientations of the edges of G. A nice relation between frequency assignments and Hamiltonian paths of G was first pointed out by Raychaudhuri (1994). First observe that, with any ordering $\sigma(V) = (u_1, \ldots, u_n)$, a Hamiltonian path

$$P(\sigma) = \{(u_1, u_2), (u_2, u_3), \ldots, (u_{n-1}, u_n)\}$$

of G is uniquely associated (where G is a complete graph and $\delta_{uv} \geq 0$ for $\{u, v\} \in E$). If δ_{uv} is interpreted as the length of $\{u, v\} \in E$, then the length $\delta(H^*)$ of a minimum length Hamiltonian path H^* of G is a lower bound on $Span(G)$. In fact, let f^* be an optimum assignment and let $\sigma^* = (u_1^*, \ldots, u_n^*)$ be one possible corresponding ordering. Then

$$f^*_{u_j^*} = max_{i<j} \ f^*_{u_i^*} + \delta_{u_i^* u_j^*} \geq f^*_{u_{j-1}^*} + \delta_{u_{j-1}^* u_j^*}$$

for $j = 2, \ldots, n$. Finally

$$Span(f^*) = f^*_{u^*_n} - f^*_{u^*_1} = \sum_{j=2}^{n} f^*_{u^*_j} - f^*_{u^*_{j-1}} \geq \sum_{j=2}^{n} \delta_{u^*_{j-1}u^*_j} = \delta(P(\sigma^*)) \geq \delta(H^*).$$

In order to satisfy an increasing traffic demand, the number of TRXs installed in a same BS had to be increased; in fact, for practical instances, the number of frequencies to be assigned to a BS ranges from 1 to several units (up to ten or more). In the graph model introduced so far, every TRX is represented by a vertex v of G. However, as for their interferential behavior, TRXs belonging to the same BS are indistinguishable. This yields to a more compact representation $G = (V, E, \delta, m)$, where each vertex v of G corresponds to a BS, while $m \in R^{|V|}$ is a *multiplicity* vector with m_v denoting, for each $v \in V$, the number of frequencies to be assigned to v. The FAP is then the problem of assigning m_v frequencies to every vertex of G so that (i) *every* frequency f_v assigned to v and *every* frequency f_u assigned to u satisfy $|f_v - f_u| \geq \delta_{uv}$ and (ii) the difference between the largest and the smallest frequencies assigned (*Span*) is minimized. This version of the FAP was very popular up to the 1990s; the most famous set of benchmark instances, the Philadelphia instances (FAP website, 2000), are actually instances of this problem. Whilst the majority of solution methods use demand multiplicity in a straightforward way by simply splitting each (BS) vertex v into m_v "twin" vertices (the TRXs), a few models (and algorithms) account for it explicitly, see e.g. Janssen and Wentzell (2000) and Jaumard et al. (2002). The introduction of multiplicity led to a natural extension of the classical Hamiltonian paths to the more general m-walks, i.e., walks "passing" precisely m_v times through every vertex $v \in V$ (a Hamiltonian path is an m-walk with $m = 1_{|V|}$). In fact, one can show (Avenali et al., 2002) that the length of a minimum length m-walk is a lower bound on the span of any (multiple) frequency assignment. These observations led to the definition of suitable integer linear programming (ILP) formulations, with variables associated with edges and walks of the interference graph. These formulations can be exploited to produce lower bounds (Allen et al., 1999; Janssen and Wentzell, 2000) or to provide the basis for an effective Branch-and-Cut solution algorithm (Avenali et al., 2002).

In the late 1980s and in the 1990s the number of subscribers to GSM operators grew to be very large and the available band rapidly became inadequate to allow for interference-free frequency plans: in addition to this, frequencies were now sold by national regulators in blocks rather than in single units. The objective of planning shifted then from minimizing the number of frequencies to that of maximizing the quality of service, which in turn corresponds to minimizing (a measure of) the overall interference of the network. This last objective gives rise to the so called *Minimum Interference Frequency Assignment Problem* (MI-FAP) which can be viewed as a generalization of the well-known *max k-cut* problem on edge-weighted graphs. Here, rather than making use of an intermediate graph-based representation of this problem (*interference graph*), we prefer to refer to a standard 0-1 linear programming formulation.

The basic version of the MI-FAP takes only into account *pairwise* interference, i.e., the interference occurring between a couple of interfering TRXs. Interference is measured as the number of unsatisfied requests of connection. Specifically, if v and w are potentially interfering TRXs and f, g two available frequencies (not necessarily distinct), then we associate a penalty p_{vwfg} to represent the interference (cost) generated when v is assigned to f and w is assigned to g. Then the problem becomes that of finding a frequency assignment which minimizes the sum of the penalty costs.

In order to describe a 0-1 linear program for MI-FAP, we introduce a binary variable x_{vf} for every vertex v and available frequency $f \in F$:

$$x_{vf} = \begin{cases} 1 & \text{if frequency } f \in F \text{ is assigned to vertex } v \in V \\ 0 & \text{otherwise.} \end{cases}$$

Since it is easy to see that the contribution to the objective value of the interference between v and w can be expressed as $\sum_{f,g \in F} p_{vwfg} x_{vf} x_{wg}$, the objective function can be written as

$$\min \sum_{\{v,w\} \in E} \sum_{f,g \in F} p_{vwfg} x_{vf} x_{wg}. \tag{31.15}$$

In order to linearize the quadratic terms $x_{vf} x_{wg}$, we define the variables $z_{vwfg} = x_{vf} x_{wg}$ for all $v, w \in V$ and all $f, g \in F$, i.e.,

$$z_{vwfg} = \begin{cases} 1 & \text{if } x_{vf} \cdot x_{wg} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

To enforce z_{vwfg} to be one when $x_{vf} = x_{wg} = 1$, we add the following constraints to the formulation:

$$x_{vf} + x_{wg} \leq 1 + z_{vwfg} \qquad \forall \{v,w\} \in E, \forall f, g \in F. \tag{31.16}$$

By substitution, the quadratic form of the objective function (31.15) becomes the following linear expression:

$$\min \sum_{\{v,w\} \in E} \sum_{f,g \in F} p_{vwfg} z_{vwfg}.$$

Finally, the requirement that $m(v)$ frequencies have to be assigned to each vertex v is modeled by the following *multiplicity constraints*:

$$\sum_{f \in F} x_{vf} = m(v) \qquad \forall v \in V.$$

If only co-channel interference is involved, i.e., $p_{vwfg} = 0$ holds whenever $f \neq g$, then MI-FAP reduces to the *max k*-cut problem: given an edge-weighted graph $G = (V, E, \delta)$, find a partition of V into k classes so that the sum of the weights of crossing edges is maximized. We can solve our special instance of MI-FAP by letting $k = |F|$, solve the max k-cut problem on G and then assign to every vertex in a same class the same frequency from F, while assigning different frequencies to vertices belonging to different classes. Several algorithms exploit this natural correspondence. A special mention deserves the innovative approach proposed by (Eisenblätter, 2002) to compute strong upper bounds for MI-FAP by solving a semidefinite programming relaxation of a suitable ILP formulation of the corresponding *max k*$-$cut problem.

The MI-FAP model is certainly the version of FAP mostly addressed in the recent literature, mainly for its direct applicability to the solution of large practical instances of GSM network planning problems. Thus, a huge number of solution approaches, both exact and approximate, have been proposed in the last years. Among heuristic approaches, the most successful appear to be variants of Simulated Annealing as reported in the FAPweb benchmark instances section of (FAP website, 2000). Exact methods for MI-FAP include implicit enumeration as well as polyhedral approaches. In particular, the Branch-and-Cut proposed in Aardal et al. (1996) makes use of the ILP formulation above described, which can be strengthen by adding clique inequalities derived from the packing constraints (31.16).

The MI-FAP 0-1 model introduced so far can be exploited as a paradigm for several (constraint or objective based) variations. Indeed, the standard model for the FAP displays in practice a very rich speciation in order to adapt to a multitude of different technological or quality requirements and operator objectives. Two remarkable variants to the standard MI-FAP accounts for the *cumulative interference*, i.e., the (total) noise generated by multiple interfering transmitters on the service area of a target one (Fischetti et al., 2000; Capone and Trubian, 1999). In Capone and Trubian (1999), the interference generated by the BSs is evaluated on the points of a spatial grid and a quality threshold on the *SIR* is considered. Since the resulting problem formulation is quite different from the MI-FAP 0-1 model, in the following we describe the approach adopted in Fischetti et al. (2000) where only the total noise generated on the area covered by each TRX is considered.

If we denote by I_{vwfg} the noise generated by w on v when $f \in F$ is assigned to v and $g \in F$ is assigned to w, then the total noise produced by all neighboring vertices $N(v) = \{w : \{v,w\} \in E\}$ on a frequency f for $v \in V$ is simply computed as

$$\sum_{w \in N(v)} \sum_{g \in F} I_{vwfg} x_{wg}.$$

In order to control cumulative interference, we define for each $v \in V$ a "local" threshold L_v, which is the maximum interference acceptable for v. Then the following constraints are introduced:

$$\sum_{w \in N(v)} \sum_{g \in F} I_{vwfg} x_{wg} x_{vf} \leq L_v \qquad \forall v \in V, \forall f \in F.$$

These are non-linear constraints, but again we can easily linearize them by introducing suitable 0-1 variables. The resulting ILP model is the basis of a Branch-and-Cut algorithm exploited in Fischetti et al. (2000) for the solution of large real-life instances.

31.5 JOINT COVERAGE AND CAPACITY PLANNING

As mentioned in Section 31.2, while the problem of planning a second generation cellular system can be subdivided into a coverage and a capacity planning subproblems, such a two-phase approach is not appropriate for third generation systems that are based on a CDMA radio access scheme. Since the signal quality depends on all the communications in the systems, the critical issues of radio planning and coverage optimization must be tackled jointly.

Due to the many issues that can affect system performance and the huge costs of the service licenses that service operators have to face, there is an acute need for planning tools that help designing, expanding, and configuring third generation systems, like UMTS, in an efficient way. In practice, optimizing BS configuration can often be more critical than BS location since the set of candidate sites may be very limited due to authority constraints on new antenna installation and on electromagnetic pollution in urban areas (Cappelli and Tarricone, 2002).

In the *UMTS network planning problem*, given a set $S = \{1,\ldots,m\}$ of candidate sites, a set $I = \{1,\ldots,n\}$ of test points with the corresponding required number u_i of active connections for each TP i (which is a function of the traffic demand) and the propagation matrix G (providing channel attenuation between CSs and TPs), one has to select a subset of CSs where to install BSs together with their configurations so as to optimize an appropriate objective function which takes into account traffic coverage and installation costs. Due to the peculiarities of the CDMA radio access scheme, the *SIR* constraints and power limits must also be considered.

Most of the early work on planning CDMA systems still relies on classical coverage models and does not appropriately account for interference. For instance, in Calégari et al. (1997) a simple model based on the minimum dominating set problem is considered, while in Lee and Kang (2000) the traffic capacity is also taken into account and the resulting classical capacitated facility location problem is tackled with a Tabu Search algorithm. In Chamaret et al. (1997) a different approach is adopted: in a graph with nodes corresponding to the CSs and edges to pairs of CSs whose BSs would have coverage areas with too much overlap, a maximum independent set of vertices is searched for. In Galota et al. (2001) a simplified model for locating BSs in CDMA-based UMTS networks is considered and a polynomial time approximation scheme is presented. Although intra-cell interference is taken into account, the interference among BSs (inter-cell one) is neglected.

Akl et al. (2001) address the problem of locating a prescribed number k of BSs and of optimizing their configurations in the uplink direction, assuming a power-based PC. The model, which includes pilot signals but assumes that BS locations are continuous decision variables, is tacked by Branch-and-Bound. In Mathar and Schmeink (2001) a discrete mathematical programming model is given for locating up to k BSs, considering the downlink direction and with simplified *SIR*-constraints. A Branch-and-Bound algorithm is used to tackle instances with up to 1100 TPs but very small values of k, namely $k < 8$. Pointers to other work on planning third generation cellular systems can be found in Amaldi et al. (2003a), Amaldi et al. (2005a), and Eisenblätter et al. (2003).

The mixed integer programming (MIP) model we present is based on Amaldi et al. (2005a). The subproblem of locating BSs for the uplink direction is investigated in Amaldi et al. (2003a) with power-based as well as *SIR*-based PC mechanisms. The extensions with BS configurations and with downlink communications are addressed in (Amaldi et al., 2002) and, respectively, in (Amaldi et al., 2003b).

The overall model takes into account signal-quality constraints in both uplink and downlink directions, and assumes a *SIR*-based power control mechanism. BS configurations include, among others, maximum emission power, antenna height, antenna tilt, and sector orientation if directive antennas are co-located in the same CS. Since these BS characteristics have an impact on the attenuation matrix G, g_{ijk}, with $0 < g_{ijk} \leq 1$, denotes the gain factor of the radio link between TP i and a BS installed in CS j with configuration k, where $i \in I$, $j \in S$, $k \in K_j$.

Soft-handover is not considered explicitly, that is, each TP is assigned to at most one BS. Since soft-handover tends to increase *SIR* values, we may be too conservative but this can be compensated for by decreasing the value of SIR_{min}. For the sake of simplicity, we also assume that the number of connections assigned to any BS does not exceed the number of available spreading codes. In uplink, there is a very large number of nonorthogonal codes, while in downlink, where there are at most *SF* orthogonal codes, standard cardinality constraints can be added to the model.

A mixed integer programming formulation of the UMTS network planning problem involves the location variables y_{jk}, with j in S and k in K_j, as well as the assignment variables:

$$x_{ijk} = \begin{cases} 1 & \text{if TP } i \text{ is assigned to a BS in CS } j \text{ with configuration } k \\ 0 & \text{otherwise,} \end{cases}$$

with i in I, j in S and k in K_j.

If we assume a *SIR*-based power control mechanism, we also need for every TP $i \in I$ the continuous variables p_i^{up} to indicate the power emitted by the mobile in TP i towards the BS it is assigned to (uplink direction) and p_i^{dw} to indicate the power received at each TP i from the BS it is assigned to (downlink direction).

To aim at a trade-off between maximizing the total traffic covered and minimizing the total installation costs, the objective function is:

$$\max \lambda \sum_{i \in I} \sum_{j \in J} \sum_{k \in K_j} x_{ijk} - \sum_{j \in J} \sum_{k \in K_j} c_{jk} y_{jk}, \tag{31.17}$$

where $\lambda > 0$ is a trade-off parameter between the two contrasting objectives. Alternatively a budget can be imposed on the total installation costs.

The first three groups of constraints of the formulation ensure the coherence of the location and assignment variables. Every TP i can be assigned to at most one BS:

$$\sum_{j \in S} \sum_{k \in K_j} x_{ijk} \leq 1 \qquad \forall i \in I \tag{31.18}$$

and at most one BS configuration can be selected for CS j:

$$\sum_{k \in K_j} y_{jk} \leq 1 \qquad \forall j \in S. \tag{31.19}$$

If for a given CS j we have $y_{jk} = 0$ for all $k \in K_j$, no BS is installed in CS j. A TP i can be assigned to a CS j only if a BS with some configuration k, $k \in K_j$, has been installed in j:

$$\sum_{k \in K_j} x_{ijk} \leq \sum_{k \in K_j} y_{jk} \qquad \forall i \in I, j \in S. \tag{31.20}$$

In uplink, the power emitted by any mobile terminal at TP i cannot exceed a maximum power P_i^{max-up}:

$$0 \le p_i^{up} \le P_i^{max-up} \qquad \forall i \in I. \tag{31.21}$$

In downlink, besides the limit on the total power emitted by each BS j, we also consider an upper bound on the power used for every connection:

$$0 \le p_i^{dw} \le \sum_{j \in S} \sum_{k \in K_j} P^{max-dw}\, g_{ijk}\, x_{ijk} \qquad \forall i \in I, \tag{31.22}$$

where P^{max-dw} denotes the maximum power per connection. According to constraints (31.18) at most one of the x_{ijk} variables in (31.22) is equal to 1 and the right-hand side amounts to the maximum power available in downlink. Thus BSs cannot use too much of their power for transmission towards mobiles with bad propagation conditions (Holma and Toskala, 2000).

Let us now turn to the *SIR* constraints, which express the signal quality requirements, and consider first the uplink direction. Given the *SIR*-based PC mechanism, for each triple of BS $j \in S$ with configuration $k \in K_j$ and TP $i \in I$ we have the uplink *SIR* constraint:

$$\frac{p_i^{up}\, g_{ijk}\, x_{ijk}}{\sum_{h \in I} u_h\, p_h^{up}\, g_{hjk} \sum_{t \in S} \sum_{l \in K_t} x_{htl} - p_i^{up}\, g_{ijk} + \eta_j^{bs}} = SIR_{tar}\, x_{ijk}, \tag{31.23}$$

where η_j^{bs} denotes the thermal noise at BS j. For any single connection between a TP i and a BS installed in CS j with configuration k, the numerator of the left-hand-side term corresponds to the power of the relevant signal arriving from TP h at CS j with BS configuration k while the denominator amounts to the total interference due to all other active connections in the system. The triple summation term expresses the total power received at the BS in j with configuration k from all TPs h that are served. Indeed, $p_h^{up} g_{hjk}$ indicates the power received at the BS j from TP h and, according to (31.18), $\sum_{t \in S} \sum_{l \in K_t} x_{htl}$ is equal to 1 if and only if TP h is assigned to a BS, namely is served. The total interference, is the obtained by just subtracting the received power of the relevant signal.

In downlink, for each triple of TP $i \in I$ and BS $j \in S$ with configuration $k \in K_j$, we have the *SIR* constraint:

$$\frac{p_i^{dw}\, x_{ijk}}{\alpha \left(\sum_{h \in I} u_h g_{ijk} \dfrac{p_h^{dw}}{g_{hjk}}\, x_{hjk} - p_i^{dw} \right) + \sum_{\substack{l \in S \\ l \neq j}} \sum_{z \in K_l} \sum_{h \in I} u_h g_{ilz} \dfrac{p_h^{dw}}{g_{hlz}}\, x_{hlz} + \eta_i^{mt}} = SIR_{tar}\, x_{ijk}, \tag{31.24}$$

where η_i^{mt} denotes the thermal noise of mobile terminal at TP i. For any single connection between a BS located in CS j with configuration k and a TP i, the numerator of the left-hand-side term corresponds to the power of the relevant signal received at TP i from the BS j (definition of p_i^{dw}) and the denominator amounts to the total interference due to all other active connections in the system. The interpretation of (31.24) is similar to that of (31.23) except for the orthogonality loss factor α in the *SIR* formula (31.1), which is strictly smaller than 1 in downlink.

Thus, constraints (31.23) and (31.24) ensure that if a connection is active between a TP i and a BS j with configuration k (i.e., $x_{ijk} = 1$) then the corresponding *SIR* value is equal to SIR_{tar}.

Note that the resulting model is a mixed integer program with nonlinear *SIR* constraints since they contain products of assignment variables (x_{ijk} and y_{jk}) and power variables (p_i^{up} and p_i^{dw}).

If we assume a power-based PC mechanism instead of a *SIR*-based one, the model can be substantially simplified (Amaldi et al., 2005a). Indeed, the powers p_i^{up} emitted from any TP i in uplink and the powers p_i^{dw} received at any TP i from the BS they are assigned to, are no longer variable. Since all emitted powers are adjusted so as to guarantee a received power of P_{tar}, p_i^{up} and p_i^{dw}, for all $i \in I$, just depend on the value of P_{tar} and on the propagation factor of the corresponding radio links. To obtain the simplified model, we take into account that:

$$p_i^{up} = \sum_{j \in S} \sum_{k \in K_j} \frac{P_{tar}}{g_{ijk}} x_{ijk} \qquad\qquad p_i^{dw} = P_{tar}, \tag{31.25}$$

and in the *SIR* constraints we require that the left-hand-side terms of equations (31.23) and (31.24) are greater or equal to $SIR_{min}\, x_{ijk}$.

The resulting *SIR* constraints, which have the general form

$$\frac{P_{tar}}{(\alpha I_{in} + I_{out} + \eta)} \geq SIR_{min}\, x_{ijk}, \tag{31.26}$$

where P_{tar} is a constant, and I_{in} and I_{out} are linear functions in the x and y variables, can be linearized as follows:

$$(\alpha I_{in} + I_{out} + \eta) \leq \frac{1}{SIR_{min}} + M_{ijk}(1 - x_{ijk}) \tag{31.27}$$

for large enough values of the constants M_{ijk}.

The simplified model for locating BSs with fixed configurations considering the uplink direction and a power-based PC mechanism is described in Chapter 24. It is worth pointing out that even the linearization of this simplified model yields integer linear programs that are very challenging computationally. In practice, even small instances of the overall problem with both uplink and downlink directions and *SIR*-based PC mechanism are beyond the reach of state-of-the-art exact methods.

Heuristics for the overall model must also face very high computational requirements. Indeed, just computing the transmitted powers corresponding to any given set of active BSs (**y**) and given assignment of TPs to these active BSs (**x**) involves finding a solution of a large linear system consisting of the *SIR* equations (31.23) and (31.24) while satisfying the power limit constraints (31.21) and (31.22). Fortunately this critical computation, which has to be carried out after every variable modification, can be speeded-up substantially by adapting (Amaldi et al., 2005a) a recently proposed iterative method (Berg, 2002). An alternative method to compute power levels and determine the feasibility of a given assignment of TPs to BSs is proposed in Catrein et al. (2004).

A Tabu Search procedure, which starts from an initial solution obtained with GRASP and explores the solution space through suitable switches of the y variable values, provides good quality solutions of relevant-size instances (with up to 200 CSs and 1000 TPs) in reasonable computing time (Amaldi et al., 2005a;c). A two-stage approach can be adopted: solutions of a simpler model, which considers a power-based PC mechanism and only the uplink direction, are exploited as good initial solutions for the overall uplink and downlink model with a SIR-based PC mechanism. Since in the uplink direction the model with power-based PC is a quite accurate approximation of the model with *SIR*-based PC, the insight gained from solving the former model help drastically reducing the computing times for tackling the overall model without significantly affecting the solution quality. However, the computational requirements to tackle larger instances grow very fast with the number of discretization values of the configuration parameters.

The above MIP model and algorithm can be easily extended to include pilot signals and to deal explicitly with directive antennas, that is BSs with several sectors (Amaldi et al., 2005a;c).

For additional modeling aspects such as hand-over and the limited number of codes the reader is referred to Eisenblätter et al. (2003) and to the other references related to the Momentum IST-EU project (IST-EC website, 1999). The overall model described in Eisenblätter et al. (2003) considers both network cost and service quality in the objective function and includes uplink and downlink *SIR* constraints, *SIR*-based power control, pilot signal, antenna configurations and a limited number of codes assigned to each connection. Since more detailed models are less tractable even with heuristics, the huge resulting mathematical programs are beyond the reach of state-of-the-art MIP solvers except for very small-size instances. For realistic data scenarios see for instance IST-EC website (1999).

The actual challenge is to find a reasonable trade-off between an accurate description of the UMTS network planning problem and a computationally tractable model. This is a critical issue if we want to consider a representative set of traffic scenarios (snapshots).

Eisenblätter and Geerdes (2005) make a step in this direction by trying to derive reliable average transmitted powers from average traffic intensities instead of individual snapshots. By considering average coupling matrices aimed at capturing the essential coverage and cell coupling properties of the radio network, the network design problem is casted in terms of designing a “good” average coupling matrix.

31.6 CONCLUDING REMARKS

In this chapter, we gave an overview of the most relevant optimization problems arising in planning cellular systems and we described some optimization approaches that have been developed to tackle them. These problems have stimulated interesting lines of research which not only improved the way these wireless systems are planned but also led to significant advances in discrete optimization, as for instance in the case of frequency assignment. The peculiarities and increasing complexity of new telecommunication systems shifts attention towards new interesting challenges. A relevant challenge is the development of an integrated and computationally efficient approach to coverage and capacity planning for third and fourth generation cellular systems. Extensions of the classical set covering and maximum coverage problems which explicitly account for interference (Amaldi et al., 2005b) and signal quality, deserve special attention and are leading to new research threads.

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