WLAN Coverage Planning: Optimization Models and Algorithms

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***Abstract*— The impressive market spread of IEEE 802.11 based Wireless Local Area Networks (WLANs) is calling for quantitative approaches in the network planning procedure. It is common belief that such networks have the potentials to replace traditional indoor wired local networks and allow flexible access outdoor, eventually competing with classical cellular systems (GSM, $GPRS$, $UMTS$, etc.). The appropriate positioning of the Access Points (AP) is crucial to determine the network effectiveness. In a companion paper we argue that previously proposed approaches to coverage planning neglect the features of the IEEE 802.11 access mechanism, which limits system capacity when access points coverage areas overlap. In this paper we describe the optimization models with hyperbolic and quadratic objective functions that directly accounts system capacity and we propose heuristics combining greedy and local search phases. Computational results show that our heuristics provide near-optimal solutions within a reasonable amount of time.**

Keywords WLAN, radio planning, coverage problem, set covering problem, 802.11, wireless hot spots.

I. Introduction

WLAN technology has had and is still having a surprising diffusion in the market of telecommunications. WLAN hot spots are creeping up day by day [1] and almost all portable devices like PDAs, laptops etc. come equipped with 802.11 network interface card and adapters. This amazing success is mainly due to the simplicity of the solution, its cost effectiveness, and, last but not least, the increasing demand for "anywhere, anytime" connectivity. A WLAN is basically constituted by one or more wireless Access Points (APs) connected to the backbone network which provide wireless connectivity to the covered area [2].

In many situations, the deployment of a single AP is not enough to provide the required connectivity. As an example, large facilities, such as an office complex, apartment buildings, hospitals, university campuses, or warehouses generally require many cooperating APs in order to provide the required services to the end users.

In order to access the network, a user terminal needs to receive the radio transmission of an AP at an adequate level of power. A simple way to plan radio coverage is to consider a set of possible positions of user terminals (Test Points, TPs) in the service area and a set of AP candidate sites (CSs). A subset of CSs in which to install APs has then to be selected so as to guarantee a high enough signal level at all TPs. The problem of minimizing the number of candidate AP sites that are able to cover all TPs amounts to a well-known combinatorial optimization problem, namely the minimum cardinality set covering problem [3]. This problem is NP-hard and heuristics are usually adopted to obtain sub-optimal solutions.

However, not all feasible solutions (subsets of AP candidate sites that are able to cover all TPs) provide the same system capacity and level of service. Due to the WLAN medium access mechanism, if a user terminal is covered by more than one AP and is transmitting/receiving to/from one of them, the other APs are prevented to transmit/receive to/from other users [4]. Therefore, the overlaps between the subsets of TPs covered by different APs should be taken into account during the radio planning phase.

Classical methods for coverage planning based on random search heuristics can be applied to the problem [5]. In [6] the authors propose a formulation driven jointly by the maximization of the signal quality in the service area and by the minimization of the areas with a poor signal quality. The objective function comes from a combination of the above objectives. Rodrigues et al. [7] propose an integer linear programming (ILP) formulation in which the signal quality at the test points is maximized. This formulation, which does not require full coverage, is solved by using the state-of-the-art CPLEX ILP commercial solver. In [8] a traffic intensity is assigned to each TP and a formulation aimed at maximizing the channel utilization of each AP is proposed. This formulation turns out to be a special case of the capacitated facility location problem. In general, all the above-mentioned works focus on the problem of achieving high coverage level in terms of received signal quality. Very few of them consider network capacity as an optimization objective.

In [9] we proposed novel mathematical programming formulations for the WLAN planning problem which take into account the coverage overlap between APs and its impact on the network capacity. In particular, we formalized the planning problem by using hyperbolic and quadratic objective functions. Since the proposed formulations are hard to (NP-hard) tackle, in this paper we show how the special underlying structure makes it possible to devise effective heuristics based on combined greedy and local search procedures to provide near-optimal solutions within a reasonable amount of time. After briefly describing the formulations, the heuristics are presented in Section III. Computational results are reported and discussed in Section IV, and concluding remarks are contained in Section V.

II. Planning Problem Formulations

The access mechanism of IEEE 802.11 WLANs is based on the "listen before talk" approach, i.e., each station willing to use the shared resource listens to the channel for ongoing communications before attempting its own access. If the channel is sensed busy the station refrains from transmitting (*Carrier Sensing Multiple Access*, CSMA).

The peculiar characteristics of 802.11b access mechanism affect the coverage planning process and the planning procedure should take into account the incidence of overlapping regions, beside all the other optimization parameters.

In [9] the planning problem is formalized as follows. Let $J = \{1, \dots, n\}$ denote the set of candidate sites to host access points and $I = \{1, \dots m\}$ be the set of users. For each $j \in J$ a subset of users $I_j \subseteq I$ is given. This subset represents the users which can use the access point j. A second family of subsets I'_j, $j \in J$, representing the users affected by the interference of access point j, can be also defined. Clearly $I_j \subseteq I'_j$.

Each user is assumed to be connected to a single AP whose capacity is shared by all users within its coverage range. Furthermore the overall capacity of an AP is assumed to be equal to 1. Due to the multiple access mechanism, a user that is in the interfering range of a set of AP blocks the transmissions to/from all APs in this set. Therefore, the capacity offered by the network to each user can be measured as the ratio between 1 and the number of users in the interference range of the set of APs the considered user can interfere with.

Assuming uniform traffic for all users, the network capacity can be measured by the following expression:

$$c(S) = \sum_{i \in I(S)} \frac{1}{| \cup_{j \in S : i \in I_j} I'_j |}. \tag{1}$$

where $S \subseteq J$ is the subset of CSs in which APs are installed and $I(S)$ the subset of users covered that is included in some $I_j, j \in S$.

As in any covering problem, the basic decision variables indicate which subsets are included in the solution:

$$x_j = \begin{cases} 1 & \text{if an AP is installed in } j \\ 0 & \text{otherwise} \end{cases}$$

To measure the cardinality of the union of subsets containing each element, we also need the following variables which clearly depend on S (and hence on x's):

$$y_{ih} = \begin{cases} 1 & \text{if elements } i \text{ and } h \text{ appear together in some } I_j \\ & \text{with } j \in S \\ 0 & \text{otherwise} \end{cases}$$

Consider the usual elements-subsets incidence matrix A where $a_{ij} = 1$ if element i belongs to subset j, and 0 otherwise, for each $i \in I$ and $j \in J$. The hyperbolic formulation of the problem aiming at maximizing the capacity is:

$$PCH : \max \quad \sum_{i \in I} \frac{1}{\sum_{h \in I} y_{ih}} \tag{2}$$

$$subject\ \ to :$$

$$\sum_j a_{ij} x_j \geq 1 \qquad i \in I \tag{3}$$

$$a_{ij} a_{hj} x_j \leq y_{ih} \qquad j \in J, i, h \in I \tag{4}$$

$$x_j \in \{0, 1\} \qquad j \in J \tag{5}$$

$$y_{ih} \geq 0 \qquad i, h \in I \tag{6}$$

where (3) imposes the complete coverage and (4) defines the variables y_{ih}.

Since even small-size mathematical programs with hyperbolic objective functions are very challenging (see [10]), the AP location problem can be approximated in terms of quadratic programming. Consider an arbitrary subset I_j in a given solution $S \subseteq J$. If I_j does not intersect any other subset in the solution (or is the only subset in S), its contribution to the network capacity amounts to $1/|I_j|$ for each element of I_j, which gives a total contribution over all TPs in I_j equal to 1. To account for this fact we consider for each subset I_j a coefficient $c_j = 1$. If a solution contains two subsets I_j and I_ℓ with an empty intersection, the contribution to the network capacity due to I_j and I_ℓ is equal to $c_j + c_\ell$. Conversely, if I_j and I_ℓ do intersect, the above contribution must be decreased by an amount which depends on the cardinality of their intersection. In particular, the contribution of the intersection must be subtracted from the capacity of the two subsets and a new evaluation of the contribution of the intersection considering the union of the two subsets must be added to the capacity measure. To estimate the decrease in network capacity due to the overlapping between a pair of selected subsets I_j and I_ℓ, we define the coefficients:

$$q_{j\ell} = -\frac{|I_j \cap I_\ell|}{|I_j|} - \frac{|I_j \cap I_\ell|}{|I_\ell|} + \frac{|I_j \cap I_\ell|}{|I_j \cup I_\ell|}. \tag{7}$$

Note that these coefficients assume values between -1 (when $I_j \subseteq I_\ell$ or $I_\ell \subseteq I_j$) and 0 (when the two subsets are disjoint) and are symmetric, that is $q_{j\ell} = q_{\ell j}$.

The problem of locating APs so as to guarantee full coverage can then be approximated by the following quadratic program:

$$QPC : \max \quad \frac{1}{2} \sum_{j \in J} \sum_{\ell \in J, \ell \neq j} q_{j\ell} x_j x_\ell + \sum_{j \in J} c_j x_j \tag{8}$$

$$subject\ \ to :$$

$$\sum_j a_{ij} x_j \geq 1 \qquad i \in I \tag{9}$$

$$x_j \in \{0, 1\} \qquad j \in J \tag{10}$$

which turns out to be an interesting Quadratic set Covering Problem. It is worth noting that this formulation contains a substantially smaller number of variables than the hyperbolic one. Moreover, it is easy to verify that the objective function coincides with the network capacity if each TP belongs at most to two different selected subsets I_j and it provides a lower bound if l-tuples of selected subsets have a nonempty intersection. v

III. Heuristics

Since the two above formulations are hard to tackle even for small instances, we developed effective heuristics able to provide near-optimal solutions in a reasonable amount of time.

All proposed algorithms are composed of two phases: in the first one a greedy approach is used to build a feasible solution and in the second one the resulting solution is improved through local search. The greedy phase starts from a empty solution and iteratively adds to the current solution the candidate site which maximizes a certain benefit function. This function measures the benefit achieved when adding a candidate site to a current partial solution. The benefit function is adaptively computed for each candidate site that is not yet included in the current solution.

The general structure of the proposed heuristics can be summarized as follows:

```
PROCEDURE Heuristic(A)
  S = ∅;
  BuildUpSolution(A, S);
  LocalSearch(S);
  RETURN(S)
END Heuristic
```

where S is the set of candidate sites in which APs are installed and A is the incidence matrix defined in II. Function *BuildUpSolution* implements the greedy phase of the heuristic which iteratively converge to a feasible solution S. Function *LocalSearch* refines the solution S through a local search.

A. Greedy Phase

The greedy phase of the proposed heuristics starts off from a NULL solution ($S = \emptyset$) and keeps adding iteratively one CS at a time. The procedure stops when all the TPs are covered by the CS in set S. At each iteration, the CS which maximize a greedy *benefit function* is added to the solution. A pseudo code implementation of the greedy procedure is reported hereafter:

```
PROCEDURE BuildUpSolution(A, S)
  Best_CS = PickBestCS(A);
  S = S ∪ Best_CS;
  Covered_TPs = Covered_TPs ∪ I_Best_CS;
  WHILE Covered_TPs != ALL_TPs
    GreedyStep(A, Covered_TPs,S);
END BuildUpSolution;
```

The function `PickBestCS` chooses the first CS to be added to the solution. The idea is to choose the AP whose coverage area has the smallest overlap with the coverage areas of all the other CS. The pseudo code of this function is not reported for the sake of brevity.

The `GreedyStep` function, which represents the core of the greedy phase, returns the next CS to be added to the solution set. The pseudo code of this function is the following:

```
PROCEDURE GreedyStep(A, Covered_TPs, S)
  MaxFunction = 0;
  DO FOR j ∉ S
    IF Benefit_function_j > MaxFunction;
      CS_ToAdd = j;
      MaxFunction=Benefit_function_j;
    FI
  OD
  S = S ∪ CS_ToAdd;
  Covered_TPs = Covered_TPs ∪ I_CS_ToAdd
END GreedyStep;
```

where `I_CS_ToAdd` following the notation of Section II is the set of TPs covered by the$j$-th CS.

The CS to be added to the solution is the one with the highest *benefit function* which is calculated as follows:

$$\texttt{Benefit\_Function\_j} = \frac{\Delta_{OF}}{ATP_j}$$

where Δ_{OF} denotes the increase in the objective function if an AP is added in CS j and ATP_j the corresponding increase in the total number of TPs covered. The greedy algorithm is adaptive since both Δ_{OF} and ATP_j are re-calculated at each iteration.

Since each iteration adds to the solution a CS which covers one TP at least, the procedure *BuildUpSolution* requires $\min(n, m)$ steps to converge at most, where n and m are the TP number and the CS number respectively.

B. Local Search

The local search phase takes as a input the solution S provided by the greedy phase and tries to enhance it. In particular, the neighborhood of solution S is explored to check for the presence of a better solution. In our case, starting from S we eliminate first one, then two CS belonging to S itself and we apply to the perturbed solution S_p the *BuildUpSolution* procedure described in previous section. The final solution S^* is the one with the highest objective function, otherwise $S^* = S$.

The pseudo code of the local search is:

```
PROCEDURE LocalSearch(A,S)
  MaxOF=ComputeOF(A, S);
  DO
  Enhanced=FALSE;
  DO FOR j ∈ S
    S = S \ {j};
    Covered_TPs = Covered_TPs \ {I_j};
    BuildUpSolution(A,Covered_TPs,S);
    NewOF=ComputeOF(A,S);
    IF NewOF > MaxOF
      MaxSOL=S;
      MaxOF=NewOF
      Enhanced=TRUE;
    FI
    DO FOREACH i IN S AND i>j
      S = S \ i,j;
      Covered_TPs=Covered_TPs \ {I_i, I_j};
      BuildUpSolution(A, Covered_TPs,S);
      NewOF=ComputeOF(A, S);
      IF NewOF > MaxOF
        MaxSOL=S;
```

```
        MaxOF=NewOF
        Enhanced=TRUE;
      FI
    OD
  OD
  IF Enhanced
    S=MaxSOL;
  FI
  WHILE Enhanced;
END LocalSearch;
```

Obviously the function `ComputeOF` has different implementations according to the formulation of the planning problem we are applying the heuristic to (hyperbolic, quadratic).

The same heuristic approach can be extended with slight modifications to hyperbolic and quadratic formulations of the planning problem without full coverage constraint. For the sake of brevity, the details of this extension are not described in this paper.

IV. Computational Results

In order to test the effectiveness of the above algorithms we have implemented a instance generator able to create synthetic instances representing WLANs. The software takes as input the following parameters:

- the edge of the square area to be simulated (L)
- the number of Candidate Sites ($CSNumber$), i.e., the positions where a AP can be installed
- the number of Test Points ($TPNumber$), i.e., the end users
- the value of each AP's coverage range, expressed in meters (r).

Each AP is assumed to have a circular coverage region with radius r.

According to the above parameters, the generating tool randomly draws the positions for the $CSNumber$ candidate sites and of the $TPNumber$ test points. An instance is not feasible when no solution covering all the TPs can be found. In order to generate feasible instances, the generator does the following: firstly, the positions of the CS are randomly generated within the simulated area, secondly each TP is forced to belong to the coverage range of one CS at least.

Using the tool described above we have generated a set of *"uniform"* instances where all the APs to be installed have uniform value of coverage radius r. All the results in the following have been obtained averaging on 10 instances of the same type.

The first step of our analysis is to test the effectiveness of the proposed approaches by comparing their results with the optimum values of the two objective functions, hyperbolic and quadratic. Since the quadratic and hyperbolic problems are solved at optimum by enumeration, we are forced to limit the comparison to relatively small uniform instances with $CSNumber = 10, 20$.

Table I reports the comparison between the optimum values of the different objective functions and the ones obtained through the heuristics. The optimal values have been obtained through enumeration. The results have been compared when varying the number of CS $(10, 20)$ and the value of the coverage radius ($r = 50, 100, 200m$). The number of TP to be covered is set to 100.

The left hand part of the table, named $OPTIMUM$ reports the optimum values of the hyperbolic (PCH) and quadratic (QPC) objective functions. The other two parts on the right report the results obtained applying the heuristic approach to the hyperbolic and quadratic formulation respectively. The solution providing the heuristic optimum for one formulation is used to calculate the objective functions of all the other formulations. For example, referring to the part of the table named $HEURISTIC\ PCH$, the column with the terms in bold reports the value of the PCH objective function when looking for a heuristic solution for PCH problem, while the column named QPC reports the value of the QPC objective function computed using the heuristic solution of the PCH problem. Same thing for the part of the table named $HEURISTIC\ QPC$.

In most cases the heuristics come out with the optimum values of the objective function. A slight discrepancy from the optimum just happens for high sized instances ($CS = 20$) and high coverage radius ($r = 200m$). In these cases the number of feasible solutions is greater and the heuristic approach presents slight differences with respect to the optimum.

Once validated the effectiveness of the proposed heuristics in predicting the optimum values of the hyperbolic and quadratic objective functions, let's compare the results obtained with the heuristics themselves and the classical minimum cardinality set covering approach. The Set Covering Problem (SCP) aims at minimizing the cost of installed CS with the full coverage constraint. In our case all the CS are supposed to have the same installation cost, thus the SCP tend to install the smallest number of CS, without considering network capacity in the objective function definition.

Table II reports the values of the objective functions calculated with the optimum solution of a classical set covering problem. On the other hand, Tables III and IV report the values of the objective functions when running the hyperbolic and quadratic heuristics respectively. All the three tables II, III and IV refer to the case where 300 Test Points are deployed within the simulation area. As in Table I, the solution obtained solving one particular formulation is used to calculate the values of the objective functions of the other two formulations.

From the tables, the classical set covering approach does install a smaller number of Access Points than the heuristic approaches. On the other hand, the set covering solutions provide smaller values of capacity. It is clear from this comparison the difference between the planning approaches: on one side the set covering tends to optimize the installation costs, thus reducing the number of installed APs, on the other side our approach privileges solutions with higher network capacities.

V. Concluding remarks

We have addressed the problem of coverage planning in WLANs, which is starting to attract the attention of both industry and research community. In particular, we have

		OPTIMAL		HEURISTIC PCH		HEURISTIC QPC	
		PCH	**QPC**	**PCH**	**QPC**	**PCH**	**QPC**
CS=10	r=50m	9.334	9.323	**9.334**	9.323	9.334	**9.323**
	r=100m	7.424	7.262	**7.424**	7.251	7.424	**7.262**
	r=200m	4.344	4.06	**4.344**	3.708	4.338	**4.06**
CS=20	r=50m	17.187	17.105	**17.187**	17.105	17.187	**17.105**
	r=100m	12.243	11.852	**12.243**	11.508	12.243	**11.812**
	r=200m	5.494	5.238	**5.47**	4.71	5.407	**5.192**

TABLE I

TPNUMBER=100, UNIFORM INSTANCES, RANDOMLY GENERATED POSITIONS OF CS AND TPS WITHIN 1KM EDGE SQUARE AREA, COMPARISON BETWEEN THE OPTIMAL VALUES OF THE OBJECTIVE FUNCTIONS AND THE VALUES CALCULATED BY THE HYPERBOLIC AND QUADRATIC HEURISTICS.

	Radius	**PCH**	**QPC**	**SCP**
CS=30	50m	24.1	23.9	28.9
	100m	14.196	12.63	23.1
	200m	5.23	4.588	9.7
CS=40	50m	29.799	29.372	38
	100m	15.34	13.502	26.6
	200m	5.368	4.86	9.5
CS=50	50m	34.047	33.359	44.8
	100m	16.49	14.945	27.8
	200m	5.278	4.714	9.5

TABLE II

TPNUMBER=300, UNIFORM INSTANCES, EXACT SOLUTION OF THE SET COVERING PROBLEM.

	Radius	**PCH**	**QPC**	**SCP**
CS=30	50m	24.2	24	29.1
	100m	14.3	12.548	24.1
	200m	5.7	4.089	11.4
CS=40	50m	29.943	29.49	38.3
	100m	15.76	13.639	27.9
	200m	5.995	4.509	11.8
CS=50	50m	34.216	33.233	46.1
	100m	17.1	14.642	30.1
	200m	6.286	4.765	12.3

TABLE III

TPNUMBER=300, UNIFORM INSTANCES, VALUES OF THE VARIOUS OBJECTIVE FUNCTIONS CALCULATED WITH THE SOLUTION OF THE HYPERBOLIC HEURISTIC.

proposed and analyzed effective heuristics to tackle hyperbolic and quadratic formulations of this problem which aim at maximizing the overall network capacity. Our combined greedy and local search algorithms turn out to provide near-optimum solutions in a reasonable amount of computing time. In contrast, the classical approach based on the minimum cardinality set covering problem tends to yield networks with poor overall capacity. This stresses the need for appropriate planning models and procedures that are specific to WLANs.

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	Radius	**PCH**	**QPC**	**SCP**
CS=30	50m	24.2	24.005	29.1
	100m	14.249	12.766	23.6
	200m	5.671	5.192	10.6
CS=40	50m	29.943	29.517	38.2
	100m	15.603	14.162	27.3
	200m	5.902	5.529	10.9
CS=50	50m	34.211	33.58	45.6
	100m	16.859	15.644	29.1
	200m	6.174	5.869	11

TABLE IV

TPNUMBER=300, UNIFORM INSTANCES, VALUES OF THE VARIOUS OBJECTIVE FUNCTIONS CALCULATED WITH THE SOLUTION OF THE HEURISTIC HEURISTIC.

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