

Performance of Dynamic-Frame-Aloha protocols: Closing the gap with tree protocols

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Abstract—The Dynamic-Frame-Aloha protocol, largely studied in the 60s in the field of random access satellite systems, is nowadays commonly applied also to Radio Frequency Identification systems to orchestrate the transmissions from the tags to the reader. In a nutshell, tags respond to reader’s interrogation in slots randomly chosen in a frame whose size is dynamically set by the reader according to the current backlog (remaining tags to be resolved). In this paper, we explore the performance of the DFA protocol under Poisson-distributed population of tags when different strategies are adopted in setting the frame length and estimating the traffic backlog. We further analytically characterize the best strategy in the two cases where the frame is entirely explored or a new frame can be restarted at any slot.

I. INTRODUCTION

Random access protocols have played a fundamental role in communication systems starting with the appearance of the Aloha protocol back to 1970 [1], [2]. Since then, a variety of such protocols have been adopted for satellite networks, radio access networks, local area networks, and, more recently, to Radio Frequency Identification (RFID) systems. Although research on random access protocols is nowadays mature and consolidated, many issues still remain open when classical random access approaches are applied to ‘newer’ environments as the RFID’s.

The most famous representative of random access protocols is the Aloha, which must be stabilized in order to correctly operate. Stabilization can be achieved if some form of “channel feedback” is available, i.e., if the outcome of transmissions, EMPTY, SUCCESS, or COLLIDED, is known at each slot. Based on such information, the transmission probability can be adapted in Slotted-Aloha (S-Aloha) reaching up the theoretical throughput of $e^{-1} \simeq 0.368$ [pkt/slot] [3], [4], [5].

A different approach in orchestrating the access procedure is implemented by the tree-based protocols dated back in 1979 [6], [7]. Similarly to Aloha, channel feedback is required thus leading to intrinsically stable operation. Different than Aloha, colliding terminals are iteratively split into sub-groups in such a way that, in the end, each sub-group is composed of a single terminal only, thus ensuring collision-free transmissions. The simplest tree-based protocol yields a throughput of 0.347 [pkt/slot] under Poisson arrivals. Improvements have been attained along two ways; first, by

distinguishing retransmissions of collided packets with respect to new arrivals until all collision have been solved [6]; this gives rise to the so called Blocked Access (BA) protocols in which the algorithm operates in cycles where each cycle solves collisions of packets arrived in the previous cycle. A further improvement is attained by treating at each cycle a number of packets with average γ , where γ is optimized to provide maximum throughput. Along this way, the maximum attained throughput of 0.487 [pkt/slot] is reached by the Gallager-Tzibakov protocol [8], [7].

The BA mechanism can also be applied to S-Aloha, thus leading to the Frame Aloha (F-Aloha) protocol [9], [10]. Similarly to S-Aloha, time is divided into time slots for transmission of a single packet, and slots are grouped into frames. Differently than S-ALOHA, a terminal is allowed to transmit only one packet per frame, in a randomly chosen slot, and only terminals that have collided in the previous frame can re-schedule their transmissions in the current frame. Unfortunately, F-Aloha shares the same instability as S-Aloha. A technique to achieve stability is to dynamically adapt the frame length according to an estimate of the traffic (packet) backlog, hence the name Dynamic Frame Aloha (DF-Aloha). Along this way Schoute has proposed in [10] a backlog estimation technique that allows to reach a throughput of 0.427 [pkt/slot] under Poisson arrivals.

DF-Aloha has been recently adapted to RFID systems, where a reader interrogates a set of passive tags in order to identify each of them [11], [12]. Collisions may occur among the responses of the tags and DF-Aloha is used to arbitrate collisions so that all tags can be finally identified, i.e., transmit their own identifier to the reader successfully. The access protocol is centralized and the reader acts as central station. The performance measure commonly adopted in the RFID systems is still the throughput, fraction of tags identified in a slot, but it is commonly referred to as “efficiency” in the reference literature. Similarly to other environments, an estimate \hat{n} of the remaining tags to be identified (backlog) n is required to set the frame length in order to maximize efficiency.

A distinctive feature of the RFID environment is that the number N of tags to be identified is a constant which is usually unknown or known in probability. A further novelty is the

capability of restarting a new frame each time the exploration of the current frame is no longer considered optimal.

If the exact backlog were available at each time, then the asymptotical efficiency of the S-Aloha as the traffic goes to infinity, $0.368 [pkt/slot]$, could be reached. However, the exact backlog value cannot be available in RFID systems. Moreover, also the available estimates usually coupled to DF-Aloha protocol do perform poorly in RFID environment. Several works have addressed the enhancement of backlog estimation techniques in RFID systems running DF-Aloha protocols either resorting to Bayesian estimates or to maximum likelihood ones. In [13] the "a posteriori" probability distribution of the backlog n is evaluated using all the past observation on the channel. The "a posteriori" probability is used to find the frame length that maximizes the throughput of the next slot. In practice, the procedure in [13] uses analytical expressions that are difficult to evaluate, although a closed-form expression of probability involved is given in [14].

Recently, [15], [16] have pointed out the non-optimality of the procedures that use the backlog estimate to determine the frame length. In fact, most of the procedures set the frame length equal to the backlog estimate inherently postulating that this setting maximizes the efficiency of the procedure. It is well known that, considering a single frame, the maximum throughput is achieved when the frame length r is equal to the terminal backlog size n . However, since the entire backlog is serviced in a period of several frames, and since the efficiency of single frames changes with the length of the frame itself, it is not obvious how to set that length of the first and subsequent frames. If the backlog size n is known, it has been numerically shown that, in order to maximize the efficiency/throughput on the entire procedure, the frame length must be set equal to the backlog size n at each frame. In [15] the authors suggest a procedure to numerically find the best frame-length setting procedure when the initial population size N is only known in distribution. They provide also some examples that show the non optimality of the cited belief. In [16] the authors refer to the case in which the frame can be restarted and show that even with known backlog size n , setting the frame length n is non-optimal. The optimality procedures given in both these papers, however, are not simple to attain in practice, and also the goal of finding the best performance in specific cases is still open.

In this paper we evaluate the performance of DF-Aloha for RFID environment where the number of tags to be resolved is Poisson-distributed under different backlog estimation mechanisms when the possibility of restarting the frame is also contemplated. We find that, when the entire frame is completely explored, the Schoute's mechanism is very efficient, its performance being practically equal to the one provided by more refined and complex Bayesian estimation techniques and very close to the absolute optimum.

When frame restart is allowed, we demonstrate a practical way to attain the Bayesian estimate proposed in [13] and find that it provides an efficiency as high as $0.4687 [pkt/slot]$, a figure very close to the maximum ever observed in this

environment with tree-based protocols, that is, $0.487 [pkt/slot]$ [8][7]. We also propose an estimation mechanism by far simpler than the Bayesian one with a negligible performance loss. Finally we propose a procedure to find upper and lower bounds to the absolute optimum performance which are $0.469 [pkt/slot]$ and $0.4739 [pkt/slot]$, respectively.

The paper is organized as follows. In Section II, we analyze DF-Aloha when the entire frame is explored, and discuss Shoute's, Bayesian and Optimum cases. In Section III, we analyze DF-Aloha when the frame can be restarted, and evaluate the Bayesian estimation procedure. We also show a simpler but effective estimation procedure called Backlog Lower Bound, and finally show how to evaluate the absolute optimum performance. Our concluding remarks are reported in Section IV.

II. DYNAMIC FRAME-ALOHA

The average number of slots $L(N)$ needed to solve N collisions with frames of constant length r obeys the following recursive relation [10]:

$$L(N) = r + \sum_{s=0}^m p_N(s)L(N-s), \quad N \geq 2, \quad (1)$$

where s is the number of successful slots in the frame, $m = \min(N-2, r-1)$, and $p_N(s)$ is the distribution of the number of successes s . Relation (1) can be solved with respect to $L(N)$ yielding

$$L(N) = \frac{r + \sum_{s=1}^m p_N(s)L(N-s)}{1 - p_N(0)}, \quad N \geq 2, \quad (2)$$

This can be used recursively to derive all values $L(N)$ starting from the initial values

$$L(0) = L(1) = r.$$

As already mentioned in the previous section, F-Aloha needs stabilization to avoid reaching asymptotical null efficiency, that is,

$$\lim_{N \rightarrow \infty} \frac{N}{L(N)} = 0$$

for any finite r . Stable behavior can be obtained by adapting dynamically the frame length to the current backlog at the beginning of the frame. In fact, it is well known that the maximum throughput in a frame is achieved when the frame length r is equal to the terminal backlog size n . Equation (2) can be used to show numerically that, if N is known, in order to minimize $L(n)$, i.e., to maximize efficiency, the frame length must be set equal to the backlog size n at each frame. Efficiency results for this case are shown in the first row of Table I, where we can appreciate the fact that the performance decreases as N increases, asymptotically reaching the well known value of e^{-1} .

In our reference scenario, N is known in distribution. A procedure commonly used in this case is to derive an estimation \hat{n} of the backlog size at the beginning of each frame and set the frame length to this value, i.e. $r = \hat{n}$. Many backlog estimation mechanisms have been proposed. In

Table I

Values of throughput $n/L(n, r)$ versus n for Dynamic Frame Aloha, when n is estimated with Schoute's method. The two right-most columns have been evaluated by simulation.

	$N = 2$	$N = 3$	$N = 5$	$N = 10$	$N = 20$	$N = 30$	$N = 100$	$N = 500$	$N = 1000$
$N/L^*(N)$	0.500	0.471	0.441	0.413	0.395	0.387	0.375	0.370	0.369
$N/L(N, N)$	0.500	0.468	0.434	0.407	0.391	0.385	0.374	0.369	0.369
$N/L(N, 1)$	0.400	0.391	0.353	0.342	0.330	0.324	0.317	0.312	0.312
$N/L(N, 10)$	0.192	0.269	0.367	0.407	0.368	0.349	0.323	0.314	0.312
$N/L(N, 100)$	0.020	0.030	0.498	0.098	0.185	0.253	0.374	0.327	0.319

[10], Schoute aims at maximizing the throughput of a satellite channel running DF-Aloha, where traffic is assumed to be represented by a Poisson process with intensity γ [packet/slot]. The throughput to be maximized is then given by:

$$\eta_\gamma = \frac{\gamma}{L_\gamma} \quad (3)$$

where

$$L_\gamma = \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} e^{-\gamma} L(n). \quad (4)$$

Schoute proposes to estimate the backlog \hat{n} by counting the number of collided slots c at the end of the previous frame. Assuming that the procedure is able to keep a frame size equal to the backlog n , the number of terminals transmitting in a slot can be approximated by a Poisson distribution of average 1, such that the average number of terminals in a collided slot is:

$$H = \frac{1 - e^{-1}}{1 - 2e^{-1}} \simeq 2.39 \quad (5)$$

and the estimate is $\hat{n} = \text{round}(Hc)$. The average number of slots $L(N, r)$ to solve N collisions starting with a frame of length r is proven to be:

$$L(N, r) = r + \sum_{s=0}^m \sum_{c=0}^{\lfloor N/2 \rfloor} p_{s,c}(N, r) L(N - s, \hat{r}) \quad N \geq 2, \quad (6)$$

where $p_{s,c}(N, r)$ is the joint distribution of the number of successes s and collisions c , $m = \min\{N - 2, r - 1\}$, and $\hat{r} = N - s$ is function of N, r, s, c . Equations (6) can be solved starting from $L(0, r) = L(1, r) = r$.

In Table I, we have re-derived the throughput for different values of N with initial frame length $r_0 = N, r_0 = 1, r_0 = 10$ and $r_0 = 100$. Values up to $N = 100$ have been evaluated using (6), whereas values for $N = 500$ and $N = 1000$ have been obtained by simulating the algorithm. These values represent also the performance of an RFID system based on DF-Aloha with the Schoute estimation mechanism, when the population size is N . As we have already mentioned, this mechanism is tailored for a multiple access system with an average arrival rate per slot close to one and, therefore, we do not expect it to be optimal for the RFID environment.

Table II shows the optimum γ and throughput with Poisson distribution as evaluated by (3) with $r_0 = 1, 2, 3$. We see that the absolute optimum is for $r_0 = 1, \gamma = 1.11$ and the corresponding throughput is 0.4271 [10].

Table II

Throughput of Schoute's method when the initial number of terminals n is Poisson distributed with average γ , for different values of the starting frame length r_0 .

	$r_0 = 1$	$r_0 = 2$	$r_0 = 3$
γ_{opt}	1.11	2.15	3.1
η_{opt}	0.4271	0.4207	0.4157

Table III

Throughput of a population that is Poisson distributed with average γ , in the cases of Bayesian estimation, optimum strategy, and ideal estimation.

	Bayesian	Optimum	Ideal
γ_{opt}	1.11	1.12	1.24
η_{opt}	0.4271	0.4275	0.4377

A. Bayesian estimation

In the work of Floerkemeier [13] a way is suggested to evaluate the probability

$$P(E, S, C | j, r, n) \quad (7)$$

of observing exactly E empty, S successful, and C collided slots in j observed slots of a frame composed of r slots, being n the number of transmitting terminals, and, in our case, $j = r$. The evidence of the past frames from 1 to t , $z_{1:t}$, can then be defined as a set of vector reporting the outcome of all the slots of all the frames up to frame t . Eq. (7) can finally be used to evaluate the probability that n terminals are transmitting in the current frame:

$$P(n | z_{1:t}) = \alpha P(z_t | n, z_{1:t-1}) P(n | z_{1:t-1}) \quad (8)$$

where α denotes a normalizing constant. Since outcomes in frame t , given the number of terminals transmitting, is independent of previous frames, Eq. (8) reduces to

$$P(n | z_{1:t}) = \alpha P(z_t | n) P(n | z_{1:t-1}) \quad (9)$$

Equation (9), provides a recurrent expression by which the "a posteriori" probability distribution of the original population size n at frame t can be derived by the same expression at frame $t - 1$. It allows to evaluate the "a posteriori" probability distribution of the original population size n , conditioned to all the past history, starting from the "a priori" distribution of the number of transmitting terminals.

In Table III we show the optimum γ and throughput with Poisson distribution (3) for the Bayesian estimate \hat{n} of the backlog. The Bayesian strategy that takes into account all the history does not improve the performance with respect to the Schoute's estimate, a simple estimate based only the observation of the current frame.

B. Optimum strategy

In [15] it has been shown that setting $r = \hat{n}$ is not necessarily an optimal strategy. Authors suggest an approach based on Markov chains to find the optimal strategy. Unfortunately the numerical approach is rather involved, so we have resorted to an almost exhaustive investigation by exploring the tree of the different strategies.

A strategy is made of the frame lengths chosen at the end of a frame, and it depends on the past history. The set of the strategies is

$$\mathcal{R} = \{\text{NewFrame}(r)\}_{r=2}^{r_{\max}},$$

where "NewFrame"(r) means that a new frame of length r must be started, and r_{\max} is the maximum length for a frame. The number of participating terminals is generally known through a probability mass function, also indicated with *beliefs*, $\{n\} = \{P(N = n)\}_{n=0}^{N_{\max}}$. The expected number of slots needed for solving $\{n\}$, starting with a frame of length r is

$$L(\{n\}, r) = r + \sum_{s=0}^m \sum_{c=0}^{\lfloor N_{\max}/2 \rfloor} p_{s,c}(\{n\}, r) \min_{t \in \mathcal{R}} L(\{n\}_{s,c}, t), \quad (10)$$

where $m = \min\{N_{\max} - 2, r - 1\}$, the joint probability $p_{s,c}(\{n\}, r)$ given the beliefs and r is

$$p_{s,c}(\{n\}, r) = \sum_{n=0}^{N_{\max}} P(n) p_{s,c}(n, r),$$

being $p_{s,c}(n, r)$ the same appearing in (6), and

$$\{n\}_{s,c} \propto \begin{cases} P(n+s)p_{s,c}(n+s, r) & 0 \leq n \leq N_{\max} - s \\ 0 & N_{\max} - s < n \leq N_{\max} \end{cases}$$

are the updated beliefs.

The recursive evaluation of (10) induces the tree of the strategies. Each node of the tree has at most $|\mathcal{R}| = r_{\max} - 1$ branches, and the leaves of the tree are those nodes in which n is known, in fact the best strategy for known n is $r = n$.

Since the visit of all the branches of the tree may become infeasible, we resort to a method that yields a lower bound L_{LB} and an upper bound L_{UB} to L . We use a pruning procedure that drops some of the branches of the tree. Considering the set of pruned branches that leave the same node with a backlog n , the best pruned strategy is the one that presents a length l_o such that $l_o = \min_i l_i$. The pruning procedure prevents from knowing l_o ; nevertheless, we can always find a lower bound l_{lb} and an upper bound l_{ub} to l_o , that can be used with the pruned tree to evaluate L_{LB} and L_{UB} . In our case we have assumed l_{lb} as the performance of the ideal protocol in which n is known, and whose efficiencies are reported in Table I, whereas l_{ub} is assumed as the performance of Schoute's protocol. The pruning strategy takes into account the probability of reaching that particular node and stops the exploration of its branches if this probability is under a given threshold. Diminishing this threshold increases the number of nodes visited and narrows the difference between bounds.

We have evaluated the lower bound and the upper bound of the best strategy, when starting with a frame of length 1 and with an initial backlog distribution represented by a Poisson distribution, truncated to $N_{\max} = 8$. The threshold probability used for pruning the strategy tree has been set to 10^{-4} . With this setting the two bounds coincide up to the fourth decimal digit. Then we have chosen γ so that the coincident bounds are minimal. The results are shown in Table III. Again, we observe an almost zero improvement with respect to the Bayesian/Schoute's estimates, a surprising result. In the same Table we have reported also the performance of the Ideal case, that refers to a perfect estimate of the backlog. This result has been derived from the data that appears in row one in Table I. Its performance, 0.4377 [pkt/slot], constitutes the ultimate upper bound to the performance of any estimation procedure.

III. DF-ALOHA WITH FRAME RESTART

In this section we consider the capability of DF-Aloha of restarting a new frame when the further exploration of the current one is no longer considered useful.

A. Bayesian estimation

We follow the work of Floerkemeier [13] already cited in Section II-A. Here equations (8) and (9) are enriched with the evidence $o_{1:j}$ from the first j slots in the current frame, in addition to the evidence $z_{1:t}$ from previous frames. Namely, $o_{1:j}$ is the vector which includes the observations of the current frame from the first slot to slot j , with each element o_i expressing the observed status of slot i ($o_i \in \{E, S, C\}$). We can write:

$$P(n|z_{1:t}, o_{1:j}) = \alpha P(n|z_{1:t}, o_{1:j-1}) P(o_j|n, z_{1:t}, o_{1:j-1}) \quad (11)$$

where α denotes a normalizing constant. Since consecutive frames are independent given the number of terminals transmitting, the following holds

$$P(o_j|n, z_{1:t}, o_{1:j-1}) = P(o_j|n, o_{1:j-1}). \quad (12)$$

Again, equation (12), provides a recurrent expression by which the "a posteriori" probability distribution of the original population size n , at slot j frame t can be derived by the same expression at slot $j-1$ of frame t , starting from the "a priori" distribution of the number of transmitting terminals. The "a posteriori" probability p_n is then used to find the frame length r that maximizes the throughput of the next slot

$$E[S(r)] = \sum_n S(n, r) p_n = \sum_n \frac{n}{r} \left(1 - \frac{1}{r}\right)^{n-1} p_n. \quad (13)$$

This maximization is carried out slot by slot and a new frame is restarted whenever the frame length so determined changes with respect to the old one.

Unfortunately in [13] the expression for (12) is provided only by way of transforms, a method hardly suited to evaluate distributions when n is large. It is based on probability (7) that is hard to evaluate when $j \neq r$. A close-form expression of probability (7) is given in [14], but even this formula has scarce

Table IV

Throughput of a Poisson population with average γ , with frame restart and three different strategies.

	Bayesian	modification 1	Floerkemeier
γ_{opt}	1.11	1.16	1.18
η_{opt}	0.4624	0.4680	0.4687

practical utility. Therefore, to derive results on this procedure, we have proposed a new evaluation method that evaluates (7) iteratively in r , the number of slots in the frame. To ease up presentation, the proposed procedure is illustrated in Appendix A.

Throughput (3) of this procedure is reported in Table IV. In the case of the first column a new frame is restarted if \hat{n} does not match the number of remaining slots in the frame. We also report the performance of two other cases that behave slightly better. In the first modification we do not allow to interrupt the second frame, which is composed of two slots. This provides a better estimate even with a slight penalty in the instantaneous throughput, and results in higher efficiency. In the case denoted as Floerkemeier we maintain the first modification and restart a new frame according to the maximum throughput measure (13) as suggested in [13].

B. Backlog Lower Bound

The estimation procedure exposed in the previous section may be too complex to be implemented. However, we have observed that most of the gain of the Bayesian technique lies in avoiding the exploration of the last slot of the frame when a collision is certain. A simple mechanism capable to provide a lower bound on the backlog size can retain the same advantage. Thus, we adopt the following mechanism, named Backlog Lower Bound (BLB), that represents both a lower bound and good estimate on the backlog size n :

$$\begin{cases} \hat{n}_1 &= 2 \\ \hat{n}_k &= \max\{\hat{n}_{k-1} - s_k, 2c_k\} \end{cases} \quad (14)$$

being s_k and c_k respectively the number of successes and observed collisions in frame k up to that point. Here the *max* operation presents the advantage of including a relevant part of the memory of the past frames. To get a lower bound of current backlog, the number of colliding transmissions in a collided slot can be set to 2, thus leading to a backlog estimate of $\hat{n} = 2c_k$. This choice is adequate since we have verified that if it is used in place of the Schoute's estimate $2.39c_k$, owing to the small γ , the performance of the Schoute's protocol is practically not changed.

The BLB protocol operates as follows: the last slot of the frame k gives rise to a collision whenever we have $\hat{n}_k - s_k \geq 2$ and $c_k = 0$, and, to avoid a waste of a slot, a new frame is started; otherwise, the last slot is observed, \hat{n}_k is updated and a new frame is started. In both cases the length of the new frame is $r_{k+1} = \hat{n}_{k+1}$.

The performance of this algorithm can be evaluated by suitably changing relation (6). Denoting by $L(n, r, b)$ the expected number of slots needed to resolve n terminals starting

with a frame of length r and lower bound b for the backlog, the following holds

$$\begin{aligned} L(n, r, b) &= \\ &= r + \sum_{s=0}^m \sum_{c=2}^{\lfloor n/2 \rfloor} p_{s,c}(n, r) L(n-s, \hat{r}, \max\{b-s, 2c\}) \\ &\quad + \sum_{s=0}^{b-2} p_{s,1}(n, r) \left(-\frac{1}{r} + L(n-s, \hat{r}, \max\{b-s, 2\}) \right) \\ &\quad + \sum_{s=b-1}^m p_{s,1}(n, r) L(n-s, \hat{r}, \max\{b-s, 2\}) \quad n \geq 2, \end{aligned}$$

where $\hat{r} = \max\{b-s, 2c\}$ is the estimate for the new frame length, $m = \min\{n-2, r-1\}$. The double summation of (6) has been rearranged to isolate the case $c=1$: when $c=1$ and $0 \leq s \leq b-2$ the last slot can be saved with probability r^{-1} , hence the term $-1/r$ is added in the second summation.

In Table VI we have reported the throughput under Poisson statistics, and the value of γ that maximizes such throughput. As we can see this suboptimal mechanism provides a Poisson performance only 2% below the Bayes mechanism.

C. Optimum strategy

The performance evaluation of the optimal strategy under frame restart goes along the same lines those presented in Section II-B.

If the current frame can be interrupted, then at each slot there is available a set of strategies \mathcal{S} made as follows

$$\mathcal{S} = \{\text{Continue}, \{\text{NewFrame}(r)\}_{r=2}^{r_{\max}}\},$$

where the strategy "Continue" means that the current frame must be further explored, and "NewFrame(r)" means that a new frame of length r must be set. The total number of choices that can be taken at each slot is $|\mathcal{S}| = r_{\max}$, but this number can be significantly lowered by neglecting the strategies that are surely suboptimal at a certain slot.

It is evident that the optimal decision to be taken after having observed a slot must depend on the entire past history, being the history made of the slots' outcomes and past decisions. Specifically, the past history serves to update the beliefs about the backlog, $\{n\}$, and about the number of terminals that will transmit in the rest of the current frame, $\{R\}$, that are the only statistics needed to take the optimal decision. These beliefs are inferred from the slots' outcomes and they are recursively updated as described by (11), (12) and in Appendices A and B.

Being in the j -th slot of a frame with length r , and having beliefs for the backlog $\{n\}$ and for the remaining terminals $\{R\}$, the average number of slots needed to conclude the inventory is

$$\begin{aligned} L(\{n\}, \{R\}, r, j) &= 1 \\ &+ \sum_{o \in \{E, S, C\}} p(O_j = o | \{R\}, r) \min_{s \in S} L(\{n\}_{s,o}, \{R\}_{s,o}, r_s, j_s), \end{aligned} \quad (15)$$

Table V

Values of upper and lower bounds of throughput $N/L(N)$ versus N with the optimal strategy. Optimum frame choices r_o are also indicated.

$N/L(N)$	$N = 2$ $r_o = 2$	$N = 3$ $r_o = 2$	$N = 4$ $r_o = 3$	$N = 5$ $r_o = 4$	$N = 6$ $r_o = 5$	$N = 7$ $r_o = 6$	$N = 8$ $r_o = 7$
Lower Bound	0.6666	0.6	0.5547	0.5235	0.5025	0.4864	0.4738
Upper Bound	0.6666	0.6	0.5549	0.5246	0.5035	0.4876	0.4752

Table VI

Throughput of a Poisson population with average γ , and frame restart under optimal strategies and ideal estimate.

	BLB	Bayesian	Optimum LB	Optimum UB	ideal
γ	1.19	1.18	1.2	1.2	1.51
η_{opt}	0.4583	0.4687	0.4693	0.4739	0.515

where O_j is the observation in slot j , $\{n\}_{s,0}$ and $\{R\}_{s,o}$ are the beliefs updated according to the specific slot outcome and the strategy adopted after the j -th slot. The new frame length is $r_s = r$ if $s = \text{"Continue"}$ and it is $r_s = t$ if $s = \text{"NewFrame}(t)$ ". The next slot index is $j_s = j + 1$ if $s = \text{"Continue"}$ and it is $j_s = 1$ if $s = \text{"NewFrame}(t)$ ". If the beliefs indicate that all terminals have been identified, then L equals zero. The probabilities appearing in (15) are

$$\begin{aligned}
 p(O_j = E|\{R\}, r) &= \sum_{n=0}^{N_{\max}} \left(1 - \frac{1}{r-j+1}\right)^n R_n \\
 p(O_j = S|\{R\}, r) &= \sum_{n=0}^{N_{\max}-1} \frac{(n+1)(r-j)^n}{(r-j+1)^{n+1}} R_{n+1} \\
 p(O_j = C|\{R\}, r) &= 1 - p(O_j = E|\{R\}, r) \\
 &\quad - p(O_j = S|\{R\}, r).
 \end{aligned}$$

Note that (15) can be evaluated in a recursive manner, where each function call makes at most $3|S| = 3r_{\max}$ calls. Since the visit of all the branches of the call tree may become infeasible, we adopt a pruning procedure as the one exposed in Section II-B, that allows to determine a lower bound L_{LB} and an upper bound L_{UB} to the actual value of L .

We have first evaluated the Ideal case where N is known. In this case we have assumed zero as l_{LB} and the Backlog Lower Bound procedure to obtain l_{UB} . Results are shown in Table V, where we can verify that, for $2 < n \leq 8$, the optimal frame length is $r_o = n - 1$, a fact already cited in [16], that differentiates this case with frame restart with respect to the complete exploration of the frame, whose optimal strategy is always setting $r_o = n$.

In the second row of Table VI we have reported the lower bound and the upper bound of the optimum strategy, when starting with a frame of length 1 and with an initial backlog distribution represented by a Poisson distribution truncated to $N_{\max} = 8$, with mean value γ (first row). For the evaluation of the lower bound we have used the value of L obtained for the case of Ideal shown in Table V. For the upper bound we have used the value of L obtained by Backlog Lower Bound procedure.

For sake of comparison in Table VI we have also reported the throughput of the Bayesian estimate already given in

section III-A. We have also reported the Poisson throughput of the ideal procedure using the values reported in Table V. The results shows that there is small room for an improvement with respect to the Bayesian estimate.

Finally, Figure 1 shows the initial part of the optimal strategy tree corresponding to the bounds reported. The nodes of the tree represent different stages of the exploration of the frames. In each node a string represents the outcomes of slots (S,E,C), spaces representing the frame boundaries, the number in brackets the length of the next slot, and the dash indicates the further exploration of the frame. We see that the second frame is always composed by two slots that are always explored except when a collision is certain.

IV. CONCLUSIONS

In this paper we have investigated the performance of the Dynamic Frame Aloha protocol under a population that is Poisson distributed, for different strategies in setting the frame length and backlog estimation mechanisms, in the two cases in which either the frame is entirely explored or a new frame can be restarted at any slot. We have re-derived the performance with estimation mechanism already proposed in literature, such as the Shoute's and Bayesian estimates. We also have proposed a new estimate, for the Frame restart case, that is simpler than the Bayesian one but almost as efficient. We have also devised a new method to determine the performance of absolute best strategies. We have found that, if the frame is entirely explored, the Shoute's estimate performs very well, exactly as Bayesian does, reaching 0.4271 [pkt/slot], very close to the absolute optimum of 0.4275 [pkt/slot]. When the frame can be restarted, the Bayesian estimate features 0.4687 [pkt/slot], while our proposal, much simpler by far, can reach 0.4583 [pkt/slot]. In this case, the optimum performance lies somewhere between 0.4693 [pkt/slot] and 0.4739 [pkt/slot]. The attained results show that, contrary to common belief, Dynamic Frame Aloha can reach efficiency very close to the best observed in random access protocols, that is, 0.487 [pkt/slot], reached by the Gallager-Tzybakov version of the Tree Protocols.

APPENDIX A

Referring to (12), we want to compute the expression $P(o_j|n, o_{1:j-1})$ that can be written as

$$P(o_j|n, r, o_{1:j-1}) = \frac{P(o_{1:j}|n, r)}{P(o_{1:j-1}|n, r)}$$

having indicated also the frame length r , $j \leq r$. If we denote by n_j the number of terminals transmitting in the first j slots

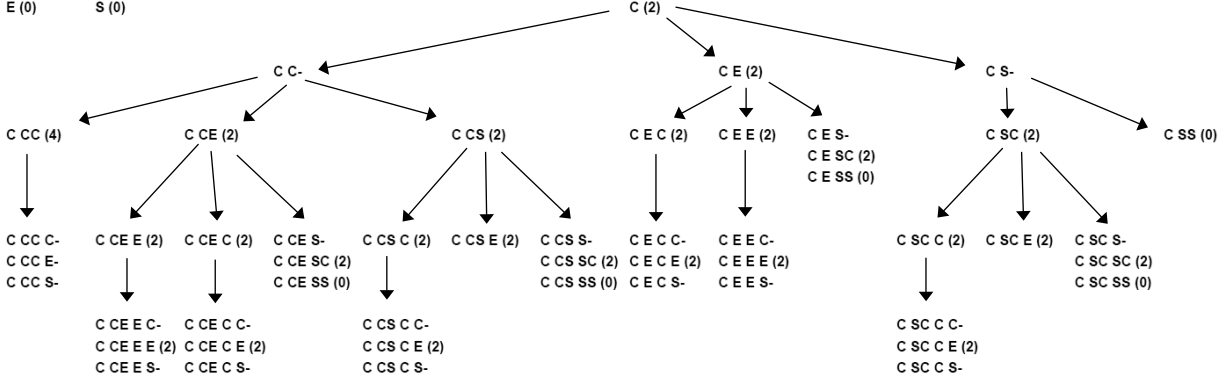


Figure 1. The initial part of the optimal strategy tree when the population is the truncated Poisson distributed with average 1.2.

of the frame we have

$$P(o_{1:j}|n, r) = \sum_{n_j=0}^n P(o_{1:j}|n_j, n, r) \cdot P(n_j|n, r) = \sum_{n_j=0}^n P(o_{1:j}|n_j, j) \cdot \binom{n}{n_j} \left(\frac{j}{r}\right)^{n_j} \left(1 - \frac{j}{r}\right)^{n-n_j}.$$

Now we show that term $P(o_{1:j}|n_j, j)$ can be easily derived when $P(o_{1:j-1}|n_{j-1}, j-1)$ is known, thus allowing a recursive evaluation in j for each n_j , starting from $P(o_1|n_1, 1)$.

We write

$$P(o_{1:j}|n_j, j) = P(o_{1:j-1}|o_j, n_j, j)P(o_j|n_j, j),$$

Terms $P(o_j|n_j, j)$ are given by the binomial expressions

$$P(E|n_j, j) = \left(1 - \frac{1}{j}\right)^{n_j},$$

$$P(S|n_j, j) = \frac{n_j}{j} \left(1 - \frac{1}{j}\right)^{n_j-1},$$

$$P(C|n_j, j) = 1 - P(E|n_j, j) - P(S|n_j, j).$$

Terms $P(o_{1:j-1}|y_j, n_j, j)$ are evaluated as

$$P(o_{1:j-1}|O_j = E, n_j, j) = P(o_{1:j-1}|n_{j-1} = n_j, j-1),$$

$$P(o_{1:j-1}|O_j = S, n_j, j) = P(o_{1:j-1}|n_{j-1} = n_j - 1, j-1),$$

$$P(o_{1:j-1}|O_j = C, n_j, j) = \frac{1}{P(O_j = C|n_j, j)} \times$$

$$\times \sum_{x=0}^{n_j-2} P(o_{1:j-1}|n_{j-1} = x, j-1) \binom{n_j}{x} \left(1 - \frac{1}{j}\right)^x \left(\frac{1}{j}\right)^{n_j-x}.$$

The initialization of the iterative method is

$$P(E|n_1, r = 1) = 1 \text{ for } n_1 = 0 \text{ and } 0 \text{ elsewhere,}$$

$$P(S|n_1, r = 1) = 1 \text{ for } n_1 = 1 \text{ and } 0 \text{ elsewhere}$$

$$P(C|n_1, r = 1) = 1 \text{ for } n_1 > 1 \text{ and } 0 \text{ elsewhere.}$$

APPENDIX B

Let us denote with

- Y_i the random variable that counts the terminals that transmit from the $(i+1)$ -th slot to the last slot of the frame, and by y_i its current value;
- X_i the number of terminals that transmit in the i -th slot of the frame, and by x_i its current value;
- O_i the outcome of the i -th slot of the frame, and by o_i its current value, that can assume one of the values $\{E, S, C\}$.

Given the *a priori* distribution of the number of remaining terminals at the beginning of the frame

$$P(Y_0 = n) \quad 0 \leq n \leq N_{\max},$$

the distribution of Y_1 given the number of terminals X_1 that have transmitted in the first slot is

$$\begin{aligned} P(Y_1 = n|X_1 = x) &= P(Y_0 = n+x|X_1 = x) \\ &\propto P(X_1 = x|Y_0 = n+x) \cdot P(Y_0 = n+x) \\ &= \binom{n+x}{x} \left(\frac{1}{r}\right)^x \left(1 - \frac{1}{r}\right)^n P(Y_0 = n+x) \end{aligned}$$

for $n = 0, \dots, N_{\max} - x$. Note that typically the reader is not able to identify the number of colliding terminals in a slot, hence we have to find the distribution of Y_1 conditioned to the available observation O_1 , that is easily found by observing that

$$P(Y_1 = n|O_1 = E) = P(Y_1 = n|X_1 = 0),$$

$$P(Y_1 = n|O_1 = S) = P(Y_1 = n|X_1 = 1),$$

and

$$\begin{aligned} P(Y_1 = n|O_1 = C) &= P(Y_1 = n|X_1 > 1) \\ &= \sum_{x=2}^{N_{\max}-n} P(X_1 = x)P(Y_1 = n|X_1 = x) \\ &= \sum_{x=2}^{N_{\max}-n} \binom{n+x}{x} \left(\frac{1}{r}\right)^x \left(1 - \frac{1}{r}\right)^n P(Y_0 = n+x). \end{aligned}$$

Generalizing, given the distribution

$$P(y_{k-1}|o_1, o_2, \dots, o_{k-1}) = P(y_{k-1}|\mathbf{o}_{k-1})$$

the distribution of Y_k , having observed the slots up to the k -th, results into

$$\begin{aligned} & P(Y_k = n|\mathbf{o}_{k-1}, X_k = x) \\ &= P(Y_{k-1} = n + x|\mathbf{o}_{k-1}, X_k = x) \\ &\propto P(X_k = x|\mathbf{o}_{k-1}, Y_{k-1} = n + x)P(Y_{k-1} = n + x|\mathbf{o}_{k-1}) \\ &= \binom{n+x}{x} \frac{(r-k)^n}{(r-k+1)^{n+x}} P(Y_{k-1} = n + x|\mathbf{o}_{k-1}). \end{aligned}$$

Since the observations are drawn from the set $\{E, S, C\}$, the updating reads as follows

$$\begin{aligned} P(Y_k = n|\mathbf{o}_{k-1}, o_k = E) &\propto \\ &\propto \left(\frac{r-k}{r-k+1} \right)^n P(Y_{k-1} = n|\mathbf{o}_{k-1}), \end{aligned}$$

$$\begin{aligned} P(Y_k = n|\mathbf{o}_{k-1}, o_k = S) &\propto \\ &\propto \frac{(n+1)(r-k)^n}{(r-k+1)^{n+1}} P(Y_{k-1} = n+1|\mathbf{o}_{k-1}), \end{aligned}$$

$$\begin{aligned} P(Y_k = n|\mathbf{o}_{k-1}, o_k = C) &= P(Y_k = n|\mathbf{o}_{k-1}, X_k > 1) \\ &= \sum_{x=2}^{N_{\max}-n} P(X_k = x|\mathbf{o}_{k-1})P(Y_{k-1} = n+x|\mathbf{o}_{k-1}, X_k = x) \\ &= \sum_{x=2}^{N_{\max}-n} \binom{n+x}{x} \frac{(r-k)^n}{(r-k+1)^{n+x}} P(Y_{k-1} = n+x|\mathbf{o}_{k-1}). \end{aligned}$$

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