On Spectrum Selection Games in Cognitive Radio Networks

Ilaria Malanchini, Matteo Cesana, Nicola Gatti
Dipartimento di Elettronica e Informazione
Politecnico di Milano, Milan, Italy
Email: {malanchini, cesana, ngatti}@elet.polimi.it

***Abstract*—Cognitive Radio Networks aim at enhancing spectrum utilization by allowing cognitive devices to opportunistically access vast portions of the spectrum. To reach such ambitious goal, cognitive terminals must be geared with enhanced *spectrum management* capabilities including the detection of unused spectrum holes (*spectrum sensing*), the characterization of available bands (*spectrum decision*), the coordination with other cognitive devices in the access phase (*spectrum sharing*), and the capability to handover towards other spectrum holes when licensed users kick in or if a better spectrum opportunity becomes available (*spectrum mobility*).**

In this paper, a game theoretic framework is proposed to evaluate *spectrum management* functionalities in Cognitive Radio Networks. The spectrum selection process is cast as a non-cooperative game among secondary users who can opportunistically select the "best" spectrum opportunity, under the tight constraint not to harm primary licensed users. Different quality measures for the spectrum opportunities are considered and evaluated in the game framework, including the spectrum bandwidth, and the spectrum opportunity holding time. The cost of spectrum mobility is also accounted in the analytical framework. Numerical results are reported to assess the quality of the game equilibria.

I. Introduction

Cognitive Radio Networks (CRNs) are emerging as a viable solution to solve spectrum shortage/efficiency problems. CRNs are based on cognitive devices [1] which are able to configure their transmission parameters (e.g., frequency band, waveforms, etc.) on the fly depending on the surrounding environment, consequently exploiting under-utilized spectrum portions. The motivation for cognitive radio stems from various measurements of spectrum utilization which generally show that spectrum is under-utilized, in the sense that the typical duty cycle of spectrum usage at a fixed frequency and at a random geographical location is low. This means that there are many "holes" in the radio spectrum that may be exploited for use by wireless users other than the spectrum licensee. To this extent, even "unlicensed" (secondary) cognitive radio users can be allowed to use licensed spectrum, provided that they do not interfere with any primary licensed user.

The achievable efficiency of CRNs depends on the specific cognitive capabilities (and functionalities) of the network devices. As a first step of the *cognitive cycle* [2], cognitive nodes must be able to detect unused (or under-utilized) spectrum portions (*spectrum sensing*), and to characterize them on the basis of several parameters (e.g., bandwidth, holding time), which can be collected through local observations or statistical information on primary activity (*spectrum decision*). Once spectrum holes have been identified and characterized, one of the most challenging issues concerns the exploitation of the available resources. Depending on the specific quality of service requirements, several functions can be adopted by users to select the *best available spectrum portion*, and to coordinate with other cognitive devices in the access phase (*spectrum sharing*). Finally, secondary cognitive devices should support the capability to handover towards other spectrum holes when licensed users kick in or if a "better" spectrum opportunity becomes available (*spectrum mobility*).

In this work, we address the CRN scenario where greedy and selfish secondary users opportunistically exploit the spectrum portions vacated by primary users. To this extent, we propose a non-cooperative game theoretic framework to study the inherent competition among secondary users in the cognitive spectrum selection process. Namely, the proposed framework accounts for:

- the time-varying radio environment in terms of availability and quality of the spectrum portions (*spectrum decision*);
- the interference among secondary users (*spectrum sharing*);
- the cost associated to spectrum handover (*spectrum mobility*).

Since one of the most important aspects of cognitive terminals resides in their rationality, game theory may be conveniently used to provide formal tools to model the interactions and competitions among users, to derive equilibrium criteria and to study the optimality and the stability of the solution [3].

We take here a constructive approach by analyzing, at first, a static game in which *spectrum mobility* is neglected, and secondary users evaluate different spectrum opportunities, considering both the quality-of-service and the corresponding congestion level. Then, we move to a dynamic game formulation which accounts for the temporal evolution of the system, the corresponding time-varying primary users activity, and the costs associated to spectrum mobility. In both cases, we characterize and derive the Nash equilibria of the games, and we thoroughly comment on their quality.

The paper is organized as follows: in Section II, we briefly overview previous works on the issue of spectrum management, highlighting the novelty of our contribution. Section III defines the problem statement. In Section IV and

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

Section V we formalize and analyze the static (one-shot) game and the dynamic game, respectively. Concluding remarks and comments on future activities are reported in Section VI.

II. Related Work

The problem of spectrum management has been widely investigated in the last few years, especially focusing on the different functionalities of the cognitive cycle. The main issues of spectrum sensing are highlighted in [4], whereas an optimal framework is proposed in [5] with the aim of addressing both the interference avoidance and the spectrum efficiency problem. In [6], the concept of *time-spectrum block* is introduced to model the spectrum allocation problem in cognitive networks, and a distributed protocol is presented to enable users to share the spectrum holes. The problem of spectrum management is addressed also in [7], where a central spectrum policy server coordinates spectrum demands, whereas distributed approaches are adopted in [8] and in [9]. A comprehensive survey is provided in [10].

As pointed out [10], the issue of spectrum decision, which is of paramount importance to determine the success of CRNs, is still scarcely addressed in the literature, with the notably exception of [11] and [12]. In [11], spectrum matching algorithms are presented to support QoS for cognitive radio users. The spectrum decision is based on statistical characteristics of spectrum bands. In [12], the authors propose and compare three schemes for frequency channel selection that are based on the load and interference perceived in each channel by users. In our work, we follow a similar approach, but, in addition, we analyze the interaction among users and the temporal evolution of the system.

Most of the aforementioned work addresses the problem of spectrum management proposing protocols and algorithms aimed at handling specific functionalities of cognitive nodes. Differently, we provide here a comprehensive theoretical framework to analyze the spectrum management problem. Since we thoroughly resort to game theory, it is worth commenting on other game theoretic approaches applied to cognitive radio networks. Solutions resorting to game theory for modeling the interactions among cognitive users in sharing the same spectrum portions may be generally distinguished with respect to the inspiration of the game models they resort to; namely, in cooperative and non-cooperative game models.

Cooperative game models assume the cooperation among users, who decide to coordinate each other in order to maximize the global utility of the system. In [13], the authors analyze a multi-hop wireless scenario where nodes need to agree on a fair allocation of spectrum. In [14], the authors focus on the spectrum access problem, and propose a resource allocation algorithm to maximize spectrum efficiency. Both these works leverage the concept of Nash Bargaining Solution to guarantee fairness and optimality. Nevertheless, in several network scenarios, the cooperation among users postulates the existence of a common control channel to be used to distribute cooperation signalling traffic. On the other side, works based on non-cooperative game models drop the cooperation assumption and study competitive scenarios where players are selfish entities. As an example, the authors of [3] cast the spectrum sharing as a non cooperative game, and derive general guidelines to implement local (and greedy) algorithms to drive the sharing decision of the users. Similarly, in this paper we target non-cooperative scenarios where the secondary cognitive users act selfishly to maximize local utility functions. Different from the aforementioned piece of work, we include in the game formulation also aspects of spectrum decision and spectrum mobility, besides spectrum sharing ones.

A relevant aspect that can be further considered in the definition of secondary users strategies is the temporal evolution of the system. This can be modeled using extensive-form games. In [15], the authors propose non-cooperative repeated game in which users leverage the outcome of previous game stages to improve their own revenues. In [16], the authors model the problem of spectrum sharing as an oligopoly market competition and use a Cournot game to obtain the spectrum allocation for secondary users. Since users are supposed to have partial knowledge on other users' strategies, an extensive-form game is proposed, in which players adapt their strategies by observing the evolution of the game. Secondary users coexistence issue is considered also in [17]. Different from these valuable approaches, we consider the variability of the underlying primary users, which forces time varying access strategies (Spectrum OPportunities, SOPs) for the secondary users (players), and we characterize the temporal evolution of the spectrum management process by secondary users.

III. Spectrum Selection Game Model

We consider a scenario composed of primary and secondary users sharing a given portion of the spectrum, which is subdivided into orthogonal channels (sub-bands), throughout the paper referred to as Spectrum OPportunities (SOP) (see Fig. 1). Each primary user is licensed to transmit arbitrarily on a specific sub-band (SOP). Time is divided into epochs which can be defined as the time period where the activity of primary users does not change[1][2]. Secondary users can opportunistically access SOP which are vacant in a given epoch, with the firm constraint to handover whenever the SOP gets occupied by a licensed primary user.

For the sake of presentation, the interference relations among users can be modeled through interference ranges, i.e., when two users are closer than these ranges, they interfere. If a primary user is transmitting on a given SOP, any secondary user which is closer than the primary interference range (R_p) must remain silent on that SOP. In a similar way, two secondary users do interfere when transmitting on the same SOP if they are closer than the secondary interference range (R_s).

We denote by $\mathcal{B}$ the set of SOPs, by $\mathcal{N}$ the set of secondary users, and by $\mathcal{B}_i$ ($i \in \mathcal{N}$) the subset of spectrum opportunities available for secondary user i, i.e., SOP with no primary users or with an active primary user that is out of primary

[1] Conversely, the primary users' activities can change from epoch to epoch.

[2] Note that we are not concerned here with the timescale of epochs, which depends on the time variability of the primary users' activity.

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

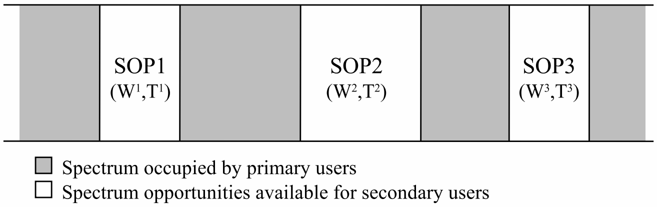


Fig. 1. Example of spectrum opportunities characterized by different parameters (e.g., bandwidth W and holding time T).

interference range. In a time-varying scenario, due to the primary activity, the set $\mathcal{B}_i$ can change at each epoch. The activity of each primary user is modeled as binary traffic source driven by the evolution of two-state Markov chain, with parameters p_j, probability for primary user j to become active given that it was not active at the previous epoch, and q_j, probability to deactivate given that it was active in the previous epoch. Finally, we denote by $\mathcal{N}_i$ the subset of secondary users that interfere with i.

In our game model, the secondary users are the players and their strategy space is composed of the available SOPs. (Hereafter, we use the terms "users" and "players" to refer to secondary users.) We assume each user to be rational, choosing the SOP that maximizes the perceived quality-of-service (QoS), which depends on the features of each SOP, in terms of SOP bandwidth and SOP congestion, i.e., number of other users sharing the same SOP. To this extent, we associate to each possible strategy (available SOP) a cost function $c_i(k, x_i^k)$ that is specific for user i, for the chosen spectrum opportunity k and, in general, depends on the aforementioned factors (i.e., spectrum opportunity characteristics). Moreover, $c_i(k, x_i^k)$ is monotonically increasing in x_i^k, that is the congestion level of resource k perceived by user i (number of users selecting SOP k and belonging to $\mathcal{N}_i$).

The Spectrum Selection Game (SSG) can be formally defined as:

$$SSG = \langle \mathcal{N}, \{\mathcal{B}_i\}_{i\in\mathcal{N}}, \{c_i(k, x_i^k)\}_{i\in\mathcal{N},k\in\mathcal{B}_i} \rangle.$$

By definition, the generic end user i selfishly plays the strategy $\overline{k} \in \mathcal{B}_i$ which minimizes her experienced cost function, that is:

$$\overline{k} = \operatorname*{argmin}_{k\in\mathcal{B}_i} c_i(k, x_i^k).$$

It can be shown that SSG belongs to the class of *congestion game* [18]. Specifically, it can be reduced to a *crowding game* [19], i.e., a *single-choice* (each player selects only one resource) congestion game with *player-specific* cost functions (each player can have a different cost function, with the constraint to be increasing in the level of congestion). Moreover, the game is *non-weighted*, since users congest the resources with the same weight and with linear cost functions. For the sake of brevity, we skip the formal reduction of our game to this class of games and we report only the sketch of the proof in Appendix A. The class of congestion games with resource reuse and the same result obtained in Appendix A are differently presented also in [20].

From this equivalence it directly comes that also our game does admit at least one pure-strategy Nash equilibrium[3] [19] for any cost function $c_i(k, x_i^k)$ that is increasing in the level of the congestion. Therefore, we can safely limit us to search for pure strategy equilibria.

IV. Analysis of the Spectrum Selection Game

In this section, we consider the static game, representing a single time epoch, in which available spectrum opportunities for each user are a fixed subset of $\mathcal{B}$. In Section IV-A we define different cost functions, i.e., different implementations of $c_i(k, x_i^k)$ in the Spectrum Selection Game, accounting for SOP quality parameters related to *spectrum decision* and *spectrum sharing*. In Section IV-B we focus on the characterization of the corresponding Nash equilibria, by resorting to mathematical programming formulation of the SSG; first, we evaluate different quality equilibria, obtained with a specific cost function, using the well-known concepts of Price of Stability (PoS) and Price of Anarchy (PoA), then, we compare equilibria associated to different cost functions.

A. Assessing the Quality of SOP

In general, the definition of a cost function for secondary users depends on the amount of available information they have. At first, we consider the case where the number of users sharing the same SOP is known. In this case, users may select a SOP on the basis of a cost function that depends on the number of interferers. Formally, we have the following interference-based cost function:

$$c_i(k, x_i^k) = x_i^k \tag{1}$$

Nevertheless, also other pieces of information may be available at the secondary users; as an example, SUs may get to know the nominal bandwidth associated to the given SOPs, as well as the average SOP availability time. Such type of information can be obtained by the secondary users through direct observations of the channel, and/or implementing some type of predicting/learning techniques. In this work, we are not concerned on how the information can be obtained, but, on the other end, we aim at evaluating the impact of this further knowledge on the secondary users spectrum management policies.

Since we are dealing with cost functions, we consider parameters that are in inverse proportion to SOP quality parameters, the bandwidth and the holding time (larger the bandwidth and the holding time, lower the cost function). In general, these parameters are *user-specific*, i.e., they are different for each user. For this reason, we introduce the parameter W_i^k, that is in inverse proportion to the bandwidth that user i can obtain from SOP k, and T_i^k, that is in inverse proportion to the average holding time of SOP k for user i.

We consider two different cost functions that include the number of interferers, the bandwidth and the expected holding

[3] A Nash equilibrium is a set of strategies such that no player has incentive to unilaterally change her action.

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

time. The first cost function is just a linear combination of the three factors (interference, bandwidth and holding time):

$$c_i(k, x_i^k) = \lambda_i x_i^k W_i^k + (1 - \lambda_i) T_i^k \quad (2)$$

The first term represents the portion of bandwidth perceived by each user, that is the total available bandwidth divided among x_i^k users (we recall that W_i^k is proportional to the inverse of the bandwidth). The second term is simply the expected holding time. The weights λ_i represent the preference of user i to the bandwidth with respect to the holding time.

The second cost function we consider is the product of the three defined parameters:

$$c_i(k, x_i^k) = x_i^k W_i^k T_i^k \quad (3)$$

In general, neither of these two cost functions overcomes the other, a priori. In fact, we can observe that the cost function (3), represents the total amount of bandwidth that can be used by each user, defined as: [Bandwidth $\cdot$ Time/Interfering Users] whereas cost function (2) does not reflect this property. On the other side, cost function (2) allows users to give a preference between bandwidth and holding time, whereas the other does not.

B. Experimental Evaluation

The experimental setting used in our simulations is a network deployed on a square area with edge L, composed by m primary users and m corresponding bands, n secondary users and circular interference radii $R_p = R_s = r$. We have implemented an instance generator that randomly draws the position of users and generates primary traffic on the basis of activity parameter p_j.

For the sake of simplicity, we assume that spectrum characteristics are the same for all users, i.e., W_i^k and T_i^k do not depend on i, and secondary users have the same preference, i.e., the same weight λ.

Inspired by simulation scenario used in [5], we classify available spectrum bands in six classes, resulting from the combination of *low/medium/high activity* and *low/high opportunity*. The level activity depends on p (larger p, higher activity) whereas high/low opportunity represent the spectrum bands with $p < q$ and $p > q$, respectively. Table I reports the parameters associated to each one of the $m = 18$ considered bands.

Clearly, the achievable throughput that a user can obtain using a specific SOP depends not only on the bandwidth, but also on other parameters related to the specific radio technology, such as the modulation scheme, the coding rate, and so on . In this scenario, we do not consider these transmission details, and we only make the reasonable assumption that the actual throughput is somehow proportional to the available bandwidth. Finally we remark that the average holding time can be calculated as the inverse of p. Given the average bandwidth and holding time, we derive parameters W^k and T^k as specified in Table I.

As mentioned before, to study the quality of equilibria we evaluate the PoS and the PoA. More precisely, the PoS (PoA) is the ratio between the best-case (worst-case) social cost at a Nash equilibrium and the optimal social cost that can be achieved by a central authority. The social cost is defined as the sum of the costs of all the users. To find the best/worst equilibrium and the optimal solution of our game we resort to a mathematical programming model, whose formulation is provided in Appendix B. We have formalized the problem with AMPL [21] and solved it with CPLEX commercial solver [22]. All the results reported are averaged on 100 randomly generated instances, varying the position of users, and varying the primary activity according to parameters reported in Table I.

Table II shows the results obtained on a uniform topology with $n = 20$ secondary users, in case $L = 500$ and $r = 100$ meters. We report the average parameters ($\overline{x}_i^k$, $\overline{T}^k$, $\overline{W}^k$) obtained by users at the best Nash equilibrium solution, using the the three proposed cost functions and varying λ for the function (2). Moreover, we evaluate also the actual bandwidth [*KHz*] and the holding time [*sec*] perceived by users. Finally, to compare the best/worst equilibria with the optimal solution, we report the value of PoS and PoA.

We can observe that using cost function (1) only the number of interferers is minimized, whereas W^k and T^k are in general greater than one. On the other side, using cost function (2), we can give different weights to bandwidth and holding time, varying the value of λ. Finally, cost function (3) provides a good tradeoff among all the considered factors.

We can further observe that PoS and PoA are both very close to one. This means that there is no difference, in terms of social cost, between the optimal solution, that could be reach with a coordination among users, and the equilibria that users reach playing selfishly.

In Fig. 2 we report the probability for a generic secondary user to occupy one of the eighteen SOPs numbered as in Table I. First, we can observe that using cost function (1) users spread almost evenly among different SOPs, since they are considered equal in terms of bandwidth and holding time. In contrast, using cost function (2) with $\lambda = 1$, users choose based on SOP's bandwidth and interference level only, thus high bandwidth SOPs are favored. Finally, using cost function (2) with $\lambda = 0.5$ and cost function (3), users take a tradeoff choice still favoring SOPs with high bandwidth and high holding time. In general, we can also observe that SOPs with lower activity are preferred by secondary users.

V. DYNAMIC SPECTRUM MANAGEMENT AS A MULTI-STAGE GAME

In this section, we consider a time-varying scenario, as the one shown in Fig. 3. In this case, secondary users are required not only to classify different SOPs and choose the best available one, but also to move to a new SOP whenever a primary user kicks in, or switching is still more convenient (*spectrum mobility*). In Section V-A we define the repeated game, proposing a new cost function that users can adopt playing the game. Then, in Section V-B, we evaluate the performance of the proposed model.

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

TABLE I
SPECTRUM OPPORTUNITIES FEATURES

Spectrum Class	Low Activity						Medium Activity						High Activity					
	Low Opportunity			High Opportunity			Low Opportunity			High Opportunity			Low Opportunity			High Opportunity		
Spectrum band k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
p	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5	0.5	0.5	0.5	0.5	0.8	0.8	0.8	0.8	0.8	0.8
q	0.1	0.1	0.1	0.5	0.5	0.5	0.3	0.3	0.3	0.8	0.8	0.8	0.3	0.3	0.3	0.9	0.9	0.9
Bandwidth [KHz]	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70	250	100	70
W^k	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5	1	2.5	3.5
Holding Time [sec]	5	5	5	5	5	5	2	2	2	2	2	2	1.25	1.25	1.25	1.25	1.25	1.25
T^k	1	1	1	1	1	1	2.5	2.5	2.5	2.5	2.5	2.5	4	4	4	4	4	4

TABLE II
RESULTS OBTAINED ON A UNIFORM TOPOLOGY WITH $n = 20$ SECONDARY USERS, $L = 500$ AND $r = 100$ METERS

Cost Function	(1)	(2)											(3)
		$\lambda = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
$\overline{x}_i^k$	1.000	3.250	1.220	1.220	1.220	1.220	1.220	1.030	1.030	1.000	1.000	1.000	1.220
$\overline{W}^k$	2.186	1.008	1.135	1.135	1.135	1.135	1.098	1.000	1.000	1.000	1.000	1.000	1.060
Bandwidth [KHz]	150.83	249.25	236.50	236.50	236.50	236.50	240.25	250.00	250.00	250.00	250.00	250.00	244.00
$\overline{T}^k$	2.250	1.000	1.000	1.000	1.000	1.000	1.038	1.278	1.278	1.323	1.323	2.125	1.075
Holding Time [sec]	3.250	5.000	5.000	5.000	5.000	5.000	4.925	4.445	4.445	4.389	4.389	3.270	4.850
PoS	1.000	1.000	1.004	1.008	1.012	1.015	1.025	1.004	1.006	1.000	1.000	1.000	1.043
PoA	1.000	1.000	1.030	1.059	1.086	1.111	1.116	1.206	1.091	1.042	1.022	1.000	1.092

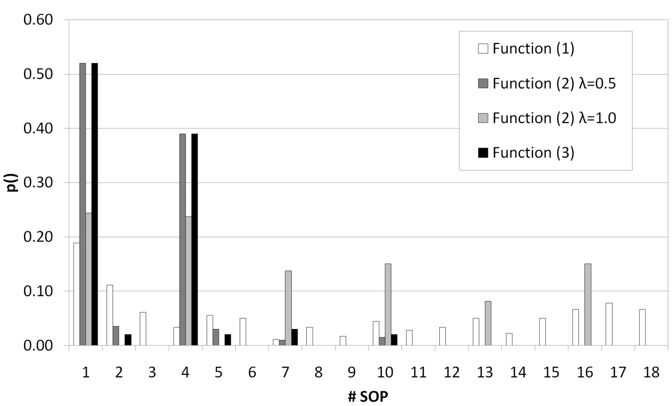


Fig. 2. Probability for a generic secondary user to occupy a SOP when varying the cost function.

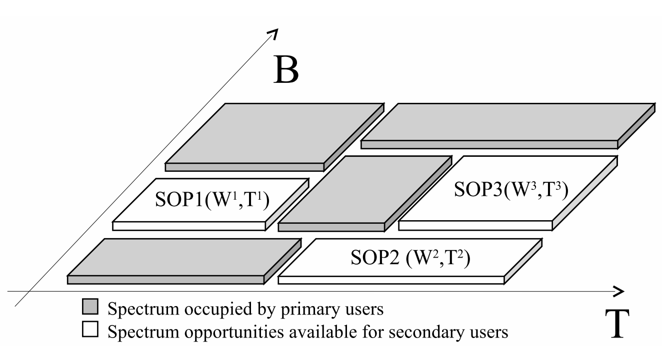


Fig. 3. Example of spectrum opportunities in a time-varying scenario.

A. Game Model

To capture the temporal evolution of the system, we repeat the previous game, i.e., the *static game*, multiple times. We assume that time is divided into epochs, and at each epoch primary users can stochastically activate or deactivate as described in Section III. In this way, the subset of SOPs available for each user can change. Playing the new stage of the game, we introduce a switching cost K_{mk}, that a user has to pay if it decides to change the spectrum opportunity from m to k. This cost can represent the switching delay experienced by a user switching over a wide frequency range. We assume $K_{mk} = 0$ whenever $m = k$, i.e., when user does not switch to a new SOP.

Therefore, the cost function each user tries to minimize is the following:

$$\widetilde{c}_i(k, x_i^k) = c_i(k, x_i^k) + K_{mk} \tag{4}$$

where m is the SOP used by the user i in the previous epoch and $c_i(k, x_i^k)$ may be anyone of the cost function previously defined. A multi-stage game is essentially an extensive-form game and therefore the appropriate solution concept is *subgame perfect equilibrium*, i.e., a strategy that is of equilibrium in each possible subgame. In our case, a subgame is representative of every possible state (e.g., characterized by primary users' activation and secondary users' undertaken actions) wherein the game can be. Due to the impressive number of possible subgames and the unavailability of information over the future states to users, the computation of a subgame perfect equilibrium is not practical in our case. Each user computes its optimal strategy on-line stage by stage on the basis of the past stages but ignoring the possible future evolution of the game.

B. Experimental Evaluation

We report here an experimental evaluation of the multi-stage game using a setting similar to the one used in Section IV-B. In this analysis, the instance generator randomly draws the position of users and generates primary traffic on the basis of activity parameters p_j and q_j over a time composed by E epochs.

We report here on the case in which users adopt the cost function (4), with $c_i(k, x_i^k)$ replaced by the function (3).

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

Moreover, the switching cost is assumed to be fixed, namely $K_{mk} = K$, $\forall m, k \neq m \in \mathcal{B}$, whereas $K_{mk} = 0$, $\forall m, k = m \in \mathcal{B}$. We consider a simulation scenario composed by $n = 20$ secondary users, $m = 9$ primary users, using a subset of $m = 9$ spectrum opportunities defined in Table I, and $E = 10$ epochs.

Fig. 4 reports the comparison between the two cases where the best Nash equilibrium and the best solution (with respect to the social cost) are derived. Namely, the average cost function (3), the SOP quality (proportional to the inverse of (3) and defined as [Bandwidth · Time/Interfering Users]), and the average switching probability are plotted versus the switching cost in Fig. 4.a, 4.b, and 4.c, respectively. Increasing the switching cost K, the average per-user cost $c_i(k, x_i^k)$ (i.e., without K) increases, whereas the average switching probability decreases. This means that the users tends to reduce the number of handovers, but "loosing" in terms of interference, bandwidth and holding time of the chosen SOP. Notably, the switching probability tends to zero as K keeps increasing. Finally, we observe that as the switching cost increases, the average SOP quality perceived by the secondary users decreases. In other words, secondary users tends to remain on the chosen SOP even if better ones become available, due to the high switching cost.

VI. Concluding Remarks and Future Work

In this paper, we propose a framework to evaluate *spectrum management* functionalities in Cognitive Radio Networks. Since one of the most important aspects of cognitive terminals resides in their rationality, we resort to a game theoretical approach to model the spectrum access process among secondary users who can opportunistically select the "best" available channel without interfering primary users. The proposed framework can be adopted by secondary users in order to characterize different spectrum opportunities, share available bands with other users and evaluate the possibility to move in a new channel whenever it is necessary or convenient. Moreover, we consider a time varying scenario, and the cost of spectrum handover is introduced in the game.

Future works aim at extending simulation scenarios, in order to consider different kind of users, coexisting in the same area and playing with different cost functions. Furthermore, a deeper analysis and comparison of cost functions can be done in terms of actual throughput perceived by users.

References

[1] J. M. III and J. G.Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, 1999.

[2] I. Akyildiz, W.-Y. Lee, M. Vuran, and S. Mohanty, "A survey on spectrum management in cognitive radio networks," *IEEE COMMUN MAG*, vol. 46, no. 4, pp. 40–48, 2008.

[3] Z. Ji and K. Liu, "Dynamic spectrum sharing: A game theoretical overview," *IEEE COMMUN MAG*, vol. 45, no. 5, pp. 88–94, 2007.

[4] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. Asilomar SSC*, vol. 1, 2004, pp. 772–776.

[5] W.-Y. Lee and I. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," *IEEE T WIREL COMMUN*, vol. 7, no. 10, pp. 3845–3857, 2008.

[6] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda, and Y. Wu, "Allocating dynamic time-spectrum blocks in cognitive radio networks," in *Proc. ACM MobiHoc*, 2007, pp. 130–139.

[7] O. Ileri, D. Samardzija, and N. Mandayam, "Demand responsive pricing and competitive spectrum allocation via a spectrum server," in *Proc. IEEE DySPAN*, 2005, pp. 194–202.

[8] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE J SEL AREA COMM*, vol. 25, no. 3, pp. 589–600, 2007.

[9] L. Cao and H. Zheng, "Distributed spectrum allocation via local bargaining," in *Proc. IEEE SECON*, 2005, pp. 475–486.

[10] I. F. Akyildiz, W.-Y. Lee, and K. R. Chowdhury, "CRAHNs: Cognitive radio ad hoc networks," *Ad Hoc Networks*, vol. 7, no. 5, pp. 810–836, 2009.

[11] O. Jo and D.-H. Cho, "Efficient spectrum matching based on spectrum characteristics in cognitive radio systems," in *Proc. WTS*, 2008, pp. 230–235.

[12] L. Marin and L. Giupponi, "Performance evaluation of spectrum decision schemes for a cognitive ad-hoc network," in *Proc. IEEE PIMRC*, 2008, pp. 1–5.

[13] J. Suris, L. DaSilva, Z. Han, and A. MacKenzie, "Cooperative game theory for distributed spectrum sharing," in *Proc. ICC*, 2007, pp. 5282–5287.

[14] A. Attar, M. Nakhai, and A. Aghvami, "Cognitive radio game: A framework for efficiency, fairness and QoS guarantee," in *Proc. ICC*, 2008, pp. 4170–4174.

[15] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J SEL AREA COMM*, vol. 25, no. 3, pp. 517–528, 2007.

[16] D. Niyato and E. Hossain, "A game-theoretic approach to competitive spectrum sharing in cognitive radio networks," in *Proc. IEEE WCNC*, 2007, pp. 16–20.

[17] S. Sengupta, R. Chandramouli, S. Brahma, and M. Chatterjee, "A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks," in *Proc. IEEE GLOBECOM*, 2008, pp. 1–6.

[18] R. W. Rosenthal, "A class of games possessing pure-strategy Nash equilibria," *INT J GAME THEORY*, vol. 2, no. 1, pp. 65–67, 1973.

[19] I. Milchtaich, "Congestion games with player-specfic payoff functions," *Games and Economic Behavior*, vol. 13, no. 1, pp. 111–124, 1996.

[20] M. Liu, S. Ahmad, and Y. Wu, "Congestion games with resource reuse and applications in spectrum sharing," in *Proc. ACM GAMENETS*, 2009.

[21] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL, A modeling language for mathematical programming*, 1993.

[22] *ILOG CPLEX 10.0 user's manual*, http://www.ilog.com/products/cplex/.

Appendix A
Equivalence to a Non-Weighted Single-Choice Crowding Game

In a crowding game, players choose resources and are charged of the corresponding costs. Each resource k is characterized by a congestion level x^k, defined as the number of players that choose it, being the game non-weighted. With player-specific cost functions, the cost charged to a player i that chooses resource i is $c_i(k, x^k)$. Given this definition, the formulation of a single-choice crowding game that captures our game model is non-trivial. Assumed the SOPs to be the resources in our model, the complications are due to the fact that different players that choose a SOP can perceive different congestion levels x_i^k. Consider a scenario with three secondary users A, B, C, as shown in Fig. 5.a, and suppose the SOPs to be two. If all the users choose the same SOP k, $x_A^k = x_C^k = 2$ and $x_B^k = 3$. Anyway, our game model can be formulated as a network crowding game and, by a mathematical trick, we reduce it to a single-choice game. In this class of games, players choose paths that connect their source to their destination. The edges of the network are the resources and the players' costs are the sum of the

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.

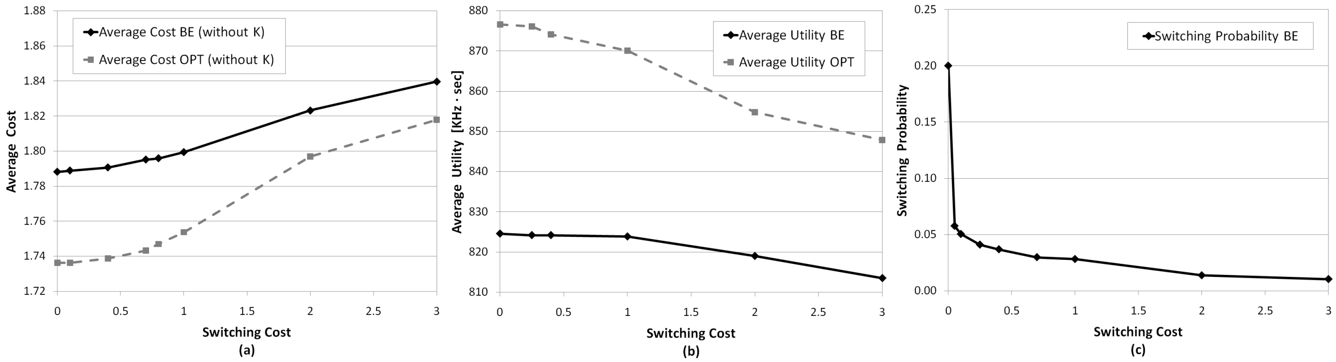
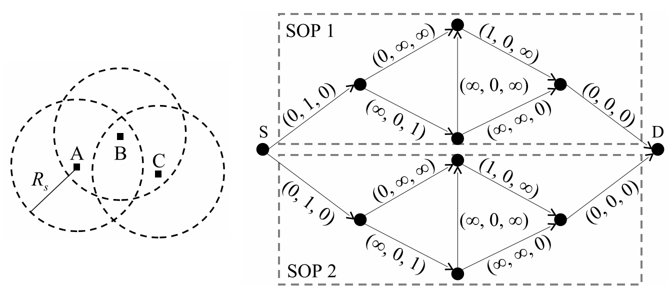


Fig. 4. Average cost (a), average SOP quality (b) and average switching probability (c) perceived by users at the best Nash equilibrium (BE) and at the optimal solution (OPT).

costs of the chosen resources. A network congestion game is generally a multiple-choice congestion game, being multiple the nodes where the players can choose. We use linear player-specific cost functions of the form $c_i(k, x^k) = a_{i,k}x^k$ where $a_{i,k}$ is a player-specific parameter. In our specific case, by opportunistically setting the parameters $a_{i,k}$s, we can produce a network congestion game that captures our game model with the property that each player makes essentially one choice. With "essentially" we mean that a player have multiple nodes wherein it can choose the next edge, but in all nodes but one there is a dominant choice independently of the other players. We report in Fig. 5.b the network congestion game capturing the situation depicted in Fig. 5.a; the triples over the edges denote the player-specific parameters as $(a_{A,k}, a_{B,k}, a_{C,k})$. In this case, the unique node without a dominant choice is the source. Consider player B: if it chooses the upper edge (SOP 1) in the source, then it will follow the path wherein $a_{B,k} = 0$. The same holds for player A and C. Given a players' strategy profile, it can be easily shown that the costs in the game depicted in Fig. 5.b are the same of the game stated in Section III. Therefore, our game is essentially a non-weighted single-choice crowding game.



(a) (b)

Fig. 5. (a) A topology with three secondary users. (b) Network congestion game representing the topology (a) with two SOPs ('S' denotes the source and 'D' the destination).

Appendix B
Mathematical Programming Formulation

We provide here a general mathematical programming formulation that we use to find best/worst Nash equilibria and the optimal solution in our congestion games. The following model can be used (and linearized) for each one of the cost functions presented in Section IV-A and for the dynamic game described in Section V-A. The switching cost K can be set to zero for solving the one shot game.

The parameters of the model are:

$$z_{ik} = \begin{cases} 1 & \text{if SU } i \text{ has chosen SOP } k \text{ in the previous epoch} \\ 0 & \text{otherwise} \end{cases}$$

We define the association of a user to a SOP by introducing a binary decision variable:

$$y_{ik} = \begin{cases} 1 & \text{if SU } i \text{ chooses SOP } k \\ 0 & \text{otherwise} \end{cases}$$

Finally, the constraints of the problem are:

$$\sum_{k \in \mathcal{B}_i} y_{ik} = 1 \qquad \forall\, i \in \mathcal{N} \tag{5}$$

$$y_{im}\left(c_i(m, x_i^m) + K(1 - z_{im})\right) \leq \left(c_i(k, x_i^k) + K(1 - z_{ik})\right) \quad \forall\, i \in \mathcal{N}, m, k \neq m \in \mathcal{B}_i \tag{6}$$

Constraints (5) guarantee the feasibility of the assignment. Constraints (6) force each user to choose the strategy (SOP) which leads to the minimum cost function, that is, they ensure that, if the single user unilaterally changes her strategy, the change does not improve her own payoff (i.e. definition of Nash equilibrium).

This formulation allows one to find equilibria that maximize/minimize the social cost. They can be found by introducing the following objective function in the formulation:

$$\min / \max \sum_{i \in \mathcal{N}} \sum_{m \in \mathcal{B}_i} y_{im}\left(c_i(m, x_i^m) + K(1 - z_{im})\right) \tag{7}$$

Moreover, to evaluate price-of-stability (PoS) and price-of-anarchy (PoA), it is necessary to provide a model that allows one to find the optimal solution, i.e. the solution that minimizes the social cost but that could not be an equilibrium. To do this, we inhibit equilibrium constraints (6) and solve the model minimizing the objective function (7).

978-1-4244-4148-8/09/$25.00 ©2009
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2009 proceedings.