Reuse efficiency of wireless access networks under physical carrier sense: A Markovian analysis

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Abstract

The setting of the physical carrier sense is critical in wireless networks, since it often has to account for contrasting objectives like limiting the overall interference while ensuring high concurrency among wireless transmissions. This paper proposes an analytical approach for evaluating the impact of carrier sense setting on the throughput efficiency of wireless access networks. A time continuous Markov chain is used to describe the system, and to further gathering performance measures in terms of throughput and collision probability. Numerical results obtained through the model and further validated against simulations are used to derive qualitative dimensioning criteria for the carrier sense under different network conditions.

1. Introduction

The efficiency of wireless systems largely depends on the capability of the wireless Medium Access Control (MAC) protocols to limit interference, while enforcing spatial reuse of the shared radio resources. The concurrent achievement of these two contrasting objectives is often challenging. On the one hand, limiting interference calls for reducing the concurrency of wireless transmissions, which, on the other hand, should be maximized to increase spatial reuse.

Commonly deployed wireless MAC protocols leverage heuristic, local solutions to solve the aforementioned problems. In particular, most of the available wireless systems resort to physical carrier sense (CS) to strike a balance between interference protection and spatial reuse augmentation [1,3]. Generally speaking, physical carrier sense observes the status of the channel to decide whether a new transmission can be performed. CS algorithms were originally devised to reduce access collisions in multiple access techniques within fully broadcast environments with exact and complete knowledge of the channel [2,15]. In wireless networks, things get a bit more complicated since the radio channel is far from being fully broadcast, and the available information on the channel status (busy/idle) is often local. In other words, carrier sense in wireless systems observes the channel status locally at the sender to decide whether the interference at the receivers is such that a new transmission is allowed.

The heuristic nature of carrier sense (locality of the channel information) leads to unavoidable well-known inefficiencies related both to interference protection and spatial reuse. Roughly speaking, carrier sense observes the received power at the transceiver and returns a “channel idle” or “channel busy” indication if such received power is below or above a certain power threshold (carrier sense threshold). Collisions due to hidden nodes [24] arise when terminals not hearing each other (i.e., received power levels below CS threshold) transmit to a terminal that is reached by both. On the other hand, remote collisions can happen due to the disruptive effect of cumulative interference coming from other ongoing communications which are not necessarily directed to the same common receiver. Finally, exposed nodes may be blocked even if they could transmit concurrently, due to the locality of the channel status information.

To this extent, the carrier sense mechanism is critical in determining the network efficiency. Intuitively, both hidden node and remote collisions reduce as the carrier sense range is increased, since the number of terminals accessing the channel decreases, thus lowering the overall network interference. On the other hand, enlarging the carrier sense range can lead to frequent “exposed nodes” cases, thus dramatically limiting the number of concurrent transmissions within the network.

Stimulated by these observations, this paper quantitatively analyzes the throughput efficiency of physical carrier sense. Namely,
we develop an analytical framework based on a time continuous Markov Chain with the purpose to evaluate the impact of the CS mechanism on the throughput efficiency of simple wireless access networks. We show how the proposed model can be used to derive network throughput, and per-user throughput, under different settings for the carrier sense mechanism. We further validate our model against simulation and use it to dimension carrier sense parameters like the carrier sense threshold (or the carrier sense range), further highlighting the unfair pattern in the achieved throughput.

The paper is organized as follows: Section 2 overviews the literature in the field. In Section 3, we discuss the reference system and the assumptions used to design the analytical framework which is fully described in Section 4. Section 5 reports numerical results on the impact of carrier sense on the reference wireless access network. Concluding remarks are given in Section 6.

2. Related work

The issue of evaluating and enhancing the spatial reuse of wireless networks is widely discussed and assessed in the literature, both under a purely theoretical point of view [11], and also with the target of designing reuse-aware MAC protocols.

Most of the published work in the field starts off from the IEEE 802.11 standard, and introduces enhancements to the access mechanism to augment the channel reuse. Within this framework, different approaches have been proposed: the FAMA protocol [10] uses both physical carrier sense and virtual carrier sense through RTS/CTS messages to ensure the acquisition of the transmission opportunity and the consequent successful transmission of the data packets. Haas et al. [12] propose a solution for access based on busy tones which eventually allows multiple parallel communications to happen in ad hoc networks. On the other side, Wu and Li [27] propose a receiver initiated access protocol. The contact point of the latter two contributions is in that they both use a signalling procedure (busy tones, floor acquisition) to let the wireless transmitters coordinate and perform parallel feasible transmissions.

On the other hand, Refs. [21,20,6] propose to tune the access mechanism of IEEE 802.11 networks on the basis of the estimate of the ongoing interference situation of the network. The central idea of this common approach is to incorporate power control in the access protocols such that successful communications are allowed with the intended receiver, while causing minimal interference to other ongoing communications.

The purpose of this paper is not to propose novel access protocols, but rather to model and investigate the reuse efficiency of wireless access networks based on pure physical carrier sense, only. Within this field, it is worth mentioning inspiring contributions on carrier sense design and modeling issues. Zhu et al. [31] study the optimal carrier sensing setting for specific ad hoc network configurations and find out that an optimal value of the carrier sense threshold does exist with respect to the network throughput. This optimal value comes from a trade off between the need of protecting the access from interference and the opposite need of reusing the radio resource. In a follow up of their work, the same authors propose a dynamic algorithm to tune the carrier sense power threshold in a wireless mesh network [30].

Along the same lines, Fuemmeler et al. [9] prove that the product of transmitted power and carrier sense sensitivity threshold should be kept constant in order to maximize the aggregate throughput of a general ad hoc network; starting off from this consideration, [22] addresses the joint problem of optimizing the carrier sense threshold (range) and the transmit power to augment the spatial reuse in single-hop ad hoc networks. The case of dynamic adaptation of carrier sense threshold for hot-spot WiFi-based access networks is considered in [25], whereas Ma et al. [17] focus on generic ad hoc architectures based on IEEE 802.11 Distributed Coordination Function (DCF), and propose a stochastic approach to determine the saturation throughput when physical carrier sense and random backoff are coupled in the access phase. The proposed approach accounts for hidden-node collisions but neglects the effect of interference accumulation from multiple sources. Recent work by Chong et al. [7] provides insightful models of Carrier Sensing Multiple Access with Collision Avoidance (CSMA/CA) under non-perfect carrier sensing procedures; the impact of false positives and detection errors is evaluated analytically and through simulation. As for the previous reference, the effect of interference accumulation is not considered, that is, concurrent transmissions always lead to collisions. Refs. [29,18] also focus on IEEE 802.11 DCF and study, mainly through simulations, the combined effect of physical carrier sensing and multi-rate transmission on the performance of wireless networks.

Besides simulation-based and analytical approaches, it is worth mentioning the experimental work carried out to assess the carrier sense efficiency under real deployments. Within this field, Jamieson et al. [14] tackle the problem in a practical way evaluating the performance of really deployed sensor networks and 802.11-based indoor networks. Brodsky and Morris [5] advocate the efficiency of physical carrier sense, by setting up a small-scale indoor testbed with off-the-shelf IEEE 802.11-based hardware.

Generally speaking, all the contributions mentioned above analyze the carrier sense in wireless networks either resorting to simulation/experiments or to static geometrical models of the channel reuse [19]. Differently, we aim here at analyzing the efficiency of physical carrier sense accounting also for temporal and spatial dynamics in the traffic generation. The two pieces of work closest to ours are Wong and Cruz [26], and Rossetto and Zorzi [23]. The former introduces a spatio-temporal model of the physical carrier sense where packet transmissions attempts are modeled as a three dimensional Poisson point process in space and time, and the interference comes from a shot noise process filtered in space and time, thus implicitly assuming an infinite number of interfering sources. The latter work relaxes the Gaussian assumption for the perceived interference by introducing the concept of “Guassian mixture interference” which distinguishes interfering nodes in dominant and far-away ones. These two papers differ from ours in two main points: first, they both address the case of general (and large) ad hoc networks, whilst we focus here on wireless access networks, second, they both resort to simplifying assumptions on the perceived interference, whereas, on the other side, our approach accounts for the exact interference (i.e., exact position and exact timing of the interfering stations).

3. Reference scenario

3.1. Network topology

The reference network architecture we use to introduce the model is composed of two wireless Access Points (APs) set at a fixed distance D (Fig. 1). We assume that the propagation time between any couple of transmitters is small compared to the packet transmission time to the point that the effectiveness of the CS in avoiding access collisions is complete, i.e., terminals are
scattered in the plane at a maximum distance such that the ratio \( r/T \), between the end-to-end propagation delay \( r \) and the average transmission time \( T \), is negligible. In other words, the only types of collisions allowed in the network are hidden and remote collisions, as introduced in Section 1.

Transmissions toward an AP are generated according to a Poisson process with intensity \( \lambda \). Once a transmission is generated, the position of the transmitting terminal is randomly chosen in the coverage area of the AP. The terminal senses the channel and, if the channel is sensed busy (i.e., the perceived power level is above the carrier sensing threshold), the transmission is refrained. Otherwise, the transmission starts and ceases after a period of time exponentially distributed with parameter \( \mu \); transmitting terminals leave the network, independently on the outcome of its transmission.

We note that the terminal vanishing assumption is common in analyzing random-access MAC protocols under the infinite-population traffic model, where new access attempts occur according to a Poisson process so that, the memory of the preceding attempts is lost.

Given a transmitter in position \( i \) and a receiver in position \( j \), the received power level at \( j \) out of a transmission from \( i \) is given by:

\[
P_t(d_{ij}) = P_t d_{ij}^{-\eta} 10^{\gamma},
\]

where \( d_{ij} \) is the distance between position \( i \) and position \( j \), \( P_t \) the transmitted power, \( \eta \) the attenuation factor and \( 10^{\gamma} \) accounts for the loss due to slow shadowing, \( \epsilon \) being a normal variate with zero mean and \( \sigma^2 \) variance.

Furthermore, if we denote by \( P_t(d_{ij}) \) the interference, i.e., the power received from another concurrent transmitter located in position \( k \), a transmission from \( i \) to \( j \) is successful if the Signal To Interference and Noise Ratio (SINR) is not below a given threshold \( \gamma \) for the duration of the transmission, i.e.,

\[
\frac{P_t(d_{ij})}{N + \sum_{k \in \text{int}} P_t(d_{ij})} \geq \gamma,
\]

where \( N \) represents the thermal noise contribution and \( \text{int} \) the set of current interference.

The thermal noise determines an ideal area around the terminal where each transmission is correctly received if no interference is present.\(^1\) The average radius of this area, called the thermal noise range, is given by:

\[
R = \left( \frac{P_t}{NY} \right)^{1/\eta}.
\]

3.2. Physical Carrier Sense model

The Physical Carrier Sense (PCS) procedure is in charge of reporting the status of the wireless medium to the MAC layer, and leverages the Clear Channel Assessment (CCA) modules implemented at the physical layer of the specific wireless technology. The CCA module measures the received energy at the Radio-Frequency front-end and consequently assesses the status of the channel (busy/idle). The decision policies implemented by the CCA module depend on the specific technology/standard. As an example, in the IEEE 802.11, four decision policies are available for CCA decisions \([1]\). Namely, CCA can issue a busy channel event notification \((i)\) if the amount of energy detected on the channel exceeds a threshold (CCA mode 1); \((ii)\) if a valid 802.11 modulated signal is detected on the channel (CCA mode 2); \((iii)\) if a valid 802.11 modulated signal detected on the channel exceeds the EDT (CCA mode 3). Finally, timer-aided carrier sense is also available \((iv)\).

Since we are interested in the efficiency of PCS independently of the specific technology/standard, we abstract the specific CCA implementation and refer to the basic Energy/Power-based clear channel assessment policy implemented in most of the carrier-sensing-based wireless technologies. Namely, we assume that the CCA returns to the MAC layer a “channel idle” indication at access time \( t \) if the following holds:

\[
\sum_{k \in \text{int}} P_t(d_{ij}) \leq P_{CS}
\]

where \( P_{CS} \) is the carrier sense threshold. In case a single interference can be active at a time, i.e., \( |\text{int}| = 1 \), the carrier sense threshold induces a blocked circular area around such an interfering terminal with radius \( R_{CS} \), where:

\[
R_{CS} = \left( \frac{P_t}{P_{CS}} \right)^{1/\eta}.
\]

On the other side, if the interference comes from multiple sources, the concept of blocked area is no longer feasible due to the interference accumulation phenomenon.

4. A Markovian framework

The system described in the previous section becomes analytically tractable if we assume a discrete set of \( n \) feasible locations for the transmitters. In our evaluations the positions of the \( n \) transmitters are randomly drawn within a semi-circular area of radius \( R \) centered in the position of the AP. However, the model can deal with any set of positions arbitrarily chosen. A terminal accessing the network chooses one out of the \( n \) positions, accesses the channel, and if the carrier sense mechanism allows it \( (i.e., \text{Eq.}(4) \text{ holds}) \), transmits. For the sake of presentation, we consider the uplink segment, i.e. communications running from the mobile terminals to the APs. The same mathematical framework can be extended also to the downlink and the mixed uplink/downlink cases.

The access attempts are modeled according to a Poisson point process of intensity \( \lambda \). We take a constructive approach here to introduce the Markovian framework and start off by considering the case where a single transmission per AP is allowed, i.e., no hidden collisions within the same hot spot are allowed. Under this assumption, we introduce the basic Markov chain (Section 4.1), and we show how the chain can be extended to account for hidden node collisions (Section 4.2). Section 4.3 reports the procedure to calculate the system throughput.

4.1. Basic Markov chain

The system can be modeled by the multi-dimensional time-continuous Markov Chain:

\[
(X(t), Y(t), V_1(t), V_2(t))
\]

where \( X(t) = \{0_0, 1_2, 2_3, \ldots, n_1\} \) and \( Y(t) = \{0_b, 1_b, 2_b, \ldots, n_b\} \) are sets of labels identifying the positions of terminals associated to

\(^1\) In practice, the shape of this area is not circular but rather depends on the shadowing effect on the signal propagation.
The Markov chain can be synthetically represented as
defined in terms of states and transitions. Values of $X$ and $Y$ are determined by the values $x$ and $y$ respectively; furthermore, in these states, both ongoing transmissions have not suffered any collisions.

- **Zone 1, 1** is composed of states $(x, y)$, $1 \leq x \leq n$, $1 \leq y \leq n$, where both terminals $a$ and $b$ are active and their positions are determined by the values $x$ and $y$ respectively; furthermore, in these states, both ongoing transmissions have not suffered any collisions.
- **Zone 1, 1** is composed of states $(x, y)$, $1 \leq x \leq n$, $1 \leq y \leq n'$, where both terminals $a$ and $b$ are active and their positions are determined by the values $x$ and $y$ respectively; furthermore, in these states, both ongoing transmissions have already suffered some collisions.

Transition intensities $q_{(x,y),(z,w)}$ between state $(x,y)$ and state $(z,w)$ (always being $(x,y) \neq (z,w)$) are as follows:

1. From state $(0,0)$, the only possible transitions are those representing the activation of a new transmission either in hot spot $a$ or $b$. Therefore:
   - $q_{(00),(00)} = \mu$ 
   - $q_{(00),(00)} = \lambda/n$ 
   - $q_{(00),(0y)} = \lambda/n$ 
   - $q_{(00),(0x)} = \lambda/n$ 

2. In zone $(1,0)$, the following sets of transitions are possible:
   - **(a)** The one toward state $(0,0)$ when the active transmission ceases:
     - $q_{(00),(00)} = \mu$
   - **(b)** The one toward zone $(1,1)$ when terminal $b$ activates without causing collisions:
     - $q_{(00),(1y)} = 1/\lambda \alpha(x,y)$ 
   - **(c)** The one toward zone $(1,1)$ when terminal $b$ activates causing a collision at terminal $a$
     - $q_{(00),(1y)} = 1/\lambda \beta(x,y)$ 
   - **(d)** The one toward zone $(1,1)$ when terminal $b$ activates causing a collision at terminal $b$
     - $q_{(00),(1y)} = 1/\lambda \eta(x,y)$
   - **(e)** The one toward zone $(1,1)$ when terminal $b$ activates causing collisions at both terminals
     - $q_{(00),(1y)} = 1/\lambda \delta(x,y)$

**Fig. 2. Simplified time continuous Markov chain representing the system in Figure (a), meaning of the different zones of the chain in Figure (b).**
3. in zone \( \{0, 1\} \), the same kind of transmissions are possible as in the previous case with a reversed role of indices \( x \) and \( y \);
4. in zone \( \{1, 0\} \), only the following sets of transitions are possible:
   (a) the one toward state \( \{0, 0\} \) when the active transmission ceases: \( q_{(00),(00)} = \mu \)
   (b) the ones toward zone \( \{1, 1\} \) when terminal \( b \) activates with no collision
      \[ q_{(xy),(0y)} = \frac{1}{\lambda} \beta(x, y) \quad 1 \leq y \leq n \]
   (c) the ones toward zone \( \{1, 1\} \) when terminal \( b \) activates with a collision at terminal \( a \) itself;
      \[ q_{(xy),(0y)} = \frac{1}{\lambda} \delta(x, y) \quad 1 \leq y \leq n' \]
5. in zone \( \{0, 1\} \), the same kind of transmissions are possible as in the previous case with reversed roles of \( x \) and \( y \);
6. in zones \( \{1, 1\}, \{1, 1\}, \{1, 1\}, \{1, 1\} \) the only possible transitions are those representing the end of an ongoing transmission:
   \[ q_{(xy),(0y)} = \mu \quad q_{(xy),(0y)} = \mu \]
   \( \alpha(x, y), \beta(x, y), \eta(x, y), \) and \( \delta(x, y) \) are binary functions whose value is evaluated for each of the \( n \) positions according to Eqs. (1), (2) and (4). In case the aforementioned functions assume null values, some states in the chain may be unreachable, e.g., when concurrent transmissions cannot exist in locations \( x \) and \( y \) because they are within the CS range.

**Example 1.** Fig. 3, shows the construction of the Markov chain under a specific network topology \((n = 1, \) i.e., just one position is considered in both APs). In Fig. 3(a), triangles represent APs, and circles the two users’ positions; such positions are feasible as for the physical carrier sense algorithm, however, if the two terminals are transmitting to the respective AP (solid arrows in the figure), the mutual interference (dashed arrows) is such that the SINR of position \( y \) is below the quality threshold \((\text{SINR}_y = 4 \text{ dB})\) whereas the SINR of position \( x \) is above the threshold \((\text{SINR}_x = 13 \text{ dB})\). The corresponding Markov chain is reported in Fig. 3(b). From state \((x, 0)\) where the only active transmission is the one from \( x \), the only possible transitions are the ones towards state \((0, 0)\) if \( x \)'s transmission ceases, or towards state \((x, y')\) if transmission from \( y \) starts transmitting concurrently. In this latter state, both transmissions are active but the one from \( y \) has suffered collision.

4.2. **Adding hidden-node collisions**

Two terminals belonging to the same hot spot are hidden if the respective transmissions are sensed by the other with a power level below the carrier sense threshold (see Eq. (4)). If such stations access the channel at the same time to transmit toward a common receiver, a collision happens at that receiver.

We show hereafter how the proposed Markov framework can be extended with limited additional complexity to account for hidden collisions. To this purpose, the following transition frequencies are added to those already described. From zones \( \{1, 0\} \) and \( \{0, 1\} \) where a non-collided transmission is active in \( a \) and \( b \) respectively, hidden node collision may drive the chain to zones \( \{1, 0\} \) and \( \{0, 1\} \) with transition intensities:

\[ q_{(xy),(0y)} = \frac{1}{\lambda} \alpha(x, y) \quad z = x' \]
\[ q_{(0y),(0y)} = \frac{1}{\lambda} \alpha(x, y) \quad k = y' \]

\( h(x) \) and \( h(y) \) being the number of nodes hidden to a station in position \( x \) (access point \( a \)) and position \( y \) (access point \( b \)), respectively.

The same transitions as above must be added from zones \( \{1, 1\} \) and \( \{1, 1\} \) to zone \( \{1, 1\} \). .

\[ q_{(xy),(0y)} = \frac{1}{\lambda} \alpha(x, y) \quad z = x', \quad 1 \leq y \leq n' \]
\[ q_{(0y),(0y)} = \frac{1}{\lambda} \alpha(x, y) \quad k = y', \quad 1 \leq x \leq n'. \]

When both transmissions are active, have not collided and a hidden node comes in, it causes a hidden collision within the same AP and may also cause a remote collision on the transmission of the other AP if the produced interference drives the corresponding SINR below the quality threshold. To formally account for such type of collisions due to interference accumulation, we introduce the following:

**Definition 1.** Given a node in position \( i \) within AP domain \( a \), a node \( k \) hidden from \( i \) is critical for a node in position \( j \) within AP domain \( b \) if the cumulative interference generated by \( i \) and \( k \) causes a collision at \( j \), i.e., iff:

\[
\frac{P_i(d_{ib})}{N + P_i(d_{ib}) + P_i(d_{ik})} \leq \gamma \\
P_i(d_{ib}) + P_i(d_{ik}) \leq P_{CS}.
\]

We can thus define the following variable \( H(x, y) \) as the number of nodes hidden to \( x \) which are critical to \( y \). Thus, the following transitions should be added originating from zone \( \{1, 1\} \):

1. the ones toward zone \( \{1, 1\} \) when a hidden node accesses the channel in AP \( a \), causes a hidden collision at \( a \) but the other transmission in AP \( b \) is unaffected
   \[ q_{(xy),(zy)} = \frac{1}{\lambda} (h(x) - H(x, y)) \]
   \[ 1 \leq x \leq n, \quad z = x', \quad 1 \leq y \leq n \]
Appendix

3. Referring to yielding bederived by solving the system of equations non-collided communication is active toward AP \( \lambda \) and \( S_a \) (measured in Erlang). Thus, the throughput of AP \( a \) is given by:

\[
q_{(x), (y)} = \frac{1}{n} \lambda (H(y, x) - H(y, x))
\]

where \( n \) is the number of positions in the location set, \( H(y, x) \) is the probability of finding the channel busy in successful transmissions (measured in Erlang), and \( q_{(x), (y)} \) can be evaluated for each of the \( n \) positions in the location set according to the Eqs. (1), (2) and (4).

**Example 2.** Referring to Fig. 4, nodes \( h_1 \) and \( h_2 \) are hidden to node \( x \) (outside \( x \)'s carrier sense range). Concurrent transmissions of the three nodes would collide at access point \( a \). Thus, we have in this case \( h(x) = 2 \). Moreover, a transmission from node \( h_1 \) would also collide with a transmission from node \( y \) to AP \( b \), that is node \( h_1 \) is critical to node \( y \), leading to \( H(x, y) = 1 \).

4.3. Throughput calculation

Due to the intrinsic asymmetry of the network, the main performance figures must be evaluated separately for the two hot spots; for the sake of simplicity, we report the throughput calculation procedure referring to AP \( a \) only, the one for AP \( b \) being identical. We consider the classical definition of throughput as the probability of finding the channel busy in successful transmissions (measured in Erlang). Thus, the throughput of AP \( a \) can be derived as:

\[
S_a = \lambda_S D_a
\]

where \( \lambda_S \) is the frequency of transmissions ending up with success and \( D_a \) is the average duration of such transmissions. Frequency \( \lambda_S \) can be evaluated as:

\[
\lambda_S = \mu \sum_{(x, y) \in A} \pi_{(x, y)}
\]

where \( A = \{1, 0\} \cup \{1, 1\} \cup \{1, 0\} \) is the set of states where a non-collided communication is active toward AP \( a \) (see Fig. 2(b)).

The stationary distribution of the chain, i.e., vector \( \pi_{(x, y)} \), can be derived by solving the system of equations

\[
\sum_{z \in T_{(z), (w)}} \pi_{(x, y)} q_{(x, y), (z, w)} = 0 \quad \forall (z, w)
\]

yielding \( \pi_{(x, y)} = 0 \) for the unreachable states.

To derive \( D_a \), we observe that the statistical characteristics of the duration of successful transmission are different from those of backlogged transmissions that contribute to channel traffic. The intuition is that longer transmissions are more likely to collide than shorter ones. This effect is common in random access systems without collision detection mechanisms, such as ALOHA [4]. To this extent, in our case, \( D_a \neq 1/\mu \). In Appendix, we propose a methodology for the correct calculation the average duration of successful transmissions, \( D_a \), on the Markov chain we have introduced.

Finally, the traffic on the channel, defined as the overall probability of finding the channel busy (in successful and unsuccessful transmissions), can be determined as: \( \lambda_C = \frac{\lambda_C}{\mu} \), where \( \lambda_C \) is the frequency of channel transmissions given by:

\[
\lambda_C = \mu \sum_{(x, y) \in \Theta} \pi_{(x, y)}
\]

and \( \Theta = \{1, 0\} \cup \{1, 1\} \cup \{1, 1\} \cup \{1\} \cup \{1\} \) (see Fig. 2(b)).

5. Numerical results and evaluation

The proposed model can be used to assess the physical carrier sense efficiency when users in fixed positions have access to the network. This can likely provide a powerful tool to support network planning and optimization procedures, when the spatial density of the traffic can be made discrete and predicted a priori, e.g., users in office buildings accessing the networks from specific positions (desks, conference rooms, etc.).

Furthermore, the model can also be adopted for gathering general information on the impact of specific topological parameters on the performance of the physical carrier sense. To this extent, general measures can be obtained by generating several instances of network configurations (randomly drawn end user positions), solving each instance with the model, and finally averaging over all the solutions. In this section, we consider the topology shown in Fig. 1 where \( D_a \) and \( R_C \) represent, respectively, the distance between the two APs and the CS range normalized to the value of the thermal noise range, \( R \), and we gather general performance evaluation by adopting a number of locations for each AP equal to \( N = 15 \), and averaging our measures on \( M = 20 \) solutions of the model. The results have been obtained considering a standard setting of the model parameters, unless otherwise specified: \( \alpha = 6 \) dB, \( \eta = -3 \), and \( \gamma = 10 \) dB.

To validate the results attained with the above procedure we have also built a C++ simulator of the two-access-point network scenario where, in accordance to the model described in Section 3, terminal locations are uniformly drawn in the plane dynamically, i.e., at every transmission attempt. The accessing terminals operate according to the very same physical carrier sensing as the one considered in the analytical model.
5.1. Impact of carrier sense on throughput performances

Figs. 5–7 show the network throughput \( S = S_a + S_b \), the hidden node collision probability and the remote collision probability versus the offered channel traffic \( G = \lambda/\mu \) in the cases where \( D = 2 \), \( \gamma = 10 \text{dB}, \eta = -3 \), for three values of the carrier sensing range, namely \( R_{CS} = 1, 2 \) and 4. In all the figures, we have also reported the points obtained by simulation, with the associated confidence intervals, which appear to confirm the correctness of the analytical procedure.

Expectedly, in the case \( R_{CS} = 4 \), the throughput saturates at \( S = 1 \), since only one transmission at a time can take place in the entire system. When the CS range decreases, the effect of parallel transmissions (channel reuse) is greater than the loss due to remote collisions (which are very few indeed) and the maximum throughput increases. A peculiar behavior is observed in cases \( R_{CS} = 1 \), where hidden-node collisions drive the throughput in an unstable region when \( G \to \infty \). From Fig. 6, no hidden node collision can occur for \( R_{CS} = 2, 4 \), since all the terminals are within CS range and only one transmission at a time can take place within the AP. Similarly, observing Fig. 7, the remote collision probability is not null only for \( R_{CS} = 1, 2 \), whereas for \( R_{CS} = 4 \) no collisions can occur, since all the terminals are within CS range and only one transmission at a time can take place within the whole network.

We further observe that the remote collision probability at first increases with the offered traffic and then levels off for \( G \to \infty \). For finite values of \( G \) any terminals, even the terminals belonging to a couple that collides, can successfully transmit by exploiting the sporadic holes in the channel occupation; on the other hand, when \( G \to \infty \), the collision probability coincides with the spatial average of the collision probabilities, that is, a couple of transmissions which are feasible from the carrier sensing point of view either collide (collision probability equal to 1) or not (collision probability equal to 0) depending on the specific topological environment.

Fig. 8(a) reports the network saturation throughput, i.e., the throughput at \( G = 1000 \), versus the CS range \( (R_{CS}) \) for different values of the distance between the two APs and for the same parameter settings of the previous figures. As clear from the figure, an optimal value of the carrier sense range (and threshold) does exist for both the network configurations considered. Such a value comes from a trade off between the need of protecting from hidden node collisions which requires high carrier sensing radius (low carrier sensing thresholds), and the need of reusing the channel.

Fig. 8(b) reports the remote collision probability in the very same cases of Fig. 8(a). As expected, if \( R_{CS} \) is sufficiently large collisions drop to zero and the maximum throughput is 1. When \( R_{CS} \) reduces, collisions increase, but, in the zone in which \( 2 < R_{CS} < 3 \) their impairing effect is more than compensated by the increasing of reuse, so that the throughput increases, reaching 1.8 in the \( D = 2 \) case (see Fig. 8(a)). A further decrease of the CS range again increases the collision probability and decreases the throughput.

5.2. Capturing the unfairness of physical carrier sense

We note here that systems based on physical carrier sense only as the one modeled in this paper, can be highly unfair. Indeed, spatial unfairness arises since terminals at different locations
experience different probabilities of sensing the channel busy or idle, and different collision probabilities. Moreover, since the access system does not distinguish between collided and successful transmissions, the unfairness is amplified at high values of the offered traffic $G$. 

An offered traffic $G$ transmission, the unfairness is amplified at high values of the offered traffic $G$. An example is shown in Fig. 9 that presents the spatial distribution of the throughput $S$ for one of the two APs. Such a figure is obtained evaluating the throughput of each of 15 locations in the first AP domain and averaging the results over different interfering locations in the second AP domain. The second AP is set at $D = 2$ on the right hand side of the figure, while the other parameters are reported in the figure caption.

In a completely fair system, a terminal insists in its transmission attempts until it gets through and, in so doing, the offered channel traffic is higher where the probability of success is smaller. To this extent, we foresee the possibility in future work of extending the proposed model to make the offered channel traffic density $g = g(P)$ dependant on the terminals’ positions $P$, thus providing uniform throughput density $s = s(P)$.

6. Concluding remarks

This paper introduces an analytical framework based on a time continuous Markov chain to assess the performance of physical carrier sense in simple wireless access networks. The reference scenario features two access points providing coverage to multiple end users, which fits many practical deployments of wireless LANs.

The proposed model allows us to calculate the throughput and the collision probability under different settings of the physical carrier sense (carrier sense range and/or power threshold). Numerical results derived from the model, and validated against simulations, confirm that an optimum setting of the physical carrier sensing does exist coming from a trade off between the need of protecting from interference (hidden node and remote collisions), and the opportunity of reusing the bandwidth among concurrent transmissions. Finally, we have shown how the model is able to capture the intrinsic geographical unfairness of the physical carrier sense procedure.

Appendix. Calculation of $D_u$

To evaluate $D_u$, we refer to the transmissions of the sole terminals that start transmitting towards a without colliding, i.e., in the states $(x, y)$. $0 \leq x \leq n, 0 \leq y$. The transmission will not collide if the chain, in its evolution does not enter the subset of states (Fig. A.10) $C = \{1, 0\} \cup \{1, 1\} \cup \{1, 1\}$ before reaching the subset of states $D = \{0, 0\} \cup \{0, 1\} \cup \{0, 1\}$. Referring to Fig. 2(a), an example of a transmission towards $a$ that ends with a collision may be the sequence $(0, 3) \rightarrow (2, 3) \rightarrow (2, 0) \rightarrow (2, 5)$. On the other hand, an example of a transmission of $a$ that ends without collision may be the sequence $(0, 3) \rightarrow (2, 3) \rightarrow (2, 0) \rightarrow (2, 5) \rightarrow (0, 5)$.

If we assume that the disjoint subsets $C$ and $D$ are absorbing sets, i.e., closed sets with no way out, then the probability of a successful transmission is the probability that the chain is absorbed in $D$, and the average transmission duration is the mean time to absorption in $D$ given that the chain is absorbed in $D$. The absorption probabilities in $D, f_{(xy)}, D$, from starting states $(x, y)$ can be obtained as the unique solution of the system of equations (see for example [13]):

$$f_{(xy)}, D = \sum_{(zw) \in \mathcal{T}} r_{(xy),(zw)} f_{(zw)}, D + \sum_{(zw) \in \mathcal{D}} r_{(xy),(zw)} \ (xy) \in \mathcal{T},$$

where $\mathcal{T}$ is the subset of states that are neither in $C$ nor in $D$, and $r_{(xy),(zw)}$ are the conditional transition probabilities, derived from the transition probabilities in the transition instants:

$$p_{(xy),(uw)} = \sum_{(wr) \in \mathcal{C}} q_{(xy),(wr)}.$$

re-normalized over the set $\mathcal{B}$ of states $(z, u)$ for which $p_{(xy),(uw)} > 0$:

$$r_{(xy),(uw)} = \frac{p_{(xy),(uw)}}{\sum_{(wr) \in \mathcal{B}} p_{(xy),(wr)}}.$$

The average probability of a correct transmission for terminal $a$ can be obtained by averaging $f_{(xy)}, D$:

$$P_a = \sum_{xy} \sum_{x=1}^{n} \pi_{(xy)} p_{(xy),(uw)} f_{(xy), D}.$$

The mean time to absorption in $D$, given that the chain is absorbed in $D$, can be obtained by dropping from the chain the states in $C$ and setting new transition intensities $q'_{(xy),(rs)}$. The new intensities are changed with respect the old ones only in those states $(x, y)$ that present transitions toward states $(z, w) \in C$. For these states we have:

$$q'_{(xy),(uw)} = 0 \ (z, w) \in C,$$

$$q'_{(xy),(uw)} = q_{(xy),(uw)} \sum_{(rs) \in C} q_{(xy),(rs)} \ (z, w) \not\in C.$$

In this way the sojourn time in states $(x, y) \in \mathcal{T}$ does not change with respect to the original chain. The mean time to absorption in $D$, $t_{(xy)}$, from starting states $(x, y)$, can be obtained as the unique
solution of the system of equations

\[ r_{xy} = V_{xy} + \sum_{(z,w) \in \mathcal{F}} r_{zw}(xy) \cdot P_{zw}(xy) \quad (x,y) \in \mathcal{F} \]

where: \( V_{xy} = \frac{1}{\sum_{(z,w) \in \mathcal{F}} \lambda_z \cdot t_{xw}} \) is the average sojourn time in \((x,y),\) and \( r'_{xy} \) are the conditional transition probabilities, derived from the transition probabilities in the transition instants:

\[ r'_{xy}(zw) = \frac{q_{xy}(zw)}{\sum_{(r,s) \in \mathcal{B}'} q_{rs}(zw)} \]

re-normalized over the set \( \mathcal{B}' \) of states \((z,w)\) for which \( p'_{xy}(zw) > 0\), that is:

\[ r'_{xy}(zw) = \frac{p'_{xy}(zw)}{\sum_{(r,s) \in \mathcal{B}'} p'_{rs}(zw)} \]

Consequently, we can write

\[ D = \sum_{(z,w) \in \mathcal{B}'} \sum_{t} \pi(t) P_{zw}(xy) r_{xy} \]

**References**


Flaminio Borgonovo received his Doctorate in Electronic Engineering from the Politecnico di Milano in 1971. In 1973, after a two-year period as a research assistant at the Laboratory of Electrical Communications of the Politecnico, he reached the Italian National Research Council(CNR), where he started research activities in the multiple-access and Local Area Networks field. In 1979 he became Associate Professor of Theory of Stochastic Processes at the Electronic Department of the Politecnico di Milano, where he was in charge of teaching probability and prototyping new Local Area access schemes. In 1990 he won a full-professor position at the Universita' di Catania. He is currently Full Professor at the Politecnico di Milano and teaches courses on Electrical Communications and Telecommunications Networks.
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