Numerical bifurcation analysis (with MatCont)

Analyze the prey-predator Rosenzweig-MacArthur model when a constant super-predator is present:

\[
\begin{align*}
\dot{x}_1 &= r x_1 \left(1 - \frac{x_1}{K}\right) - \frac{a_1 x_1}{b_1 + x_1} x_2 \\
\dot{x}_2 &= e \frac{a_1 x_1}{b_1 + x_1} x_2 - m x_2 - \frac{a_2 x_2}{b_2 + x_2} x_3
\end{align*}
\]

where \( K = r = e = a_1 = a_2 = 1 \), \( x_3 = 0.1 \) and \( m = 0.5 \).

Specifically:

- Interpret the model and discuss its properties (e.g., meaning of the variables, positivity of the model, ...)

- Analyze through simulations (done with MatCont or PPlane) the behaviour of the model for \( b_2 = 0.4 \) and \( b_1 = [0.1, 0.2, 0.3, 0.4, 0.5] \)

- Generate a bifurcation diagram in the space \((b_1, x_1, x_2)\) for \( b_2 = 0.4 \) and for \( b_2 = 0.02 \).

- Generate a two-parameter bifurcation diagram for \((b_1, b_2) \in [0, 1] \times [0, 1]\) and interpret the previous bifurcation diagrams as paths on this diagram.