Scheduling and routing in wireless multi-hop networks by column generation

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1. Introduction

Wireless multi-hop networks have attracted the attention of the research community in the last years. Two classes of wireless multi-hop networks are usually considered: mobile multi-hop networks, usually referred to as ad-hoc networks, and fixed multi-hop networks, usually referred to as wireless mesh networks (WMN). WMN are a promising solution to provide both indoor and outdoor broadband wireless connectivity in several environments without the need for expensive wired network infrastructures [1]. Since WMN are infrastructured networks, it is possible to plan the network resources assignment through a suitable optimization process. In this paper we address the problem of optimally exploiting the time resource in wireless multi-hop networks where the transmission time is shared among different packet transmissions. Although the proposed solution approach requires considerable computational time, WMN represent a practical scenario where high computational time can be tolerated within an almost static radio resource planning.

In a WMN, the set of nodes $\mathcal{N}$ represents the devices of the WMN. Every node has a single radio interface, tuned on a channel which is shared with all other nodes. It transmits in uni-casting and it is a half-duplex device thus it can be involved in at most one communication at a time. For every pair of nodes a traffic demand is given, representing the number of packets to be sent within the given time horizon. In the time division multiple access scheme (TDMA) [1], directed transmitter-receiver pairs of nodes – *links* – share the time resource. The time horizon – *frame* – is divided into time slots. To send its packets, a link must be assigned to a set of time slots, in which it is active. To guarantee the required transmission quality, a link (i, j) can be activated only if the Signal-to-Interference and Noise Ratio (SINR) at the receiver j is sufficiently high to correctly decode the signal, *i.e.* it is above a given threshold. Besides the thermal noise contribution, the SINR takes into account the effect of the cumulative interference generated at the receiver j by all the transmitting nodes different from i:

$$SINR_j = \frac{p_{ij} G_{ij}}{\eta + \sum_{(l,m) \neq (i,j)} p_{lm} G_{lj}} \tag{1}$$

where p_{ij} is the transmission power of i, η is the thermal noise and the matrix G_{ij} provides the transmission gain between all the pairs of nodes. With this interference physical model, a set of parallel transmissions on a set of links is admissible if the SINR at all receivers is above the given threshold. In this paper, we deal with the problem of assigning links to time slots – namely the problem of *scheduling* the transmissions in the frame – in order to send all the packets in the minimum number of time slots. This allows to better exploit the time resource and therefore to improve maximum achievable network throughput. Both SINR constraint and half-duplex constraint are considered: the first one guarantees that the SINR ratio is above the given threshold for all the active receivers in every time slot, while the second one implies that every node can be active either as a sender or as a receiver in at most one link in every time slot.

This paper presents different versions of the problem. In the first problem, the transmission power p_{ij} is fixed at a value $\bar{P}$ for every transmitter (Fixed Power Scheduling Problem, FPS). Varying the transmission power allows to assign more links to the same time slot, as the SINR is strictly related to the emitted power of each node: in this case the Power Control Scheduling Problem (PCS) is considered [2, 3, 4]. In FPS and PCS every link is assumed to send one packet in every time slot to which is assigned. PCS is proved to be NP-hard in [4].

Taking into account modern adaptive transmission techniques a link can change its rate, that is, it can send more than one packet in a time slot. Obviously, to guarantee the quality of signals the SINR threshold must grow according to the rate. If both power and transmission rate – and therefore SINR threshold – can be controlled, we are considering the Power and Rate Control Scheduling Problem (PRCS) [5, 6, 7]. Finally, if we allow a traffic demand to route its packets through different hops, instead of sending them on the direct arc, we are required to optimize also the routing of every demand among all the network nodes. This is the Routing and Scheduling Problem (RS) [8, 9, 10, 11]. The three different scenarios, fixed power, power control, joint power and rate control, can be taken into account also in case of routing and scheduling optimization.

Preliminary computational tests show that a formulation with an exponential number of variables, solved by column generation, is a promising approach [12] for all the considered problems. Thus, we propose a column generation approach, as in [13], to solve the continuous relaxation of the problem and to provide a lower bound of the optimal solution. The proposed column generation approach can help in producing good heuristic solutions, as well. In addition to a MIP-based column generation, a Constraint Programming (CP) based column generation is proposed [14]. So far, the CP-based column generation is applied only to FPS and PCS. In the presented computational results, the different behavior of the two column generation procedures and the related heuristic solution is tested on different versions of the problem, and the two approaches to solve the pricing problem are compared. Other works in literature consider some aspects of our problem formulation. However, none of them takes into account above described aspects all together, proposes an actual solution approach and uses a Constraint Programming technique.

The paper is organized as follows. In Section 2, the different versions of the scheduling problem, namely FPS, PCS and PRCS, are described together with their formulations. The proposed approaches are presented as well. Section 3 extends formulations and approaches to the RS. The behavior of the proposed methods is discussed in Section 4, that ends the paper pointing out remarks and suggesting further developments.

2. Scheduling

This section describes the three problems and the corresponding models of increasing complexity related to optimal scheduling in wireless multi-hop networks: FPS, optimal scheduling with fixed power; PCS, optimal scheduling with power control; PRCS, optimal scheduling with power and rate control. For the sake of brevity, the first problem is omitted, but it is shown to be a special case of problem PCS.

A wireless multi-hop network topology is formalised with a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where for each device i in the network there is a node in $\mathcal{N}$, and for each communication link (i, j) there is an arc in $\mathcal{L}$. We assumed that each origin destination pair can be active without violating SINR constraints if no other links is active simultaneously, and therefore we assume the graph $\mathcal{G}$ to be fully meshed. The number of packets a transmitter i has to send to a receiver j is denoted by R_{ij}, the transmission gain between the two devices by G_{ij}, and the SINR threshold by γ.

For the optimal scheduling problem for wireless multi-hop networks presented above, we formulate a model suitable for the column generation-based decomposition technique. The idea is to decompose the scheduling problem into two sub-problems: (i) columns selection and (ii) columns generation. Columns of the considered integer linear formulation of our problem are associated to *configurations*, i.e. sets of links that can be active simultaneously without violating the SINR constraint (1). The set of all possible configurations (columns) is denoted by $\mathcal{S}$, while the set of configurations with link (i, j) active by $\mathcal{S}_{ij}$. Column generation needs an initial subset of feasible configurations which we denote by $\mathcal{S}_0$.

In the column selection sub-problem, the so–called master problem, we introduce an integer variable λ_s for each configuration in $\mathcal{S}$; variable λ_s represents the number of slots to which configuration s is assigned. The total number of time slots is equal to the sum of variables λ_s, since during a time slot only a single configuration is active. Therefore, the problem of minimizing the total number of slots can be modeled as follows:

$$\min \quad \sum_{s \in \mathcal{S}} \lambda_s \tag{2}$$

$$\text{s.t.} \quad \sum_{s \in \mathcal{S}_{ij}} \lambda_s \geq R_{ij} \qquad \forall (i,j) \in \mathcal{L} \tag{3}$$

$$\lambda_s \in \mathbb{Z}^+ \qquad \forall s \in \mathcal{S} \tag{4}$$

Objective function (2) minimizes the number of time slots; constraints (3) guarantee that every link satisfies its own traffic demand; constraints (4) are the integrality constraints. To apply column generation, we start solving the continuous relaxation of the master problem (2)–(4) over the set $\mathcal{S}_0$, and then, using the dual variables σ_{ij} of constraints (3), iteratively generate new configurations (columns) which lead to an improvement of the objective function (2).

To generate a feasible improving configuration, we consider that all nodes transmit in uni-casting and are half-duplex, and that the active links must satisfy the SINR constraint. We introduce a binary variable z_{ij} for each arc (i, j) in $\mathcal{L}$; variable $z_{ij} = 1$ if and only if the corresponding link is active. To model power control, we introduce a continuous non-negative variable p_{ij} for each arc in $\mathcal{L}$. Variables z are used to model the uni-casting and half-duplex constraints, and are linked to the power control variables p. The pricing problem used to generate an improving configuration is the following:

$$c^* = \min \quad \left(1 - \sum_{(i,j) \in \mathcal{L}} \bar{\sigma}_{ij} z_{ij}\right) \tag{5}$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}} z_{ij} + \sum_{(j,i) \in \mathcal{L}} z_{ji} \leq 1 \qquad \forall i \in \mathcal{N} \tag{6}$$

$$p_{ij} G_{ij} \geq \gamma \left(\eta + \sum_{\substack{(h,m) \in \mathcal{L}, \\ h \neq i}} p_{hm} G_{hj} \right) z_{ij} \qquad \forall (i,j) \in \mathcal{L} \tag{7}$$

$$p_{ij} \leq P_{max} z_{ij} \qquad \forall (i,j) \in \mathcal{L} \tag{8}$$

$$z_{ij} \in \{0, 1\} \qquad \forall (i,j) \in \mathcal{L} \tag{9}$$

$$p_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{L} \tag{10}$$

Objective function (5) minimizes over the optimal dual variables $\bar{\sigma}_{ij}$ of problem (2)–(3). By Duality Theory, an improving column exists if and only if the optimal value $c^* < 0$, *i.e.*, the reduced cost is negative. Constraints (6) assign each device $i \in \mathcal{N}$ to at most one of its incident links; this realizes the uni-casting and half-duplex constraints. Constraints (7) are the SINR non-linear constraints: if link (i, j) is active, then the ratio between the received signal and the sum of the thermal noise and the amount of signal of other active links received by j must be above the given threshold γ. The power control variable is bounded by the maximal power achievable by the device. Notice that, by fixing all power control variables p_{ij} to a constant power $\bar{P}$, *i.e.*, $p_{ij} = \bar{P}, \forall (i, j) \in \mathcal{L}$, and removing constraints (8) and (10), we obtain problem FPS.

Finally, we extend this formulation to problem PRCS: optimal scheduling with power control and rate adaptation. In constraints (3) of the master problem, we add a constant v_{ijs} representing the number of packets sent over link (i, j) in configuration s, so that it becomes: $\sum_{s \in \mathcal{S}_{ij}} v_{ijs} \lambda_s \geq R_{ij}, \forall (i, j) \in \mathcal{L}$. Since higher transmission rates require higher SINR thresholds, we introduce a binary variable u_{ijw} for each link (i, j) and for each rate w belonging to the given set of admissible rates $\mathcal{W}$. The threshold γ in (7) depends on the rate used and therefore is replaced by γ_w; to different rates v_{ijs} correspond different values of γ_w. By replacing in problem (5)–(10) variables z_{ij} with u_{ijw}, threshold γ with γ_w, we get the pricing problem PRCS. For the lack of space, we omit here the full model, which can be found in [12].

The continuous relaxation of the formulations presented in the previous paragraphs are solved using column generation. The initial feasible set $\mathcal{S}_0$ is found using a heuristic algorithm which aims at generating a variety of different columns. The continuous relaxation of the restricted master problem, defined by (2),(3) with

$\lambda_s \geq 0$, is solved using a commercial linear solver (CPLEX 9.0 [15]) to provide a lower bound. The pricing (5)–(10) is solved using two different approaches: the first consists in using the CPLEX linear integer solver; the second, introduced recently in [14], consists in solving the pricing by Constraint Programming (CP) [16]. The first approach requires to linearize the SINR constraints (7) with a big-M linearization. By observing the solutions found whit this approach, it appears that the number of active links in feasible configurations is very small due to the SINR constraints. That is, the SINR constraints are tight. This motivates the approach by CP, which can handle efficiently tight constraints. Those non-linear constraints are formulated in CP as they are: implication of linear constraints. Concerning the CP-based approach, three strategies for solving the pricing and adding columns to the restricted master are devised: *CP-opt* solves the pricing to optimality and adds only the best column; *CP-all* adds all the improving columns found through the search to the optimum; *CP-first* stops the search after the first improving solution and adds the corresponding column.

Column generation provides a lower bound, as it solves the continuous relaxation of problem (2)–(4). If the solution of this problem is integer, it is optimum as well; otherwise, if it is fractional, we compute an integer upper bound by solving the integer problem over the final set of generated configurations. The gap between the two bounds is used to estimate the distance to the optimum objective value.

3. Joint routing and scheduling

The model analyzed in the previous section is here extended considering the joint optimization of scheduling and routing. The input of this new problem is only the set of demands between pairs of nodes, while the actual routing is determined by the solution of the optimization problem.

We define the matrix D_{ot} as the demand between node o to t, namely the number of packets to be routed from o to t. We add a set of variables f_{ij}^{ot} representing the number of packets routed from o to t and transmitted through the link (i, j) in a frame. The problem of minimizing the number of slots can be modeled as follows:

$$\min \quad \sum_{s \in \mathcal{S}} \lambda_s \tag{11}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}:(i,j) \in \mathcal{L}} f_{ij}^{ot} - \sum_{j \in \mathcal{N}:(j,i) \in \mathcal{L}} f_{ji}^{ot} = \begin{cases} D_{ot} & i = o \\ -D_{ot} & i = t \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}, \forall o \in \mathcal{N}, t \in \mathcal{N} \tag{12}$$

$$\sum_{s \in \mathcal{S}_{ij}} \lambda_s \geq \sum_{o \in \mathcal{N}, t \in \mathcal{N}} f_{ij}^{ot} \qquad \forall (i,j) \in \mathcal{L} \tag{13}$$

$$\lambda_s \in \mathbb{Z}^+ \qquad \forall s \in \mathcal{S} \tag{14}$$

$$f_{ij}^{ot} \in \mathbb{Z}^+ \qquad \forall (i,j) \in \mathcal{L}, \forall o \in \mathcal{N}, t \in \mathcal{N} \tag{15}$$

The constraints (12), added to the previous formulation, are flow balancing constraints. We adapt constraints (13) imposing that the number of packets the pair (i, j) transmits in a frame must be at least the sum of all the demand packet flows routed through (i, j), R_{ij} of previous section model. As a consequence of the routing, the load of each link is a problem variable determined by the paths of the traffic demands, and is not equal to the demands between its endpoints, as in the previous problems. With this approach, we obtain a multi-path interference-aware routing for the network. The models related to the pricing problem, instead, do not change from the previous section as the problem is independent from the paths followed by traffic demands.

4. Computational results and conclusions

We have tested the proposed approaches on fully connected network of several sizes from 5 up to 30; for every dimension eight instances are considered. The instances are generated by randomly locating the devices over an area of 100m^2 and randomly setting the entries of the demand matrix. Similar random instances are commonly used in the literature [4, 11]. The maximum number of packets is fixed to 15 for each link, η is set to 10^{-6}mW, $P_{max} = 30$mW, $\gamma = 10$, and four rates are considered.

Size	FPS				PCS				PRCS			
	gap [%]		time [s]		gap [%]		time [s]		gap [%]		time [s]	
	av	max	av	max	av	max	av	max	av	max	av	max
5	0	0	0.01	0.02	0	0	0.06	0.1	48	64	0.89	2.05
10	0	0	0.23	0.56	0.02	0.1	10.8	20.4	46	59	4798	10827
20	0	0	51.44	142.57	-	-	-	-	-	-	-	-
30	0	0	4675.65	7566.64	-	-	-	-	-	-	-	-

Table 1: Computational results for the FPS, PCS, and PRCS problems with the MIP-based column generation. The dash '-' is used when the time limit is exceeded.

	MIP-based		CP-opt		CP-all		CP-first	
Instance	nc	time	nc	time	nc	time	nc	time
a-10	16	29	15	4.9	14	4.93	14	1.63
b-10	33	63	42	16.4	50	11.1	54	6.98
c-10	17	51	22	9.73	24	8.86	29	2.82
d-20	-	-	68	11579	77	1383	76	1532
e-20	-	-	44	7251	58	2192	57	2370
f-20	-	-	66	14615	83	1754	82	1869

Table 2: Computational results for PCS with the CP-based column generation for some instances of dimension 10 and 20; *nc* is the number of added columns, and *time* is given in seconds. The dash '-' is used when the time limit is exceeded.

Table 1 shows computational results of MIP-based column generation for optimal scheduling with FPS, PCS, and PRCS. The results are averaged out over the eight instances of each dimension. Results show that for the fixed power case the integer heuristic solution is always the optimal one, and the computational time grows with the network size. Although the time is negligible for small size instances, it rises up to two hours for 30 nodes instances. The gap between lower and upper bound is small for power control case, as well, but the computational time exceeds the limit of eight hours even for 20 nodes instances. Also the PRCS can be solved within eight hours only for instances with at most 10 nodes. In this case, the gap is bigger, rising up to about 50%. In all the cases the computational effort is mainly spent in solving to optimality the pricing problem.

Table 2 shows experimental results using CP for solving the pricing sub-problem for PCS on some instances of dimension 10 and 20. In order to highlight the differences between the two approaches used, we report the number of generated columns nc and the computational time in seconds. Both the *CP-all* and *CP-opt* strategies do outperform the MIP approach in term of computational time, even if they generate more columns. Since this approach looks promising, we are currently developing the CP-based pricing for PRCS and the routing extension.

Table 3 shows computational results of MIP-based column generation for optimal routing and scheduling with: (FPS-R) fixed power, (PCS-R) power control, and (PRCS-R) power control and rate adaptation. The results show that the gap is negligible for the fixed power and the power control case, for any size of the tested instances. The gap increases for the power and rate control case, rising up to about 25% in the average. The computational time is negligible for the fixed power case, but it may rise up to 10 hours for the other considered scenarios, even for 15 nodes instances.

Table 4 shows the results for a particular instance of a fully connected mesh network formed by 12 nodes in which optimized routing and the one-hop routing are compared. Results show that optimizing the routing reduces the number of needed slots, in case of power control and rate and power control. The comparison between the use of a routing on the direct link and a routing optimization brings us to the conclusion that the

Size	FPS-R				PCS-R				PRCS-R			
	gap [%]		time [s]		gap [%]		time [s]		gap [%]		time [s]	
	av	max	av	max	av	max	av	max	av	max	av	max
5	0	0	0.04	0.06	0.05	0.74	0.06	0.08	17.12	33.33	1.29	4.04
10	0	0	0.20	0.29	0.86	5.00	20.21	75.30	25.17	75.00	7911.34	20057.80
15	0	0	1.40	2.61	0.64	1.69	13942.50	29731.57	18.49	26.67	17105.90	20375.31

Table 3: Computational results for FPS-R, PCS-R, and PRCS-R with the MIP-based column generation.

		Optimized routing				Direct link routing	
Total Pkts	Problem	nc	Slots LB	**Slots UB**	time[s]	**Slots UB**	time[s]
16	FPS-R	0	14.00	**14**	0.23	**14**	0.11
	PCS-R	94	9.78	**10**	117.08	**12**	0.66
	PRCS-R	50	1.85	**5**	2903.87	**7**	1.15

Table 4: Computational results for a mesh network with 12 nodes; the overall traffic demand is of 16 packets.

shortest path manner can provide a good heuristic solution in a very short time.

Computational results are promising, though the required computational time increases with the size of the instances, especially for the joint routing and scheduling problem. Since the computational effort is mainly devoted to the solution of the pricing problem, a deeper study of the pricing problem is needed to improve the overall efficiency of the proposed approaches. Furthermore, a wider version of the problem is currently taken into account, which considers a multi-channel network too.

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