Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

Optimization models and methods for planning wireless mesh networks

E. Amaldi, A. Capone, M. Cesana *, I. Filippini, F. Malucelli

Dipartimento Elettronica e Informazione, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy

ARTICLE INFO

Article history:
Received 23 January 2008
Accepted 2 February 2008
Available online 17 March 2008

Responsible Editor: E. Ekici

Keywords:
Wireless Mesh Networks
Planning
Topology
Coverage
Channel assignment
Discrete optimization

ABSTRACT

In this paper novel optimization models are proposed for planning Wireless Mesh Networks (WMNs), where the objective is to minimize the network installation cost while providing full coverage to wireless mesh clients. Our mixed integer linear programming models allow to select the number and positions of mesh routers and access points, while accurately taking into account traffic routing, interference, rate adaptation, and channel assignment. We provide the optimal solutions of three problem formulations for a set of realistic-size instances (with up to 60 mesh devices) and discuss the effect of different parameters on the characteristics of the planned networks. Moreover, we propose and evaluate a relaxation-based heuristic for large-sized network instances which jointly solves the topology/coverage planning and channel assignment problems. Finally, the quality of the planned networks is evaluated under different traffic conditions through detailed system level simulations.

1. Introduction

Wireless Mesh Networking is widely recognized as a promising and cost effective solution for providing wireless connectivity to mobile users eventually competing with wired broadband access technologies. Such success is mainly due to the high flexibility of the mesh networking paradigm which has many advantages in terms of self-configuration capability and reduced installation cost.

Wireless Mesh Networks (WMNs) are composed of a mix of fixed and mobile nodes interconnected through multihop wireless links. Unlike in flat Mobile Ad hoc Networks (MANETs), the wireless mesh network devices are hierarchically organized in terms of networking functionalities and hardware capabilities [1]. Roughly speaking, wireless mesh network devices are of three types: *Mesh Routers* (MRs), *Mesh Access Points* (MAPs) and *Mesh Clients* (MCs). The functionality of both the MRs and the MAPs is twofold: they act as classical access points towards the MCs, whereas they have the capability to set up a *Wireless Distribution System* (WDS) by connecting to other mesh routers or access points through point to point wireless links. Both MRs and MAPs are often fixed and electrically powered devices. Furthermore, the MAPs are geared with some kind of broadband wired connectivity (LAN, ADSL, fiber, etc.) and act as gateways toward the wired backbone. MCs are users terminals connected to the network through MAPs or MRs.

In this type of network architecture, a limited number of MAPs connected to the wired realm can potentially provide low-cost internet connectivity to a large number of MCs. This is substantially different from other wireless access networks (2G/3G cellular systems, WLAN hot spots), where all wireless access points are directly connected to the wired backbone.

Several features have a strong impact on the performance of a general wireless mesh network, including the number of radio interfaces for each device, the number of available radio channels, the access mechanism, the routing strategies and the specific wireless technology used to implement the mesh paradigm. Moreover, since network deployments can involve thousands of devices (MAPs, MRs, and MCs), manual tuning and re-configuration are very unpractical and the automatic planning and

* Corresponding author. Tel.: +39 02 2399 3695; fax: +39 02 2399 3413.
E-mail addresses: amaldi@elet.polimi.it (E. Amaldi), capone@elet.polimi.it (A. Capone), cesana@elet.polimi.it (M. Cesana), filippini@elet.polimi.it (I. Filippini), malucell@elet.polimi.it (F. Malucelli).

1389-1286/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.comnet.2008.02.020

optimization of the network coverage and topology is of utmost importance to efficiently provide the required services [5].

The problem of planning WMNs differs from that of planning other wireless networks, such as cellular systems [4] or WLANs [6]. In the latter cases, network planning is almost entirely driven by geographical coverage. Since wireless transceivers are also gateways towards the wired backbone, their positions/configurations depend only on *local connectivity* constraints, between end users and the closest network device.

In the case of WMNs, the traffic to/from the wired backbone has to be routed on a path connecting the MR to one MAP at least, thus, besides ensuring local connectivity between MCs and MRs (or MAPs), a multi-hop path needs to be set up in the WDS between each end user and one MAP at least (*multi-hop connectivity* constraint) [29]. Moreover, unlike 2G and 3G systems, each candidate site can host either MAPs or MRs, with different installation costs. In fact, MAPs are often more expensive than MRs since they must be directly connected to the wired backbone and might be more powerful in terms of processing and transmission capabilities. Thus, in designing WMNs one must simultaneously consider radio coverage of end users (MCs), as in classical radio planning for wireless access networks [3], and traffic routing, as in the design of wired networks [7].

In this paper, we propose optimization models for coverage and topology planning of WMNs with multiple radio interfaces and multiple orthogonal channels available at each interface. Our mathematical integer linear programming models aim at minimizing the overall installation cost by jointly optimizing the number/locations of MRs and MAPs, and the channel assignment, while taking into account both the *local* and the *multi-hop connectivity* requirements. We provide optimal solutions for a set of realistic medium-scale synthetic network instances and discuss the effect of different parameters on the characteristics of the solutions. Moreover, we present a heuristic to solve large-size instances with an accuracy of 5% with respect to the optimum. Finally, the planned instances are evaluated through detailed system level simulations with ns2 [26].

The paper is organized as follows. In Section 2 we summarize related work and comment on the novel contributions of our approach. In Section 3 we introduce a basic version of the optimization model for planning WMNs, and we report and discuss numerical results. In Section 4 we show how to account for interference in the optimization model, and we propose a heuristic algorithm to solve large-size network instances. Section 5 describes the complete optimization model accounting also for multiple radio interface and multiple channels available at each interface. Since the complete model formulation is hard to solve to optimality large networks, we also present a heuristic algorithm which jointly optimizes mesh devices type/positioning and channel assignment. In Section 6 we comment on the quality of the planning approach by obtaining the throughput of planned networks through system level simulations in ns2. Section 7 contains concluding remarks and future research directions.

2. Related work and contributions

Even if commercial solutions are already available, many aspects of WMNs are still under investigation. The intrinsic flexibility of the WMNs poses stringent research challenges at different layers and a huge research effort is nowadays devoted to the optimization of protocols and the design of algorithms to support mesh networking.

The most popular research fields on mesh networking include Medium Access Control and routing protocols, mobility management and security issues. Since WMNs are wireless multi-hop networks relying on a *Wireless Distribution System* to transport the information, the impact of the interference among different wireless links may dramatically affect the overall network capacity [19]. It is commonly recognized that the use of wireless devices (routers and access points) with multiple wireless Network Interface Cards (NICs) highly reduces the impact of the interference consequently increasing the capacity of the wireless mesh network [8–13].

Substantial previous work aims at providing optimized protocols for WMNs. In [8] So et al. propose a multi-channel MAC protocol for the single interface transceiver case, whereas in [9] the authors study the case of multiple radio interface per wireless node by adapting the channel access protocol. Das et al. present two integer linear programming models to solve the fixed channel assignment problem with multiple radio interfaces [10], while [11–13] address the joint problems of channel assignment and routing, and provide different formulations.

In most of the previous work, the network topology (MAPs and MRs locations) is assumed to be given and the goal is to optimize the channel assignment and/or the routing. In the present work, instead, we model the radio planning problem of WMNs, providing quantitative methods to optimize the number, positions and physical parameters of MRs and MAPs while controlling the overall topology of the network.

So far little attention has been devoted to planning WMNs or general fixed multi-hop wireless networks. So et al. [14] propose an optimization model for the topology planning of WMNs with non-linear constraints, aimed at minimizing the overall installation cost. The non-linear formulation is solved via Benders decomposition. In [15] Wang et al. describe a heuristic approach to tackle the same problem of MRs deployment. Unlike in our approach, these two papers consider the case of a single and pre-defined MAP and optimize the positions of MRs only. On the other hand, our approach endorses also the positioning of MAPs.

In [16], the authors focus on locating internet transit access points (corresponding to MAPs in the terminology adopted in this paper) in wireless neighborhood networks. Heuristic methods and a lower bound are provided, when adopting an interference model according to which interference perceived by a traffic flow grows linearly with the number of wireless hops traversed. The same problem and a similar modelling approach is studied in [17]. Since the positions of all the other nodes (MRs) are given and such nodes are also the only traffic ending points in the network, the problem considered in [16,17] is actually a subproblem of the one proposed in this paper, where the coverage part is omitted. Moreover, we consider a more detailed interference model based on a fluidic version [18] of the protocol interference model [19]. The problem in [16] was previously proposed in [20] for TDMA-based WMNs under the form of a slot scheduling problem.

3. Basic model

We first describe a basic version of the mathematical programming model for WMN planning and then extend it so as to capture the important aspects of interference and multiple channels. We will proceed in three steps not only to progressively add increasing levels of detail and complexity, but also because our three resulting models (basic, interference aware, multi channel ones) are useful in practice depending on the specific network scenario, as we show later on.

Let us consider the network description presented in Fig. 1.

As in other coverage problems for wireless access networks [5,21], let $S = \{1,\dots,m\}$ denote the set of candidate positions where to install mesh devices (Candidate Sites, CSs) and $I = \{1,\dots,n\}$ the set of traffic concentration points (Test Points, TPs). A special node N represents the wired backbone network. The cost associated to installing a MR in CS j is denoted by c_j, while the additional cost required to install a MAP in CS j is denoted by p_j, $j \in S$. The total cost for installing a MAP in CS j is therefore given by $(c_j + p_j)$.

The traffic generated by each TP i is given by parameter d_i, $i \in I$. The traffic capacity of the wireless link between CSs j and l is denoted by u_{jl}, $j,l \in S$, while the capacity of the radio access interface of CS j is denoted by v_j, $j \in S$. For each TP i in I, $S_i = \{j_1^{(i)}, j_2^{(i)}, \dots, j_{L_i}^{(i)}\}$ denotes the ordered subset of CSs that can cover i, where the CSs are ordered according to the non-increasing received signal strength.

The connectivity parameters define which network elements can be connected through wireless links. They depend on TPs and CSs locations and channel conditions and can be determined by using proper propagation prediction tools (e.g., empirical channel models like Hata's one [22], sophisticated ray tracing tools [23], or even field measurements). Note that propagation prediction is out of the scope of this paper and, as in most of the works on radio planning (see e.g. [3–6]), we assume that propagation

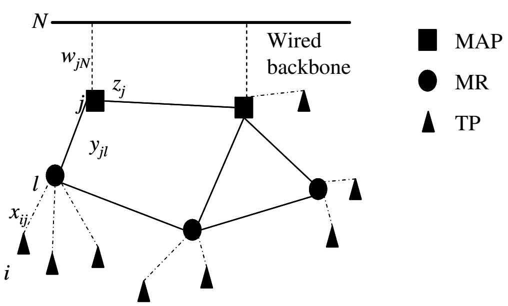


Fig. 1. WMN planning problem description.

parameters are given. In particular, we consider the coverage parameters:

$$a_{ij} = \begin{cases} 1 & \text{if a MAP or MR in CS } j \text{ covers TP } i, \\ 0 & \text{otherwise} \end{cases}$$

for each pair $i \in I$, $j \in S$, and the wireless connectivity parameters:

$$b_{jl} = \begin{cases} 1 & \text{if CS } j \text{ and } l \text{ can be connected with a link,} \\ 0 & \text{otherwise} \end{cases}$$

for each $j,l \in S$.

The decision variables of the problem include TP assignment variables x_{ij}, $i \in I, j \in S$:

$$x_{ij} = \begin{cases} 1 & \text{if TP } i \text{ is assigned to CS } j, \\ 0 & \text{otherwise,} \end{cases}$$

installation variables z_j, $j \in S$:

$$z_j = \begin{cases} 1 & \text{if a MAP or a MR is installed in CS } j \\ 0 & \text{otherwise} \end{cases}$$

wired backbone connection variables w_{jN}, $j \in S$ (if $z_j = 1$, w_{jN} denote if j is connected to the wired network N, i.e. if it is a MAP or a MR):

$$w_{jN} = \begin{cases} 1 & \text{if a MAP is installed in CS } j, \\ 0 & \text{otherwise,} \end{cases}$$

wireless connection variables y_{jl}, $j,l \in S$:

$$y_{jl} = \begin{cases} 1 & \text{if there is a wireless link between CS } j \text{ and } l, \\ 0 & \text{otherwise} \end{cases}$$

and finally flow variables f_{jl} which denote the traffic flow routed on link (j,l), where the special variable f_{jN} denotes the traffic flow on the wired link between MAP j and the backbone network.

Given the above parameters and variables, the WMN planning problem can be formulated as follows:

$$\min \sum_{j \in S} (c_j z_j + p_j w_{jN}) \tag{1}$$

s.t.

$$\sum_{j \in S} x_{ij} = 1 \quad \forall i \in I, \tag{2}$$

$$x_{ij} \leqslant z_j a_{ij} \quad \forall i \in I \ \forall j \in S, \tag{3}$$

$$\sum_{i \in I} d_i x_{ij} + \sum_{l \in S} (f_{lj} - f_{jl}) - f_{jN} = 0 \quad \forall j \in S, \tag{4}$$

$$f_{lj} + f_{jl} \leqslant u_{jl} y_{jl} \quad \forall j,l \in S, \tag{5}$$

$$\sum_{i \in I} d_i x_{ij} \leqslant v_j \quad \forall j \in S, \tag{6}$$

$$f_{jN} \leqslant M w_{jN} \quad \forall j \in S, \tag{7}$$

$$y_{jl} \leqslant z_j, y_{jl} \leqslant z_l \quad \forall j,l \in S, \tag{8}$$

$$y_{jl} \leqslant b_{jl} \quad \forall j,l \in S, \tag{9}$$

$$z_{j_\ell^{(i)}} + \sum_{h=\ell+1}^{L_i} x_{ij_h^{(i)}} \leqslant 1 \quad \forall \ell = 1,\dots,L_i - 1 \ \forall i \in I, \tag{10}$$

$$x_{ij}, z_j, y_{jl}, w_{jN} \in \{0,1\} \quad \forall i \in I, \ \forall j,l \in S. \tag{11}$$

The objective function (1) accounts for the total cost of the network including installation costs c_j and costs p_j related to the connection of MAP to the wired backbone. If for some practical reason only a MR and not a MAP can be installed in CS j, the corresponding variable w_{jN} is set to zero. Constraints (2) guarantee full coverage of all TPs, while constraints (3) are coherence constraints assuring respectively that a TP i can be assigned to CS j only if a device (MAP or MR) is installed in j and if i is within the coverage set of j.

Constraints (4), which define the flow balance in node j, are typical of classical multi-commodity flow problems. The term $\sum_{i\in I} d_i x_{ij}$ is the total traffic related to assigned TPs, $\sum_{l\in S} f_{lj}$ is the total traffic received by j from neighboring nodes, $\sum_{l\in S} f_{jl}$ is the total traffic transmitted by j to neighboring nodes, and f_{jN} is the traffic transmitted to the wired backbone. Even if these constraints assume that traffic from TPs is transmitted to the devices to which they are assigned and that this traffic is finally delivered by the network to the wired backbone, without loss of generality we can assume that d_i accounts for the sum of traffic in the uplink (from TPs to the WMN) and in the downlink (from WMN to the TPs) since radio resources are shared in the two directions.

Constraints (5) impose that the total flow on the link between device j and l does not exceed the capacity of the link itself (u_{jl}). As already mentioned above, these constraints account for the flows in either directions (f_{jl} and f_{lj}) since they share the same radio resources. Constraints (6) impose for all the MCs' traffic serviced by a network device (MAP or MR) not to exceed the capacity of the wireless link used for the access, while constraints (7) forces the flow between device j and the wired backbone to zero if device j is not a MAP. The parameter M is used to limit the capacity of the installed MAP.

Constraints (8) and (9) defines the existence of a wireless link between CS j and CS l, depending on the installation of nodes in j and l and wireless connectivity parameters b_{jl}. The constraints expressed by (10) force the assignment of a TP to the best CS in which a MAP or MR is installed according to proper sorting criteria (such as the received signal strength), while constraints (11) restrict the decision variables to take binary values.

Note that the y_{jl} variables can be eliminated from this basic formulation by combining constraints (5) and (8), that is by replacing in the right-hand side of constraints (5) y_{jl} by z_j and z_l.

The resulting model is obviously NP-hard since it includes the set covering and the multi-commodity flow problems as special cases.

The formulation presented so far considers fixed transmission rates for both wireless access interface and wireless distribution system, thus it will be referred to as *Fixed Rate Model* (FRM) throughout the paper. The FRM can be easily extended to account for transmission rate adaptation.

For the wireless distribution system, it suffices to let the parameters u_{jl} appropriately depend on the propagation conditions between CSs j and l. No other modification to the model formulation is required.

Rate adaptation in the wireless access network can be accounted for by slightly modifying constraints (6). We consider several concentric regions centered in each CS, assigning to each region a maximum rate value. All the TPs falling in one of these regions can communicate with the node in the CS by using the specific rate of the region. Obviously, regions with more favorable propagation conditions can exploit higher rates.

Formally, for any given CS j we define a set of concentric regions $R_j^k = 1, \ldots, K$ and the set $I_j^k \subseteq I$ containing all the TPs falling in the kth region of CS j. Such sets can be determined for each CS j by using the incidence variables

$$a_{ij}^k = \begin{cases} 1 & \text{if a TP } i \text{ falls within region } k \text{ of the CS } j, \\ 0 & \text{otherwise.} \end{cases}$$

For a given CS j, each of these regions is assigned a maximum capacity v_j^k. In Fig. 2 three capacity regions are defined around CS j: the nearest region to the CS has a maximum capacity $v_j^1 = 3$, while in the most external one the maximum capacity drops to $v_j^3 = 1$.

Based on the above definitions, rate adaptation in the wireless access part of the network is accounted for by substituting the constraints (6) with the following new constraints:

$$\sum_{k\in R_j} \frac{\sum_{i\in I_j^k} d_i x_{ij}}{v_j^k} \leqslant 1 \quad \forall j \in S. \tag{12}$$

The resulting model with constraints (12) will be referred to as *Rate Adaptation Model* (RAM) throughout the paper.

3.1. Numerical results

To get some insight on the proposed WMN design approach, we investigate the characteristics of the networks obtained with the basic models by evaluating the impact of different parameters like the number of CSs, the traffic demands from the MCs and the installation costs.

To this end, we have implemented a generator of WMN planning instances which considers specific parameter settings and makes some assumptions on propagation and device features. Obviously, these assumptions do not affect the proposed model which is general and can be applied to any problem instance. The instance generator considers a square area of side L = 1000 m, and it randomly draws the position of m CSs and of n = 100 TPs. The coverage area for the access part of a mesh node is assumed to be a circular

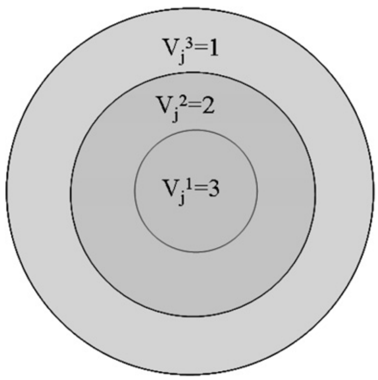


Fig. 2. Rate adaptation regions around CS j.

region with radius $R_A = 100$ m. Only feasible instances where each TP is covered by at least one CS are considered. The range of the wireless backbone links is set to $R_B = 250$ m, while the capacities of the access links, v_j, and backbone links, u_{jl}, are both set to 54 Mb/s for all j and l. The capacity M of the links connecting MAPs to the wired network is either infinite or equal to 128 Mb/s. The ratio between the cost of a MR and a MAP is β ($\beta = 1/10$ unless otherwise specified).

All the results reported in this section correspond to optimal solutions of the considered instances obtained by building the Mixed Integer Linear Programming (MILP) model with AMPL [24] and by solving it with CPLEX [25] on a workstation equipped with a INTEL Pentium (TM) processor with CPUs operating at 3 GHz, and with 1024 Mbyte of RAM.

Given the number and the positions of both CSs and TPs, the quality of the deployed WMN and the overall installation cost depend on two parameters: the traffic demand d of the MCs and the ratio between the MR and MAP installation costs β.

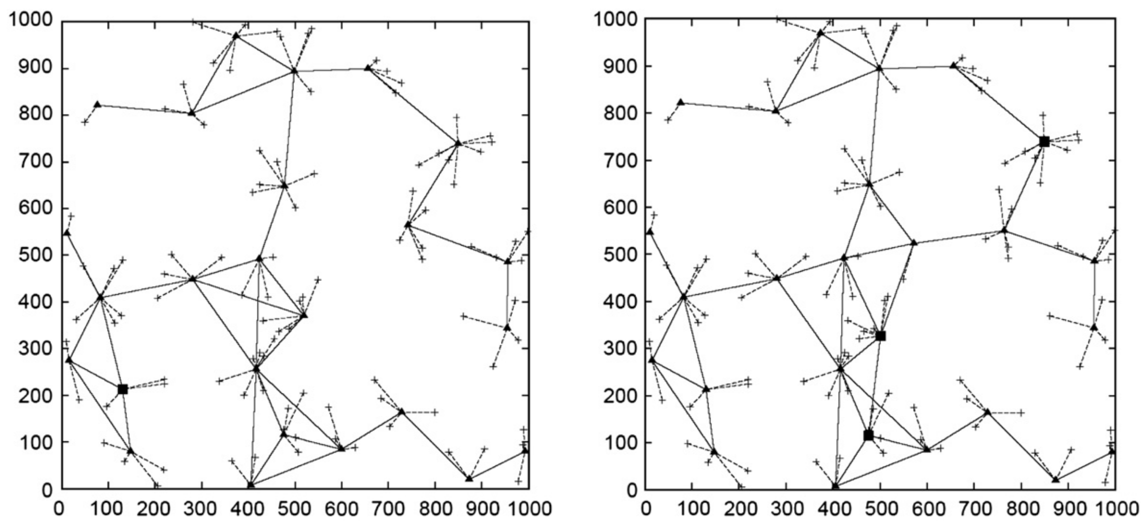
Fig. 3 reports an example of the planned networks obtained with the FRM for the same instance with two different requirements on the end user traffic, $d = 600$ Kb/s and $d = 3$ Mb/s for all MCs, and with a finite capacity of the links connecting MAPs to the wired network ($M = 128$ Mb/s). Black squares represent the installed MAPs, triangles the installed MRs and crosses the MCs positions. Dotted lines express the association between a MC and a MR, while solid lines indicate that MRs and MAPs are within communication range and the link is used for transmitting traffic. As expected, when the traffic demands increase, the number of installed MAPs increases as well in order to convey the MCs' traffic towards the wired realm. On the other hand, the total number of MRs and wireless links is almost the same in the two networks. Indeed, the overall network capacity is mainly limited by the capacity of the links connecting MAPs to the wired backbone. If we remove the MAP capacity constraints (M is unbounded) the networks obtained with the FRM, shown in Fig. 4, are quite different. In this case there is only one MAP installed for both traffic values, but the number of MRs and wireless links is significantly higher for $d = 3$Mb/s.

Tables 1 and 2 summarize the characteristics of the solutions obtained with the FRM when varying the number of CSs for the instances with $M = 128$ Mb/s and M unbounded, respectively. For each pair (m, d) the tables report the number of installed MRs, the number of installed MAPs, the number of wireless links of the WDS and the processing time to get the optimal solution. The results are averages over 20 WMN planning instances.

Table 1 suggests two main comments. First, we observe that, as in Fig. 3, also for average results the number of installed MAPs increases when the traffic demand is increased. Second, for a given traffic value d, increasing the number of CSs to 50 also increases the probability for a MC to be connected to a MAP through a multi-hop wireless path. Therefore, fewer MAPs and more MRs tend to be installed. On the other hand, for a lower number of CSs (30), more MAPs are installed since not all the MCs can be connected to the MAPs through multi-hop wireless paths. In other words, the solution space is larger for higher values of m and solutions providing connectivity at a lower cost (installing more MRs than MAPs) are favored.

Averaged results in Table 2 also confirm the behavior shown in Fig. 4 where the solutions provided by the model are such that an increase in the traffic demands implies an increase in the dimension of the WDS which is used to convey the traffic, both in terms of installed MRs and in terms of wireless links composing the WDS.

The number of installed MAPs and MRs clearly depends also on the installation cost ratio β between a simple wireless router and a mesh access point. If the cost for installing a MAP is much higher than the one spent for installing a



(a) d=600Kb/s (b) d=3Mb/s

Fig. 3. Sample WMNs planned by the FRM with increasing traffic demands of the MCs and finite capacity of the installed MAPs ($M = 128$ Mb/s).

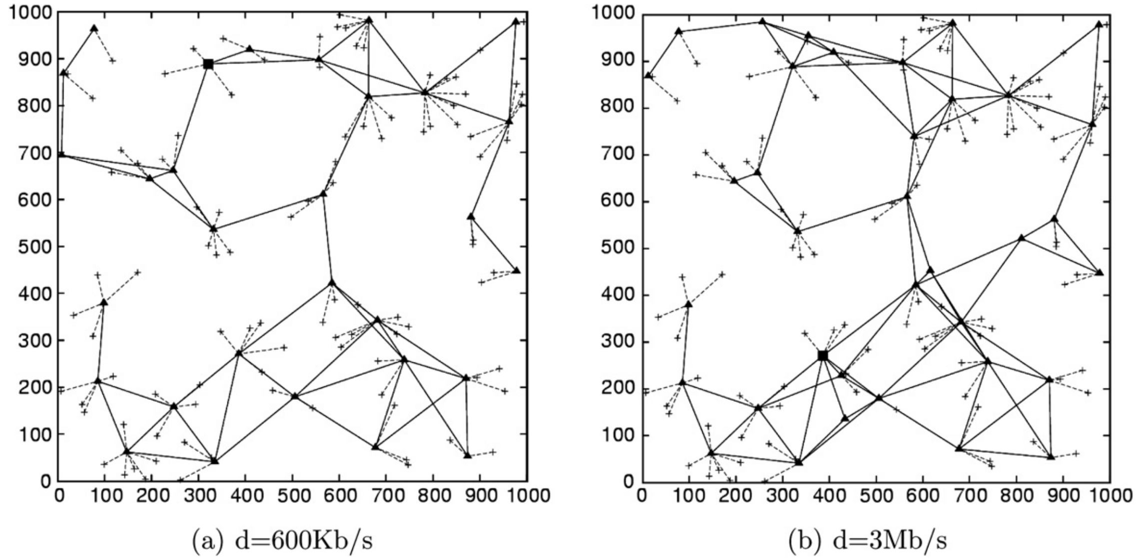


Fig. 4. Sample WMNs planned by the FRM with increasing traffic demands of the MCs and infinite capacity of the installed MAPs (*M* unbounded).

Table 1
Optimal solutions of FRM with finite capacity of the installed MAPs ($M = 128$ Mb/s)

	MAP	MR	Links	Time (s)
d = 600 Kb/s				
m = 30	2.2	23.7	21.5	0.40
m = 40	1.4	24	22.5	1.43
m = 50	1.2	24.1	22.9	4.69
d = 3 Mb/s				
m = 30	4	23.6	20.2	0.63
m = 40	3.4	23.7	21	10.93
m = 50	3.3	23.9	21.6	32.88

Table 2
Optimal solutions of FRM with infinite capacity of the installed MAPs (*M* unbounded)

	MAP	MR	Links	Time (s)
d = 600 Kb/s				
m = 30	2.3	23.4	18.6	1.28
m = 40	1.4	27.4	26.2	2.2
m = 50	1.1	28.4	29	18.16
d = 3 Mb/s				
m = 30	2.8	24.2	20.7	0.68
m = 40	1.8	28.1	27.8	5.47
m = 50	1.1	31	35.5	4.72

MR, the proposed model tends to install fewer MAPs and multiple MRs. On the other hand, when the cost ratio tends to one only MAPs tend to be installed.

Fig. 5 illustrates this effect by showing the network layout in the case of traffic demand $d = 3$ Mb/s and unbounded MAP capacity when varying the installation costs. A larger value of the parameter β leads to a planned network with multiple MAPs installed. If the installation cost of MAPs is much higher than the one of MRs, the incidence of the MAPs installation cost on the overall network cost becomes relevant and hence the model tends to install a lower number of MAPs, resorting to multi-hop paths to serve the MCs traffic.

Table 3 reports the results obtained by varying parameter β for instances with capacity *M* = 128 Mb/s or *M* unbounded. For *M* unbounded, the comments done for Fig. 5 still hold. For $M = 128$ Mb/s, the differences observed with different cost ratios are much smaller than what one could expect, since in this case the number of MAPs installed cannot be reduced too much even when their cost is high (small β) due to capacity constraints.

In the RAM we take into account the effect of adaptive transmission rate by defining three capacity regions around a MR (and MAP) and assigning the link between MC and MR (or MAP) an increasing capacity at is comes closer to the MR (or MAP) location. To obtain numerical results for this model variant, the rate values $v_i^k = v^k \ \forall j$ have been selected in order to emulate IEEE 802.11g [2] transmission according to the distance *r*: $0 \text{ m} \leqslant r \leqslant 30 \text{ m}$ $v^k = 36$ Mb/s, $30 \text{ m} < r \leqslant 60 \text{ m}$ $v^k = 18$ Mb/s, $60 \text{ m} < r \leqslant 100 \text{ m}$ $v^k = 2$ Mb/s.

The behavior of the RAM is similar to the one of the FRM in terms of sensitivity to the model parameters. Table 4 summarizes the characteristics of the solutions obtained with the RAM when varying the number of CSs. Note that the traffic values *d* used in this case are lower than the ones used for the FRM since the link rates are in the average lower. Thus, higher values of *d* may lead to unfeasible instances.

4. Interference aware model

Although wireless technologies like IEEE 802.16 mesh mode and IEEE 802.11 multi-radio mesh networks allow to limit the impact of interference by partitioning radio resources among wireless links using several frequencies or sub-carriers, the interference effect on the access capacity

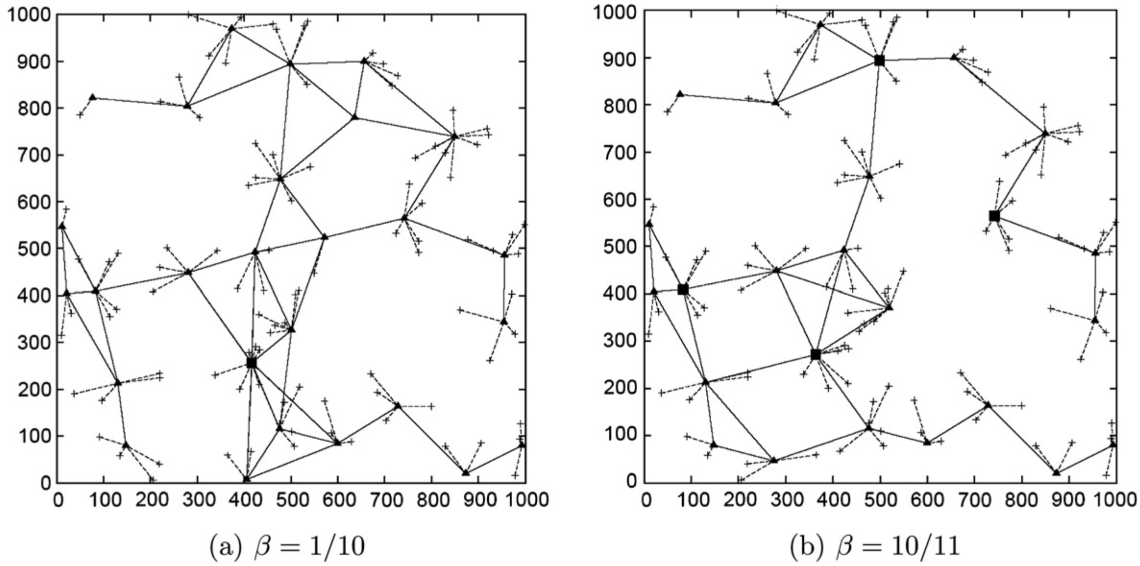


Fig. 5. Sample WMNs planned by the FRM when varying the installation cost ratio β. Infinite capacity of the installed MAPs (M unbounded).

Table 3
Solutions provided by the FRM when varying the installation cost ratio β. Number of CS $m = 50$, traffic demand d = 3 Mb/s

β	M unbounded			$M = 128$ Mb/s		
	MAP	MR	Links	MAP	MR	Links
1/10	1.1	31.0	35.5	3.1	29.0	30.4
1/2	1.8	29.2	31.3	3.4	29.4	31.0
2/3	2.5	28.3	29.1	4.1	28.3	29.3
10/11	4.0	28.1	28.5	4.6	28.1	28.6

Table 4
Quality of the solutions provided by the RAM

	MAP	MR	Links	Time (s)
d = 200 Kb/s				
m = 30	2.80	23.80	21.00	2.24
m = 40	1.70	24.60	22.90	9.66
m = 50	1.20	24.40	23.20	46.83
d = 600 Kb/s				
m = 30	2.80	25.40	22.60	2.24
m = 40	1.70	26.00	24.30	13.25
m = 50	1.20	26.10	24.90	61.37

and on the capacity of wireless distribution system (wireless links connecting mesh nodes) must be taken into account. We focus here on the case of IEEE 802.11 and assume that all MAPs and MRs share the same radio channel for the access part and use another shared channel for the backbone links. Since transmissions between MAPs or MRs and MCs occur on the same channel and the Carrier Sense Multiple Access Collision Avoidance (CSMA/CA) protocol is adopted to regulate channel access, a single transmission at a time is allowed in the coverage range [6]. The impact of interference on the access capacity can then be accounted for by modifying in the RAM constraints (12) as follows:

$$\sum_{k \in R_j} \frac{\sum_{i \in I_j^k} d_i}{v_j^k} \leqslant 1 + M_j(1 - z_j) \quad \forall j \in S, \tag{13}$$

where M_j is the smallest constant that is large enough to ensure that the inequality is always satisfied when $z_j = 0$ (regardless of the other variable values). Whenever $z_j = 1$ the left-hand side is obviously required to be at most 1. Note that here all TPs in the coverage range of a given CS are considered, rather than considering only those TPs assigned to the node in CS j as in constraints (12).

The interference limiting effect on the wireless link capacities is more difficult to account for, since it depends on the network topology and the multiple access protocol. Considering the *protocol interference model* proposed in [19], we can define sets of links that cannot be active simultaneously. These sets depend on the specific multiple access protocol considered. In the case of CSMA/CA, adopted by IEEE 802.11, each set C_{jl} associated to a link (j, l) includes all those links that are one and two hops away in the mesh-network graph (links that connect j and l to their neighbors and their neighbors to the neighbors of their neighbors). Fig. 6 illustrates an example of interference set (links in bold) associated to link (j, l).

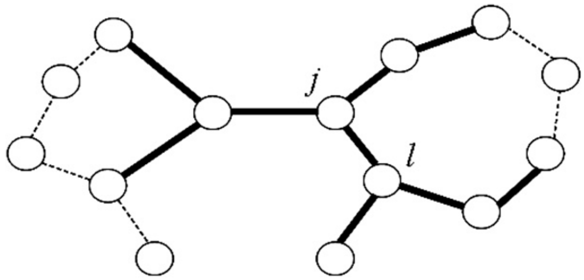


Fig. 6. Example of interference set associated to link (j, l).

To each set C_{jl} we can associate a constraint on the flows crossing its links:

$$\sum_{(k,h)\in C_{jl}} \frac{f_{kh}}{u_{kh}} \leqslant 1 + M_{jl}(1 - y_{jl}) \quad \forall j, l \in S, \tag{14}$$

where the constant M_{jl} is such that the inequality is always satisfied when $y_{jl} = 0$ and the left-hand side is required to be at most 1 when $y_{jl} = 1$. Obviously, by describing the capacity limitation due to set C_{jl} with a constraint on the flows crossing its links, we make an approximation since we consider a fluidic (continuous) model of a discrete problem [18]. However, the effect of this approximation and the one due to traffic dynamics can be accounted for by properly reducing the capacity values u_{kh}. Through simulation of IEEE 802.11 multi-hop networks we have estimated that a reduction of 5% is sufficient to achieve consistent results. Adding to the RAM model constraints (14) and replacing (12) with (13) we obtain the *Interference Aware Model* (IAM), which considers the impact of interference on both the access capacity and the wireless distribution system capacity.

Table 5 reports the results obtained with the IAM for the special case in which rate adaptation is not included. Since the parameter settings are the same adopted for the FRM, results in Tables 5 and 1 can be directly compared. The sets C_{jl} have been determined by considering the IEEE 802.11 multiple access scheme.

We observe that the number of MAPs installed is much higher and the number of links in the WDS lower with respect to the FRM case. This is due to the capacity reduction of wireless links that favors solutions where MRs and MAPs are interconnected through paths with a small number of hops. In fact, with short paths between MRs and MAPs the effect of interference is weaker. Obviously, short paths require a higher number of MAPs. Another significant difference between the FRM and the IAM results is the computing time which is much higher for IAM in most of the cases. Expectedly, this is due to the structure of constraints (14) which involve several flow variables. Similar comments also hold for the IAM with rate adaptation.

4.1. Relaxation-based heuristic

Since solving large-scale instances of IAM is computationally challenging, we have developed a heuristic approach based on the linear relaxation of the MIP formulation. The rationale of the proposed algorithm is the following: we determine an optimal solution of an easier continuous relaxation of the IAM and exploit it to derive useful information about the structure of an optimal solution of the IAM. A candidate solution of IAM is obtained by appropriately rounding the optimal solution of the IAM relaxation where all but one group of binary variables can take fractional values in [0,1]. Finally, the candidate solution for the IAM is improved by applying local changes.

The proposed algorithm proceeds in four steps.

Step 1 – *Solve Continuous Relaxation*. Solve to optimality the IAM relaxation where the MR installation variables z_j and the TP-MR assignment variables x_{ij} are fractional, and only the MAP installation variables w_{jN} are binary. Since experimentally more than 90% of the z_j are binary (within 10^{-9} precision) and all the remaining relaxed variables are very close to 0 or 1, we set the z_j that are binary to 0 or 1 and we round up or down the other z_j.

Step 2 – *Find Candidate Solution*. A candidate solution is obtained by solving the IAM relaxation with z_j fixed in Step 1 and with the variables w_{jN} and x_{ij} binary. Clearly, the flows may be infeasible with respect to the given capacities, as the MR locations have been obtained with fractional TP assignment variables x_{ij}.

Step 3 – *Refinement Phase*. Three refinement moves are applied to improve the candidate solution found in Step 2 and, in case, to make it feasible. These refinements aim at reducing the number of MAPs by installing new MRs to connect isolate regions of the Wireless Distribution System. The three moves are:

- *Add one MR*. Order the installed MRs according to increasing number of neighbors where a device has been installed. Add one MR in one of the empty neighbors of the most isolated MR and get evidence for the new WMN's feasibility by solving the IAM relaxation with z_j fixed, w_{jN} binary and x_{ij} fractional.
- *Delete one MR*. Consider CSs in the reverse order (those with the largest number of installed MRs in their 1-hop neighborhood first), delete the first one in the list and get evidence for feasibility as above.
- *Add k simultaneous MRs*. Proceed as in the "*Add one MR*" move but, instead of picking single MRs, add k random selected MRs among the best ones considered. The k value depends on the cost ratio β between MR and MAP.

Note that the number and positions of installed MAPs may change at each move since the solution of the IAM relaxation can modify the value of the binary variables w_{jN}. The three moves are combined in the following way: the first refinement phase consists in alternating N_{ADD} *Add one MR* moves and N_{DEL} *Delete one MR* moves until no improvement is achieved. Then, the second phase starts performing N_{k-ADD} *Add k simultaneous MRs* moves. If after an addition of MRs an improvement is obtained, that is, at least one MAP is deleted, the algorithm executes N_{DEL} *Delete one MR* moves.

Table 5
Optimal solutions of IAM without rate adaptation

	MAP	MR	Links	Time (s)
d = 600 Kb/s				
m = 30	3.40	22.20	19.40	4.37
m = 40	2.50	23.10	20.90	258.09
m = 50	2.30	23.40	21.60	1706.63
d = 3 Mb/s				
m = 30	8.50	22.00	14.20	4.16
m = 40	7.80	22.50	16.10	53.98
m = 50	7.70	23.40	17.80	1345.32

Notice that such moves aim at trimming possible useless MRs caused by a MAP deletion. Computational results are obtained with $N_{\text{ADD}} = 20\text{—}30$, $N_{\text{DEL}} = 10\text{—}20$, $N_{k-ADD} = 30\text{—}40$, and $k = 4$. The pseudo code of the *Refine Solution* is given in Algorithm 1.

Step 4 – Check Feasibility. The feasibility of the solution found is checked by solving the IAM model with z_j fixed after the *Refinement Phase* and all w_{jN} and x_{ij} binary. Experimentally, we always get a feasible assignment.

Algorithm 1. Refine solution

```
repeat
  for i = 1 to N_ADD do
    Add One MR
  end for
  for j = 1 to N_DEL do
    Delete One MR
  end for
until Solution improved
for i = 1 to N_k-ADD do
  Add k MR
  if Solution improved then
    for j = 1 to N_DEL do
      Delete One MR
    end for
  end if
end for
```

Table 6 reports the computational results provided by the relaxation-based heuristic as well as the optimal solutions for instances with 30, 40, and 50 CSs. All network's costs (MAP cost equal to 10 and MR cost equal to 1) and computing times are averages over 20 instances. In the first two columns, the values in the brackets indicate the relative standard deviation with respect to the average.

Our relaxation-based heuristic is very effective for large instances: it yields near-optimal solutions with a relative gap of at most 5% in less than 20% of the computing time required to solve the mixed integer linear programming IAM model.

5. Multiple channel model

As far as the interference is concerned, the basic models (FRM and RAM) and IAM can be considered two extreme cases corresponding, respectively, to those scenarios where enough channels and radio interfaces are available in the mesh nodes so that interference can be neglected, and to those scenarios where only one channel is available for the WDS. In all the other intermediate scenarios, channel assignment to mesh nodes must be taken into account in the optimization model.

Let us assume that a set F of Q channels is available and that each mesh node is equipped with B radio cards. To extend the RAM to the case with multiple channels, we consider for each pair $q \in F$ and $j \in S$ the installation variable

$$z_j^q = \begin{cases} 1 & \text{if an interface of a MAP or a MR is installed} \\ & \text{in CS } j \text{ and is assigned channel } q, \\ 0 & \text{otherwise} \end{cases}$$

for each triple $q \in F$ and $j, l \in S$, the link variable

$$y_{jl}^q = \begin{cases} 1 & \text{if there is a wireless link between CS } j \text{ 9m } l \\ & \text{on channel } q, \\ 0 & \text{otherwise,} \end{cases}$$

and the flow variable

$$f_{jl}^q = \begin{cases} 1 & \text{traffic flow between CS } j \text{ and } l \text{ on channel } q, \\ 0 & \text{otherwise.} \end{cases}$$

We also need, for each $j \in S$, a new variable t_j which takes the value 1 if a mesh node is installed in j and 0 otherwise.

By replacing in the objective function the variables z_j with the new ones t_j and by modifying the constraints of the RAM to include the channels, we obtain the MILP formulation:

$$\min \sum_{j \in S} (c_j t_j + p_j w_{jN}), \tag{15}$$

$$\sum_{j \in S} x_{ij} = 1 \quad \forall i \in I, \tag{16}$$

$$x_{ij} \leqslant \sum_{q \in F} z_j^q a_{ij} \quad \forall i \in I \ \forall j \in S, \tag{17}$$

$$\sum_{i \in I} d_i x_{ij} + \sum_{l \in S, q \in F} (f_{lj}^q - f_{jl}^q) - f_{jN} = 0 \quad \forall j \in S, \tag{18}$$

$$f_{lj}^q + f_{jl}^q \leqslant u_{jl} y_{jl}^q \quad \forall j, l \in S \ \forall q \in F, \tag{19}$$

$$\sum_{(k,h) \in C_{jl}} \frac{f_{kh}^q}{u_{kh}} \leqslant 1 + M_{jl}^q (1 - y_{jl}^q) \quad \forall j, l \in S \ \forall q \in F, \tag{20}$$

$$\sum_{k \in R_j} \frac{\sum_{i \in I_j^k} d_i}{v_j^k} \leqslant 1 + M_j^q (1 - z_j^q) \quad \forall j \in S \ \forall q \in F, \tag{21}$$

$$f_{jN} \leqslant M w_{jN} \quad \forall j \in S, \tag{22}$$

$$y_{jl}^q \leqslant z_j^q, \ y_{jl}^q \leqslant z_l^q \quad \forall j, l \in S \ \forall q \in F, \tag{23}$$

$$y_{jl}^q \leqslant b_{jl} \quad \forall j, l \in S \ \forall q \in F, \tag{24}$$

Table 6
Solutions of the relaxation-based heuristic for IAM and optimal solutions of FRM

	Heuristic value	Heuristic time	Opt value/time	% gap
d = 600 Kb/s				
m = 30	53.48 (0.097)	62.95 (0.52)	53.48/4.37	0
m = 40	48.04 (0.11)	128.5 (0.4)	46.35/258.9	3.64
m = 50	45.85 (0.09)	233.44 (0.26)	45.03/1706.63	1.82
d = 3 Mb/s				
m = 30	101.85 (0.094)	39.05 (0.27)	101/4.16	0.84
m = 40	101.95 (0.1)	89.25 (0.36)	97.11/53.98	4.95
m = 50	94.25 (0.09)	214.31 (0.24)	89.68/1345.32	5.1

Table 7
Solutions provided by the MCM without rate adaptation

	MAP	MR	Links	Time (s)
d = 600 Kb/s				
m = 30	2.25	23.75	21.60	2.99
m = 40	1.45	24.00	22.60	73.43
m = 50	1.25	24.05	22.85	393.38
d = 3 Mb/s				
m = 30	4.00	23.65	20.55	9.79
m = 40	3.40	23.75	21.30	141.18
m = 50	3.30	23.65	21.35	673.52

$$t_{j^{(i)}_\ell} + \sum_{h=\ell+1}^{L_i} x_{ij^{(i)}_h} \leqslant 1 \quad \forall \ell = 1,\ldots,L_i-1 \ \ \forall i \in I, \tag{25}$$

where M, M^q_j and M^q_{jl} are appropriate constants. To limit the maximum number of channels assigned to a mesh node by a constant B, we need the additional set of constraints:

$$\sum_{q\in F} z^q_j \leqslant B \quad \forall j \in S. \tag{26}$$

To define the new variables t_j, we also include:

$$t_j \geqslant z^q_j \quad \forall j \in S, \forall q \in F. \tag{27}$$

Finally, the variable domains are

$$x_{ij}, z^q_j, y^q_{jl}, w_{jN}, t_j \in \{0,1\}; \quad f^q_{jl}, f_{jN} \geqslant 0; \quad \forall i \in I \ \forall j,l \in S \ \forall q \in F. \tag{28}$$

The resulting model, which accounts for channel assignment to multi-radio devices, is referred to as *Multi-Channel Model* (MCM).

Table 7 reports the computational results obtained for the MCM when $Q = 11$ channels and $B = 3$ radio interfaces, which are typical values for the IEEE 802.11a technology. The formulations are solved with the MILP solver of CPLEX [25].

Overall the results are quite similar to those obtained with the FRM in terms of installed MAPs, MRs and links (see Table 1). On the other hand, MCM turns out to be much harder to solve with a state-of-the-art MILP solver than the other models. Indeed, within a computing time limit of four hours, only 80% of the instances are solved to optimality and are reported in the table. Therefore, the MCM can be directly used to plan multi-channel/multi-radio WMNs only when the number of available channels and interfaces is very limited. In the other cases, we exploit the information on the network structure provided by the single-channel FRM/RAM to design an effective heuristic algorithm.

5.1. Two-phase heuristic

We subdivide the problem described by the MCM into two subproblems. Since the networks obtained with 802.11 channels and only three interfaces are very similar to those planned without considering the interference, we first locate the MR and MAP and determine the routing flows in the Wireless Distribution System by solving the Basic model (FRM) to optimality. Then the following three steps are carried out iteratively.

Step 1 – *Color Links*. Try to color each link so that each node is incident to at most three colors (as at most three interfaces per node are allowed). Links are considered by non-increasing traffic load and the color with the smallest load in a given multi-hop neighborhood is assigned. In particular, the color is selected based on the local interference (2-hop) plus a "look-ahead" term introduced by considering up to 4-hops. In order to reduce feasibility problems due to interference sets overlapping, we want to avoid that links with heavy load and the same channel are within four hops. Therefore, even if we neglect capacity constraints when assigning channels, we look for a solution that aims at minimizing possible violations.

Step 2 – *Check Interference Feasibility*. Check the assignment's feasibility by evaluating the capacity constraint for each interference set. If no constraint is violated, the obtained network is feasible and the procedure stops. Otherwise go to *Step 3*.

Step 3 – *Increase Load and Reroute*. If any capacity constraint on the interference sets is violated, increase the traffic demands (by 1%) and solve the new FRM model (*Solve FRM*). Consider only the MR and MAP locations of this new FRM solution and then reroute original demands by solving a min-cost flow (with unit costs) on the new network defined by the z_j and w_j variables (*Re-Route Traffic*). Notice that capacities v_j and u_{lk} remain unchanged. By re-routing the demands we obtain the new traffic flows, and hence the new link loads. Go to *Step 1*.

Pseudocode for the two-phase heuristic is in Algorithm 2.

Algorithm 2. Two-phase heuristic

```
Solve FRM
Feasible Solution := FALSE
repeat
  Color links
  if Feasible interference then
    Feasible Solution := TRUE
  else
    Increase traffic demands
    Solve FRM
    Re-route traffic
  end if
until Feasible Solution = TRUE
```

Table 8 summarizes the results obtained by solving the MCM with the MILP solver and by applying the two-phase heuristic. For three instance sizes – 30, 40, and 50 CSs – with MAP cost equal to 10 and MR cost equal to 1, the total network's cost and solution time are reported, averaged on 20 instances. In the low-traffic case, we reach optimality for almost all the instances in very short computing time, that is, in a few seconds even for the largest instances. In the high-traffic case the performance and the solution time are slightly worse, however the error gap is below 5.2% and the computing time is at most 30% of that required to solve the MILP for the largest instances.

Table 8
Solutions provided by the two-phase heuristic for MCM without rate adaptation

m	Heuristic value	Heuristic time	Opt value/time	% gap
$d = 600$ Kb/s				
30	44.128 (0.15)	0.4 (1.22)	44.127/2.99	0
40	37.20 (0.15)	1.6 (0.75)	37.19/73.43	0.01
50	35.76 (0.11)	4.68 (1.06)	35.76/393.38	0
$d = 3$ Mb/s				
30	63.07 (0.12)	11.3 (1.60)	59.97/9.79	5.17
40	56.04 (0.1)	42 (1.55)	54.69/141.18	2.46
50	55.21 (0.1)	133.19 (1.65)	54.01/673.52	2.23

The proposed heuristic allows us to tackle much larger instances in reasonable computing time. In Fig. 7 we report an example of network obtained considering 1000 TPs (1 Mb/s traffic per TP) and 600 CSs on a 2000×2000 m area. The solution contains 105 installed devices: 12 of them are MAPs, while the remaining 93 are MRs. The WDS is formed by 99 wireless links and the solution time is below 1 h.

6. Network performance evaluation via simulation

The main goal of this section is to assess the impact of the assumptions made in the optimization models on the performance of the planned networks. To this end, we have run system level simulations with ns-2 [26], a discrete event simulation platform widely used by the networking research community. The main differences between the optimization models and the simulations are related to traffic and interference. Indeed, traffic is fluidic in the models and packet-based in the simulations, and the interference is modelled with interference sets in the optimization framework, whereas simulations run a realistic version of the IEEE 802.11 MAC.

In order to define the simulation scenario and get more insights on the performance of the planned WMNs, we consider the solutions provided by the IAM which is the most critical one due to the assumptions on the impact of interference on the network capacity.

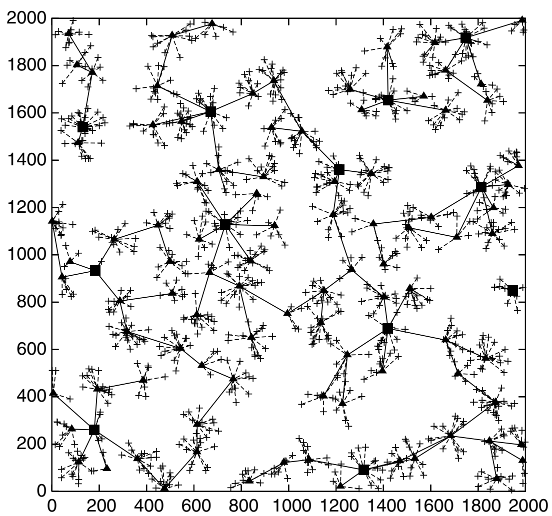


Fig. 7. Example of large network planned with the proposed heuristic for the MCM.

Given the set of installed MRs and MAPs, and the corresponding traffic demands, we first derive the routing matrix by solving the model to be reproduced in the simulations. Therefore, we solve a multi-commodity min-cost flow linear programming (LP) model using the given network topology and assigning to each link a capacity equal to the flow provided by the solution of the planning problem. As the planning model takes into consideration interference and capacity constraints when defining traffic flows in the network, the routing provided by the multi-commodity model is guaranteed to be a feasible solution.

The linear programming model considers two sets of variables: l_{ij}^{rm} variables representing the flow through the link between nodes i and j originated from node r and directed to MAP m, and a_r^m variables expressing the amount of traffic demand from node r that is routed to MAP m. In addition, we have the link capacity k_{ij} equal to the flow f_{ij} of the planning model and the total demand of node r, D_r, obtained summing all demands of TPs associated to r, that is

$$\sum_{i \in TP} d_i x_{ir}.$$

The LP model formulation is the following:

$$\min \sum_{\substack{i,j,r \in MR,\\ m \in MAP}} l_{ij}^{rm} \tag{29}$$

such that

$$\sum_{j \in MR: j \neq i} l_{ji}^{rm} - \sum_{j \in MR: j \neq i} l_{ij}^{rm} = \begin{cases} -a_r^m, & i = r, \\ a_r^m, & i = m, \\ 0 & \text{other}, \end{cases} \quad \begin{matrix} \forall i, r \in MR, \\ m \in MAP, \end{matrix} \tag{30}$$

$$\sum_{m \in MAP} a_r^m = D_r \quad \forall r \in MR, \tag{31}$$

$$\sum_{r \in MR, m \in MAP} (l_{ij}^{rm} + l_{ji}^{rm}) \leqslant k_{ij} \quad \forall i, j \in MR, \tag{32}$$

where (29) is the objective function aiming at minimizing the total flow on network links. Constraints (30) are flow balancing constraints that force the routing of traffic demands, according to the traffic partition among available MAPs. Constraints (31) ensure that traffic demand D_r of each MR r is routed to MAPs. Finally, constraints (32) limit the flow on each link to the value obtained by the planning model. The variable set l_{ji}^{rm} defines the routing of all flows into the network, from their originating MR to their destination MAPs.

By using the network topology provided by the planning model and the routing provided by the multi-commodity model, we derive a simulation scenario which is comparable to the modelled one.

We have used the CMU monarch's wireless extension [28] of ns2 version 29 running 802.11 MAC layer and NOAH routing module [27]. We have introduced modifications to NOAH module to account for route splitting based on source/destination pairs. We have run simulations with CBR traffic on top of UDP and TCP. In case of TCP we have also considered reverse routes and ACK traffic overhead. The transmitting power has been selected so as to have the same communication and interference range

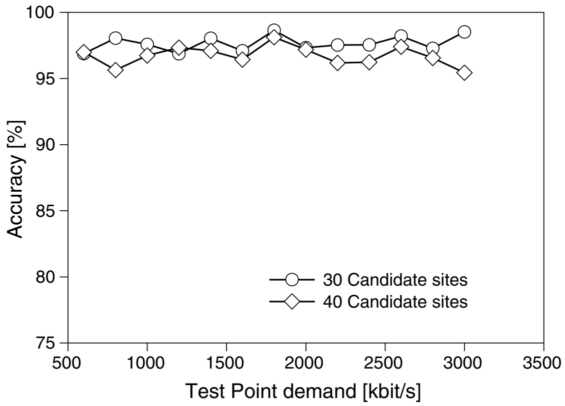


Fig. 8. Simulation results: ratio between delivered traffic and nominal traffic (UDP flows).

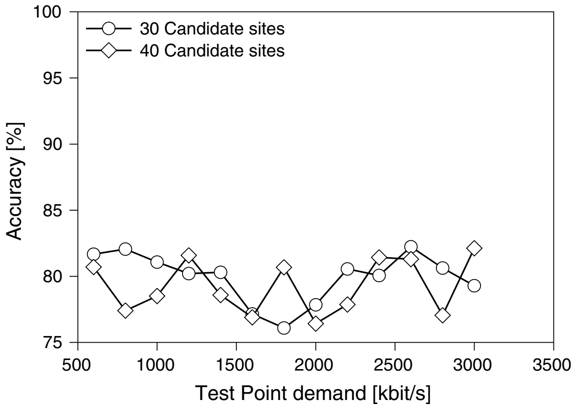


Fig. 9. Simulation results: ratio between delivered traffic and nominal traffic (TCP flows).

considered in the model. Finally, the transmission rate on wireless links have been set equal to the value considered in the model according to the adaptive modulation scheme.

Figs. 8 and 9 show the results for the UDP and TCP cases, respectively. The accuracy is computed as the ratio between the traffic received at MAPs and the total injected traffic that is equal to the nominal value used to plan the network. For instances with 30–40 CSs, the simulated scenario is able to route towards MAPs more than 95% of UDP traffic. This shows that the actual network capacity is very close to the traffic value used to plan it with the proposed optimization model. Therefore, the assumptions made in the model do not affect the quality of the planned network. With TCP traffic, instead, the ratio drops down to approximately 80%. This is mainly due to retransmissions and timeout expirations that make TCP not able to fully exploit the available capacity. It is worth noting that this well known effect can be observed on any network and is not due to an insufficient capacity of the planned network.

7. Conclusion

In this paper we addressed the WMNs planning problem in terms of deciding the types (either MAP or MR), positions and configuration of the network devices to be deployed. We have proposed optimization models aiming at minimizing the overall network installation cost while taking into account the coverage of the end users, the wireless connectivity in the wireless distribution system, the management of the traffic flows, and channels assigned to radio interfaces. Technology dependent issues such as rate adaptation and interference effect have also been considered. Moreover, we have proposed relaxation-based heuristic algorithms to obtain very good solutions in reasonable computation time.

In order to validate the optimization approach proposed, we have generated synthetic instances of WMNs and solved them to optimality using AMPL/CPLEX varying several network parameters. On the same instances we have run the proposed heuristics. Numerical results show that the models are able to capture the effect on the network configuration of all relevant parameters, providing a promising framework for the planning of WMNs. Moreover, we have shown that the heuristics are always able to provide close to optimal solutions even for large instances.

Finally, we assessed the quality of the networks planned using our optimization framework by simulation. Simulation results shows that the capacity of the planned networks is very close to the nominal value used in the model and that the impact of the assumptions made in the models is negligible.

References

[1] I.F. Akyildiz, X. Wang, A survey on wireless mesh networks, IEEE Comput. Mag. 43 (9) (2005) 23–30.
[2] http://grouper.ieee.org/groups/802/.
[3] R. Mathar, T. Niessen, Optimum positioning of base stations for cellular radio networks, Wireless Networks 6 (4) (2000) 421–428.
[4] E. Amaldi, A. Capone, F. Malucelli, Planning UMTS base station location: optimization models with power control and algorithms, IEEE Trans. Wireless Commun. 2 (5) (2003) 939–952.
[5] A. Eisenblatter, H.F. Geerd, Wireless network design: solution-oriented modeling and mathematical optimization, IEEE Wireless Commun. 13 (6) (2006) 8–14.
[6] S. Bosio, A. Capone, M. Cesana, Radio planning of wireless local area networks, ACM/IEEE Trans. Networking 15 (6) (2007) 1414–1427.
[7] M. Pioro, D. Medhi, Routing, Flow, and Capacity Design in Communication and Computer Networks, Morgan Kaufman Publishers, 2004.
[8] J. So, N. Vaidya, Multi-channel MAC for ad hoc networks: handling multi-channel hidden terminals using a single transceiver, in: Proceedings of ACM MOBIHOC 2004, pp. 222–233.
[9] A. Adya et al., A multi-radio unification protocol for IEEE 802.11 wireless networks, in: Proceedings of IEEE BROADNETS 2004, pp. 344–354.
[10] A.K Das, H.M.K. Alazemi, R. Vijayakumar, S. Roy, Optimization models for fixed channel assignment in wireless mesh networks with multiple radios, in: Proceedings of IEEE SECON 2005, pp. 463–474.
[11] M. Alicherry, R. Bhatia, L.E. Li, Multi-radio, multi-channel communication: joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks, in: Proceedings of ACM MOBICOM 2005, pp. 58–72.
[12] A. Raniwala, T. Chiueh, Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network, in: Proceedings of IEEE INFOCOM 2005, vol. 3, pp. 2223–2234.
[13] A. Raniwala, K. Gopalan, T.C. Chiueh, Centralized channel assignment and routing algorithms for multichannel wireless mesh networks, ACM MC2R 8 (2) (2004) 50–65.
[14] A. So, B. Liang, Minimum cost configuration of relay and channel infrastructure in heterogeneous wireless mesh networks, in: Proceedings of IFIP Networking, May 2007.

[15] J. Wang, B. Xie, K. Cai, D.P. Agrawal, Efficient mesh router placement in wireless mesh networks, in: Proceedings of IEEE MASS 2007, October 2007.
[16] R. Chandra, L. Qiu, K. Jain, M. Mahdian, Optimizing the placement of integration points in multi-hop wireless networks, in: Proceedings of IEEE ICNP 2004, pp. 271–282.
[17] B. He, B. Xie, D.P. Agrawal, Optimizing the internet gateway deployment in a wireless mesh network, in: Proceedings of IEEE MASS 2007, October 2007.
[18] M. Kodialam, T. Nandagopal, Characterizing the capacity region in multi-radio multi-channel wireless mesh networks, in: Proceedings of ACM MOBICOM 2005, pp. 73–87.
[19] P. Gupta, P.R. Kumar, The capacity of wireless networks, IEEE Trans. Inform. Theory 46 (2) (2000) 388–404.
[20] Y. Bejerano, Efficient integration of multi-hop wireless and wired networks with QoS constraints, in: Proceedings of ACM MOBICOM 2002, pp. 215–226.
[21] K. Tutschku, Demand-based radio network planning of cellular mobile communication systems, in: Proceedings of IEEE INFOCOM 1998, vol. 3, pp. 1054–1061.
[22] M. Hata, Empirical formula for propagation loss in land mobile radio services, IEEE Trans. Vehicular Technol. 29 (August) (1980) 317–325.
[23] J.W. McKown, R.L. Hamilton, Ray tracing as a design tool for radio networks, IEEE Network 5 (6) (1991) 27–30.
[24] R. Fourer, D.M. Gay, B.W. Kernighan, AMPL, A modeling language for mathematical programming, 1993.
[25] ILOG CPLEX 10.0 user's manual. <http://www.ilog.com/products/cplex/>.
[26] Network Simulator ns-2, Information Sciences Institute (ISI), University of Southern California. <http://nsnam.isi.edu/nsnam/>.
[27] NO Ad-Hoc Routing Agent (NOAH), Ecole Polytechnique Federale de Lausanne. <http://icapeople.epfl.ch/widmer/uwb/ns-2/noah/>.
[28] Wireless and Mobility Extensions to ns-2, Monarch Project, Rice University. <http://www.monarch.cs.rice.edu/cmu-ns.html>.
[29] E. Amaldi, A. Capone, M. Cesana, F. Malucelli, Optimization models for the radio planning of wireless mesh networks, in: Proceedings of IFIP Networking 2007, May 2007.

Edoardo Amaldi received the "Diplome" in Mathematical Engineering and the "Doctorat és Sciences" (Ph.D.) from the Swiss Federal Institute of Technology at Lausanne (EPFL) in 1988 and 1994, respectively. After three years at the School of Operations Research and Industrial Engineering, Cornell University, USA, he joined in 1998 the Dipartimento di Elettronica e Informazione (DEI), Politecnico di Milano, Italy, where he is currently an Associate Professor in Operations Research. His main research interests are in discrete optimization and in the design and analysis of algorithms, with applications in telecommunications, machine learning, image processing, and data mining. In 2005 he received an IBM Faculty Award for his work on the maximum feasible subsystem problem. He is on the editorial board of the International Transactions in Operational Research and he serves on the Program Committees of several international conferences and workshops (ESA,INOC and WEA) and since 2005 he is on the Steering Committee of WEA.

Antonio Capone is an Associate Professor at the Information and Communication Technology Department (Dipartimento di Elettronica e Informazione) of the Technical University of Milan (Politecnico di Milano). His expertise is on networking and main research activities include protocol design (MAC and routing) and performance evaluation of wireless access and multi-hop networks, traffic management and quality of service issues in IP networks, and network planning and optimization. On these topics he has published more than one hundred peer-reviewed papers in international journal and conference proceedings, and holds several patents. He received the M.S. and Ph.D. degrees in electrical engineering from the Politecnico di Milano in 1994 and 1998, respectively. In 2000 he was a visiting professor at UCLA, Computer Science department. He currently serves as editor of the Wiley Journal of Wireless Communications and Mobile Computing and the Elsevier Journal of Computer Networks. He served as guest editor of the Special Issue of the IEEE Wireless Communications magazine on "3G/4G/WLAN/ WMAN Planning and Optimization", the Special Issue of the Elsevier Ad Hoc Networks journal on "Recent research directions in wireless ad hoc networking" and as member of the technical program committee of several international conferences. He is currently involved in the scientific and technical activities of several national and European research projects, and he leads several industrial projects. He is a Senior Member of the IEEE (Communications, Computer and Vehicular Technology societies).

Matteo Cesana received his M.S. in Telecommunication Engineering and his Ph.D. in Information Engineering from the Politecnico di Milano in July 2000 and in September 2004, respectively. From September 2002 to March 2003 he has been working as a visiting researcher at the Computer Science Department of the University of California in Los Angeles (UCLA). He is now an Assistant Professor of the Electronics and Information Department of the Politecnico di Milano. His research activities are in the field of performance evaluation of cellular systems, ad-hoc networks protocol design and evaluation and wireless networks optimization. He is an editor of Ad Hoc Networks Journal (Elsevier), and he has served in the technical program committee of several international conferences. He is a regular reviewer of the main journals in the networking area. He is a member of IEEE Communication and Computer societies.

Ilario Filippini received the B.S. and M.S. degrees in Telecommunication Engineering from Politecnico di Milano, Italy, in 2003 and 2005. He is currently a Ph.D. student at Electronics and Information Department of Politecnico di Milano. His research activities are in networking and optimization, main topics are wireless multi-hop network planning, optimization and protocol design. He is a student member of the IEEE Communication Society.

Federico Malucelli (Ferrara, 7/4/62) got a Laurea in Computer Science and Ph.D. in Computer Science both from Universita' di Pisa in 1988 and 1993, respectively. Since 2003 he is full professor of Operations Research at the Politecnico di Milano. In 1998–2002 he was associate professor of Operations Research at the Politecnico of Milano, and from 1992 to 1998 research associate at Pisa and Perugia Universities. He has visited several foreign universities and research laboratories, including HP Laboratories, Palo Alto (USA), Universite' de Montreal (Canada) and Linkoping Universitet (Sweden). He has served as research unit coordinator for several nationwide MIUR and CNR research projects on optimization in telecommunications networks and transportation systems. His main research interests include: models and algorithms for combinatorial optimization problems, with applications in particular to telecommunications, transportations, logistics, and electronic circuit design. He has published more than 30 articles on international scientific journals.