This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

Joint Routing and Scheduling Optimization in Wireless Mesh Networks with Directional Antennas

Antonio Capone
Department of Electronics and Information
Politecnico di Milano
Email: capone@elet.polimi.it

Ilario Filippini
Department of Electronics and Information
Politecnico di Milano
Email: filippini@elet.polimi.it

Fabio Martignon
Department of Information Technology and Mathematical Methods
University of Bergamo
Email: fabio martignon@unibg.it

***Abstract*—Wireless Mesh Networks (WMNs) have recently emerged as a technology for next-generation wireless networking. WMNs partially replace wired backbone networks, and it is therefore reasonable to plan carefully radio resource assignment to provide quality guarantees to traffic flows.**

Directional transmissions allow to reduce radio interference, thus exploiting spatial reuse. Therefore, as a main contribution, in this paper we study the joint routing and scheduling optimization problem in Wireless Mesh Networks where nodes are equipped with directional antennas.

To this aim, we assume a Spatial reuse Time Division Multiple Access (STDMA) scheme, a dynamic power control able to vary the emitted power slot-by-slot, and a rate adaptation mechanism that sets transmission rates according to the Signal-to-Interference-and-Noise Ratio (SINR).

We provide column generation-based heuristic approaches for the proposed models in a set of realistic-size instances and discuss the impact of different parameters on the network performance. The results show that our schemes increase considerably the total traffic accepted by the network, providing bounds to the achievable performance.

***Index Terms*: - Wireless Mesh Networks, Routing, Scheduling, Optimization, Directional Antennas.**

I. Introduction

Wireless Mesh Networks (WMNs) have recently emerged as a practical solution for the broadband wireless Internet [1]. WMNs are the ideal solution to provide both indoor and outdoor broadband wireless connectivity in several environments without the need for costly wired network infrastructures.

The network nodes in WMNs, named mesh routers, provide access to mobile users, like access points in Wireless Local Area Networks, and they relay information hop by hop, like routers, using the wireless medium. Mesh routers are usually fixed and do not have energy constraints. WMNs, like wired networks, are characterized by infrequent topology changes and rare node failures. They partially replace the wired backbone network providing similar services and quality guarantees.

In recent years, directional antenna technology has been studied in WMNs [2]. The main advantage of using directional antennas with wireless multi-hop networks is the reduced interference and the possibility of having parallel transmissions among neighbors with a consequent increase of spatial reuse of radio resources.

Therefore, as a main contribution, in this paper we study the joint routing and scheduling optimization problem in WMNs where nodes are equipped with directional antennas. We assume a Spatial reuse Time Division Multiple Access (STDMA) scheme, a dynamic power control able to vary the emitted power slot-by-slot, and a rate adaptation mechanism that sets transmission rates according to the Signal-to-Interference-and Noise Ratio (SINR). Traffic quality constraints are expressed in terms of minimum required bandwidth. Since the time frame defined by the TDMA scheme is fixed, the bandwidth requirement can be translated into the number of information units (packets) that must be transmitted on each link per frame. Moreover, according to a discrete set of possible transmission rates, the number of packets that can be transmitted per time slot depends on the SINR at receivers.

To get more insights on the characteristics of the problem and the effect of different control mechanisms, we consider three different versions of the problem with increasing complexity. In the first one we assume fixed transmission power and rate, in the second one variable power and fixed rate, and finally in the third one variable power and rate. For each version we consider mesh nodes with both omnidirectional and directional antennas.

Given a number of available slots, our goal is to provide an assignment of time slots to links such that bandwidth constraints are satisfied and the number of available slots is not exceeded. To solve such problem, it is possible to look for the minimum number of needed time slots: if it is smaller than the number of available slots, a feasible assignment exists.

Since the classical compact mathematical programming formulation is very hard to solve [3], the solution approach we propose is based on an alternative problem formulation where decision variables represent compatible sets of links active in the same time slot. As variables are exponentially many, we use a column generation approach to solve the continuous relaxation of the problem which provides a lower bound of the optimal solution. In several cases the solution provided by the column generation procedure is equivalent to the integer optimum; however, to provide good solutions in reasonable time we propose two solution approaches with different computational complexity.

We analyze the proposed models in a set of realistic-size instances and discuss the effect of different parameters on the characteristics of the solution. The results show that the utilization of directional antennas and rate control schemes increase considerably the total traffic accepted by the network.

The paper is structured as follows: Section II discusses the routing and scheduling problem in WMNs, and revises related work. Section III introduces the assumptions made for the modeling approach proposed in section IV. Section V discusses numerical results that show the effect of different parameters on the characteristics of the solution. Finally, conclusions and directions for future research are presented in Section VI.

II. Wireless Mesh Networks: Routing and Scheduling Issues

In a wireless environment, the network topology depends on the nodes position and propagation conditions. We can assume that a link between two nodes (i, j) exists if transmitting a signal at maximum power in i, the Signal-to-Noise Ratio (SNR) in j is sufficiently high to correctly decode the signal. To achieve high network efficiency, parallel transmissions on more than one link must be considered by the scheduling scheme. However, parallel transmissions generate interference at receiving nodes that may affect the correct decoding. Therefore, some constraints on resource reuse must be considered to guarantee correct network operation.

A simple model that has been proposed for reuse constraints is based on a conflict graph [4]. However such model does not consider that the effect of several interfering transmissions is cumulative. A more accurate model takes into account the cumulative effect of interference evaluating the Signal-to-Interference and Noise Ratio (SINR) at receivers [4]. With this model a set of parallel transmissions on a set of links is admissible if the SINR at all receivers is above a given threshold [5].

Since SINR values depend on the power emitted by all transmitting nodes, power control mechanisms can improve resource reuse adjusting powers in order to set the SINR at receivers just above the threshold [6]. In this case a set of parallel transmissions is feasible if it is possible to find a power assignment that satisfies power limitations and provides the required SINR. Several scheduling approaches that jointly assign transmission time slots and the emitted power have been proposed considering different objectives and constraints [7], [8], [9].

A further improvement can be achieved considering also transmission rate control within a cross-layer approach. Modern adaptive modulation and coding schemes allow to adapt the transmission rate to the actual propagation and interference conditions. According to the SINR value, the best transmission rate that provides an error rate sufficiently low can be selected. Since in the considered scenario the SINR values are determined by the set of parallel transmissions and the transmission powers, it appears quite reasonable to consider transmission scheduling, power control and rate control at the same time [10], [11], [12], [13], [14].

Finally, some works deal with the routing and scheduling problem in wireless networks where nodes are equipped with directional antennas [15], [16], [17].

To the best of our knowledge, our paper is the first that tackles the joint routing and scheduling problem in WMNs where nodes are equipped with directional antennas, power control and rate adaptation are enabled, and a realistic interference model is adopted. This allows us to derive bounds to the performance achievable by such networks.

III. Modeling Assumptions

To specify the WMN scenario we are dealing with, the following definitions and assumptions are needed.

Scheduling approach

Transmissions between different node pairs can be scheduled in the same time slot only if signals at respective receivers can be correctly decoded. A signal is correctly decoded at a receiver j if the corresponding SINR_j is greater than a defined threshold γ. Actually, the following inequality must hold for every active pair of transmitter i and receiver j:

$$\text{SINR}_j = \frac{p_{ij} G_{ij}}{\eta + \sum_{l,m \neq i,j} p_{lm} G_{lj}} \geq \gamma$$

where η is the thermal noise, p_{ij} is node i's transmitted power to j and G_{ij} is the channel gain between nodes i and j. The summation expresses the interference generated by other nodes not involved in the $i \rightarrow j$ communication. When multiple transmission rates are considered, the SINR constraints are modeled requiring that SINR_j is higher than γ_w to allow a correct reception with modulation w.

Additional constraints preventing nodes to transmit and receive at the same time are added assuming that nodes are equipped with a single radio element and that a single channel is available.

Directional antenna and channel models

Directional antennas allow the concentration of the transmitted energy into a limited region and a higher reception gain from certain arriving directions. The interference reduction permits a higher channel reuse with respect to omnidirectional antennas, leading to better resource exploitation and potentially better performance.

We adopt in this paper the antenna model considered in [18], [19]: the main radiation lobe is an angular sector having width equal to α degrees, while the omnidirectional coverage around the station due to sidelobes is represented with a circle having lower radiation gain. If G_H is the radiation gain of the main lobe, the gain of the side lobes, G_L, can be assumed at least 10 dB lower [18], [19].

Since WMNs' topology is fixed we assume that each node knows its position, as well as the neighbors' one. In particular, each device is able to point its antenna's main lobe in the direction of the receiving neighbor. As a consequence, to compute the channel gain we can consider only the discrete set of possible antenna pointings among pairs of nodes.

Since the radiation patterns are non-uniform, it is necessary to distinguish between the three following cases, depending on the pointing of the two nodes' antennas: (a) the two nodes have the main lobe reciprocally pointed, (b) they both have the main lobe pointed away from the other node, (c) one node has the main lobe pointed towards the other one while the main lobe of the other node is pointed away from the first node. Let G_{ij} be the channel gain due to propagation attenuation between nodes i and j; the channel gain matrix is:

$$\tilde{G}_{ij}^{qr} = G_{ij} \cdot \begin{cases} G_H G_H & \text{if case (a) occurs} \\ G_L G_L & \text{if case (b) occurs} \\ G_H G_L & \text{if case (c) occurs,} \end{cases}$$

where q is the node pointed by the transmitting antenna at node i and r is the node pointed by the receiving antenna at j. Note that given the position of nodes i, q, j, r and the angular width of the main lobe sector, the actual values of $\tilde{G}_{ij}^{lk}$ can be pre-computed for all nodes.

IV. Mathematical Models

Given in input the network topology and the demand matrix between nodes' pairs, the goal of the models presented in the following is the optimization of demands' routing paths and the frame structure in order to minimize the frame length. We consider three transmission schemes: a fixed power (FP) scheme, a power control (PC) scheme and a power and rate control (PRC) scheme. In addition, for each scheme we consider WMN nodes equipped with either omni-directional or directional antennas.

Previous works [3] have shown that classical compact MILP (Mixed Integer Linear Programming) formulations require high computational effort: very small size instances can be already intractable with state-of-the-art solvers. Therefore, in this work we consider an alternative formulation having as decision variables the family of sets of compatible links that can be activated at the same time. Since these subsets of the whole link set are exponentially many with respect to the size of instances, we have exponentially many variables. This formulation is specially devised for a solution approach based on column generation.

The network is defined by the graph $GR = (\mathcal{N}, \mathcal{L})$, where the vertex set $\mathcal{N}$ is formed by wireless nodes and the arc set $\mathcal{L}$ is formed by wireless links, that is activable pairs of transmitter-receiver nodes. We define a *configuration* as the set of compatible active links that fulfill SINR and technology dependent constraints. Configurations define the link pattern that can be activated in every possible slot. Traffic demands are defined by triplets $d^k = (d_o^k, d_t^k, d_v^k)$ in the set $\mathcal{D}$, identifying respectively the source node, the destination node and the traffic amount, which represents the guaranteed bandwidth QoS parameter, and it is modeled as a given number of packets to be delivered within the frame time. The basic transmission rate consists in the transmission of one packet in each slot in which the transmitter is active, while with higher rates a different number of packets are transmitted according to the selected modulation.

Let us denote with $\mathcal{S}$ the set of all possible compatible configurations. Besides, $\mathcal{S}_{ij}$ denotes the set of configurations in which link (i, j) is active. To formulate the problem, an integer variable λ_s is defined for each configuration $s \in \mathcal{S}$, representing the number of slots to which s is assigned. As only one configuration can be assigned to one time slot, the number of configurations assigned to time slots is equal to the number of used slots. In addition, variables y_{ij}^k express the number of demand k packets routed on link (i, j). The problem of minimizing the number of needed slots can be therefore modeled as follows:

$$\min \quad \sum_{s \in \mathcal{S}} \lambda_s \tag{1}$$

$$\text{s.t.} \quad \sum_{(i,j) \in \mathcal{L}} y_{ij}^k - \sum_{(j,i) \in \mathcal{L}} y_{ji}^k = \begin{cases} d_v^k & i = d_o^k \\ -d_v^k & i = d_t^k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$
$$\forall i \in \mathcal{N}, d^k \in \mathcal{D}$$

$$\sum_{s \in \mathcal{S}_{ij}} W_{ij}^s \lambda_s \geq \sum_{d^k \in \mathcal{D}} y_{ij}^k \qquad \forall (i,j) \in \mathcal{L} \tag{3}$$

$$y_{ij}^k \in \mathbb{N}^+ \qquad \forall (i,j) \in \mathcal{L}, d^k \in \mathcal{D} \tag{4}$$

$$\lambda_s \in \mathbb{N}^+ \qquad \forall s \in \mathcal{S}, \tag{5}$$

where W_{ij}^s is the number of packets transmitted during a time slot at the rate at which the link (i, j) is activated in configuration s. The objective function (1) imposes the minimization of the total number of needed time slots, while constraints (2) are flow balance constraints for demand routing. Constraints (3) guarantee that each link is active in at least one slot for each packet to be sent.

To complete the mathematical description of the problem, we must provide equations describing compatible configurations. Since the omnidirectional antenna case can be easily derived from the directional antenna formulation, the following configurations take into account only directional antennas. We start by considering a feasible configuration s for the fixed power scheme. Let z_{ij} be a binary variable equal to one if link (i, j) is active in a given configuration and zero otherwise. A feasible configuration s must satisfy the following constraints:

$$\sum_{(i,j) \in \mathcal{L}} z_{ij} + \sum_{(j,i) \in \mathcal{L}} z_{ji} \leq 1 \qquad \forall i \in \mathcal{N} \tag{6}$$

$$\frac{P \tilde{G}_{ij}^{ji} z_{ij}}{\eta + \sum_{\substack{(l,m) \in \mathcal{L}: \\ l,m \neq i,j}} P \tilde{G}_{lj}^{mi} z_{lm}} \geq \gamma z_{ij} \qquad \forall (i,j) \in \mathcal{L} \tag{7}$$

$$z_{ij} \in \{0, 1\} \qquad \forall (i,j) \in \mathcal{L}, \tag{8}$$

Constraints (6) are half-duplex constraints, while (7) guarantee the required transmission quality. P is the fixed transmission power. Constraints (7) are clearly non-linear, and their linearization is discussed in the following.

Note that the column generation approach allows us to separate the routing and scheduling problem from the admissible configuration problem. The first problem, in fact, needs only to be fed with feasible configurations, that is, possible slots, disregarding the assumptions and procedures they are generated with. The admissible configuration problem includes all technological and practical aspects about the definition of sets of simultaneously active transmissions, and it is completely independent from the previous one.

Given this separation, the feasible configuration's formal definition can be easily extended to deal with the other considered transmission schemes. If power control is enabled, a non-negative variable p_{ij} is introduced for each link, and constraints (7) are changed as follows:

$$\frac{p_{ij}\tilde{G}_{ij}^{ji}z_{ij}}{\eta + \sum_{\substack{(l,m)\in\mathcal{L}:\\ l,m\neq i,j}} p_{ij}\tilde{G}_{lj}^{mi}z_{lm}} \geq \gamma z_{ij} \qquad \forall (i,j)\in\mathcal{L} \tag{9}$$

while the following constraints must be added:

$$p_{ij} \leq z_{ij}P_{MAX} \qquad \forall (i,j)\in\mathcal{L} \tag{10}$$

$$p_{ij} \in \mathbb{R}^{+} \qquad \forall (i,j)\in\mathcal{L} \tag{11}$$

where P_{MAX} represents the maximum allowed transmission power.

Finally, for the problem with power and rate control, each active pair of transmitter/receiver (i,j) can also select a transmission rate $w \in \mathcal{W}$. The admissible configuration definition must include the rate information, thus new binary variables z_{ij}^{w} are defined such that $z_{ij}^{w} = 1$ if link (i,j) is active with rate w in the considered configuration and zero otherwise. The formal configuration definition for the power and rate control problem becomes:

$$\sum_{w\in\mathcal{W}}\sum_{(i,j)\in\mathcal{L}} z_{ij}^{w} + \sum_{w\in\mathcal{W}}\sum_{(j,i)\in\mathcal{L}} z_{ji}^{w} \leq 1 \qquad \forall i\in\mathcal{N} \tag{12}$$

$$\frac{p_{ij}\tilde{G}_{ij}^{ji}z_{ij}^{w}}{\eta + \sum_{\substack{(l,m)\in\mathcal{L}:\\ l,m\neq i,j}} p_{ij}\tilde{G}_{lj}^{mi}z_{lm}^{w}} \geq \gamma_{w} z_{ij}^{w} \qquad \forall (i,j)\in\mathcal{L}, w\in\mathcal{W} \tag{13}$$

$$p_{ij} \leq P_{MAX}\sum_{w\in\mathcal{W}} z_{ij}^{w} \qquad \forall (i,j)\in\mathcal{L} \tag{14}$$

$$p_{ij} \in \mathbb{R}^{+} \qquad \forall (i,j)\in\mathcal{L} \tag{15}$$

$$z_{ij}^{w} \in \{0,1\} \qquad \forall (i,j)\in\mathcal{L}, w\in\mathcal{W}, \tag{16}$$

where γ_{w} is the required SINR to correctly decode transmissions at rate w.

Solution approach

We consider the continuous relaxation of the problem (1)-(5), *i.e.* the problem described by (1)-(3) in which λ_s and y_{ij}^{k} variables are continuous, and denote such problem as $\mathcal{P}$. The optimal solution of $\mathcal{P}$ provides a lower bound to the minimum number of needed slots. However, since the number of variables in $\mathcal{P}$ is huge, it is not possible to enumerate them all. Therefore, to solve $\mathcal{P}$ we apply a column generation approach. In the column generation only a subset of variables is considered. The problem $\mathcal{P}$ involving a subset of variables, called *master problem* $\mathcal{MP}$, is solved: the solution is optimal for the $\mathcal{MP}$ but it may not be optimal for the original problem $\mathcal{P}$ as only a subset of variables is considered. Then, we need a procedure, called *pricing*, to check if the solution found is optimal also for $\mathcal{P}$ or to find out the variables to be added to the master problem $\mathcal{MP}$ to improve the solution. The pricing procedure is based on the properties of the dual problem of $\mathcal{P}$.

We recall that, given a solution of the problem $\mathcal{MP}$, if the dual variables related to such solution are feasible for the dual problem, then the given solution is optimal for $\mathcal{P}$. Besides, each variable (constraint) of $\mathcal{MP}$ is associated to a constraint (variable) of the dual problem. Given a $\mathcal{MP}$'s variable, if the associated dual constraint is violated, this variable can produce an improvement in the $\mathcal{MP}$'s objective function if it is added to the set of the considered variables. Thus the aim of the pricing procedure is to verify whether the dual variables associated to the found $\mathcal{MP}$'s solution are feasible for the dual problem. To this end, we try to build a new feasible configuration such that the related dual constraint is violated. The variable related to such configuration, if the new configuration exists, must then be added to the considered set and the $\mathcal{MP}$ is to be solved again. The continuous relaxation optimum is reached when no configuration can be found such that the related dual constraint is violated.

We first consider the pricing problem with fixed power. Denoting with σ_{ij} the dual variables related to $\mathcal{MP}$'s constraints (3), the dual constraint associated to a given configuration s is

$$\sum_{\substack{(i,j):\\ s\in\mathcal{S}_{ij}}} \sigma_{ij} \leq 1. \tag{17}$$

To solve the pricing problem we compare two methods. In the first one, named OPT (Optimal solution), we look for a configuration s satisfying constraints (6), (7) and (8) such that the following expression is maximized:

$$\max \sum_{(i,j)\in\mathcal{L}} \sigma_{ij}z_{ij} \tag{18}$$

If the found maximum is greater than one, the variable related to such configuration must be added to the set, because it violates the dual constraint (17). In the second method, named FF (First Feasible), we maintain (18) as objective function and add the following constraint to the admissible configuration's constraint set:

$$\sum_{(i,j)\in\mathcal{L}} \sigma_{ij}z_{ij} \geq 1 \tag{19}$$

The new configuration to be inserted in the considered set is the first feasible solution of the so-obtained problem. If the problem is infeasible, no improving solution can be found, thus the pricing procedure stops.

Therefore, the pricing problem for the OPT method in the fixed power case is formulated as the problem of maximizing (18) subject to admissible configuration constraints (6), (7) and (8). In case of power control the pricing problem is the problem of maximizing (18) subject to admissible configuration constraints (6), (8), (9), (10), and (11); finally, if both power and rate control are enabled the pricing problem becomes:

$$\max \sum_{(i,j)\in\mathcal{L}}\sum_{w\in\mathcal{W}} \sigma_{ij}z_{ij}^{w} \tag{20}$$

subject to admissible configuration constraints (12), (13), (14), (15), and (16). Constraint (19), or its modification that includes z_{ij}^w variables as a replacement for z_{ij}, can be added independently to each pricing problem formulation in order to apply the FF pricing method.

The pricing problem as formulated above turns out to be non-linear for all the proposed problems. However, the formulation can be linearized and the problem can be solved to optimality with a commercial linear solver as CPLEX. The linear formulation of the SINR constraint is:

$$PG_{ij} + M(1 - z_{ij}) \geq \gamma \left(\eta + \sum_{\substack{(h,m)\in\mathcal{L},\\ i\neq h}} PG_{hj} z_{hm} \right) \quad \forall (i,j) \in \mathcal{L}, \qquad (21)$$

where M is a constant such that:

$$M \geq \gamma \left(\eta + \sum_{\substack{(h,m)\in\mathcal{L}:\\ i\neq h}} PG_{hj} z_{hm} \right) \qquad \forall (i,j) \in \mathcal{L}. \qquad (22)$$

In our paper M's value has been set as follows:

$$M = \gamma \left(\eta + (G_H)^2 G_{max} P_{MAX} \frac{|\mathcal{N}|}{2} \right),$$

where $G_{max} = \max_{(i,j)\in\mathcal{L}} G_{ij}$.

V. Numerical Results

In this section we present, first, the computational results obtained in a grid network to give some insights of our solution approach; then we show results averaged over random generated instance sets.

We consider a square area of $700m$; the maximum transmission power is set to guarantee the necessary SINR for transmissions between nodes at a given distance D, in absence of interference. When multiple rates are available, the lowest SINR threshold, corresponding to the lowest rate, is considered for D. The gain G_{ij} between nodes i and j is computed as d_{ij}^{-3}, where d_{ij} is the distance between the nodes, and the noise factor η is set to 10^{-11}. Available rates are 1, 2, 4, 8 packets per slot and the corresponding thresholds (γ_w) are, respectively, 2, 4, 8, 16. Directional antennas have a main lobe angular width, α, equal to 45 degrees, G_H is set to 10 and G_L is set to 0.8. Omnidirectional antenna results are obtained setting $G_H = G_L = 1$.

The models have been implemented in AMPL and solved using CPLEX 10.0 on a Linux powered machine at 1GHz with 2GB of RAM. The initial set of admissible configurations is heuristically generated in C, trying to add links to feasible configurations.

A. Grid Network

In this scenario, 9 nodes are deployed as shown in Fig. 1, where arrows represent demands between nodes; the number of packets to be routed (d_v^k) is indicated besides each arrow. The maximum communication distance D is set equal to the square diagonal.

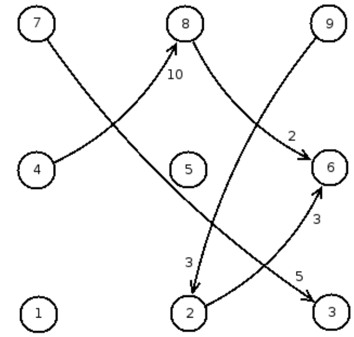

Fig. 1. Grid Network scenario.

TABLE I
SELECTED CONFIGURATIONS FOR THE GRID NETWORK OF FIG. 1.

Omnidirectional antennas			Directional antennas		
FP			**FP**		
cfs	*active links*	*slots*	*cfs*	*active links*	*slots*
1	(7,3)	4	1	(1,3) (4,2) (5,6) (8,7)	2
2	(9,2)	3	2	(1,3) (4,2) (5,8) (7,6)	1
3	(1,2) (4,8)	6	3	(1,5) (2,3) (4,8) (7,6)	1
4	(2,3) (4,8)	1	4	(1,6) (3,2) (4,8) (5,712)	1
5	(2,6) (4,8)	3	5	(2,8) (3,5) (4,1) (9,6)	1
6	(4,2) (8,6)	1	6	(4,8) (5,1) (6,2) (7,9)	2
7	(7,4) (8,6)	1	7	(4,8) (5,1) (9,6)	1
			8	(2,6) (4,8) (7,5) (9,3)	3
tot.		**19**	**tot.**		**12**
PC			**PC**		
cfs	*active links*	*slots*	*cfs*	*active links*	*slots*
1	(7,3)	2	1	(1,3) (2,5) (4,8)	2
2	(2,3) (4,8)	3	2	(1,4) (2,3) (5,7) (8,6)	1
3	(2,6) (4,8)	3	3	(1,5) (2,3) (4,7) (8,6)	1
4	(3,2) (4,8)	2	4	(1,6) (2,3) (4,8) (7,5)	2
5	(4,2) (8,6)	2	5	(3,6) (4,1) (5,8) (7,2)	1
6	(4,8) (6,3)	2	6	(3,8) (4,1) (5,6) (9,2)	1
7	(7,2) (9,6)	2	7	(4,6) (5,1) (7,8) (9,2)	1
8	(7,4) (9,2)	1	8	(4,8) (5,1) (6,3) (9,2)	1
			9	(4,8) (5,2) (7,1)	2
tot.		**17**	**tot.**		**12**
PRC			**PRC**		
cfs	*active links, rate*	*slots*	*cfs*	*active links,rate*	*slots*
1	(2,6),4	1	1	(2,3),4 (4,7),4 (5,8),4 (9,6),4	1
2	(4,8),4	2	2	(5,3),3 (6,2),3 (7,4),4 (8,9),4	1
3	(7,3),4	1	3	(3,6),4 (4,5),4 (7,8),4 (9,2),2	1
4	(8,6),4	1			
5	(9,2),4	1			
tot.		**6**	**tot.**		**3**

Table I shows the generated configurations as well as the slots in which they are used to build up the entire frame for the Fixed Power (FP), the Power Control (PC) and the Power and Rate Control (PRC) schemes, for both omnidirectional and directional antennas. Evidently, the use of directional antennas allows to generate admissible configurations with many more active links than in the omnidirectional antennas case. This leads to shorter frames, thus a better performance.

While with omnidirectional antennas the performance improves using a power control scheme instead of a fixed power scheme, with directional antennas this behavior is less evident. This is due to the fact that the interference reduction of directional antennas is sufficient to allow the activation of almost all the possible nodes, either as transmitters or as receivers. With the PRC scheme, higher rates allow to transmit multiple packets per slot, resulting in a great slots saving.

Results about routing paths, not shown for the sake of brevity, highlight two different routing strategies. With omnidirectional antennas, the paths tend to include only the direct link between the source and destination nodes, while with directional antennas they are longer, in terms of hops,

TABLE II
PARAMETER ψ VALUES FOR DIFFERENT INSTANCE SIZES.

Nodes	FP		PC		PRC	
	OMN	DIR	OMN	DIR	OMN	DIR
5	1.13	0.86	1.02	0.83	0.20	0.19
10	0.96	0.55	0.80	0.55	0.18	0.16
15	0.83	0.44	0.69	0.42	0.20	0.12

and generally packets are routed over multiple paths. The reason is again the interference reduction which permits the simultaneous activation of more links.

B. Random instances

We then considered network scenarios which are generated positioning uniformly at random 5, 10 and 15 nodes in the square area. The number of traffic demands is uniformly distributed in the 5 to 15 range. For each demand, the source and destination nodes are randomly chosen among the deployed nodes, and the number of packets to be sent is uniformly chosen between 1 and 15. Each result is averaged on 20 random instances. D is set to half the square diagonal.

To evaluate the advantage that derives from using more advanced transmission techniques and directional antennas, we define the parameter ψ as the ratio between the number of slots in a frame and the total number of packets to be routed in the network. Roughly speaking, ψ represents the average number of slots needed to transmit a packet. Table II shows the numerical results obtained for all the combinations of number of nodes, transmission techniques and antenna types. With directional antennas ψ decreases more than 45% with respect to omnidirectional antennas, thanks to the interference reduction effect. The improvement is less significant when power control is available, because such technique can reduce interference even with omnidirectional antennas. In addition, note that ψ decreases when the instance size increases, especially with directional antennas: the algorithm can choose many more alternative paths exploring a larger solution space.

Tables III, IV and V show the computational results with the considered transmission schemes for both omnidirectional and directional antennas. All tables are divided into three parts. The first one, named *Initial Solution*, is related to the solution of the joint scheduling and routing problem over the initial set of heuristically generated configurations. The average and maximum optimality percentage gap and computational time are reported. The optimality gap is computed with respect to the optimal relaxed solution given by the last $\mathcal{MP}$ iteration of the column generation procedure. The same performance figures are reported in the third part, named *Final Solution*, where the initial set of columns has been extended with the generated columns. The solving times reported in the Table are obtained solving the joint scheduling and routing problem with the columns generated with the FF method, which proved the best as discussed in the following. In the second part, *Column Generation*, we report results for the column generation. The first column of this part shows the $\mathcal{MP}$'s solution percentage improvement between the first and the last column generation iteration. Subsequent columns show the number of generated columns (column generation iterations) and the total generation time (average and maximum) for both OPT and FF methods. The generation time is almost completely due to the pricing problem, since the $\mathcal{MP}$'s solving time is negligible compared to that of the pricing problem. The three parts are repeated for both omnidirectional and directional antennas. We set time limits to have viable solutions in reasonable time: Initial Solution and Final Solution stop in 4 hours, while the whole process of column generation has been limited to 8 hours.

With the fixed power and power control schemes (Tables III and IV) the Final Solution has always a good quality, since the maximum error is about 7%. Most of the times our algorithm gives the optimal solution when compared with the continuous relaxation obtained by the column generation. Note that for small instances, with omnidirectional antennas, the Initial Solution is already an optimal solution. With larger instances and directional antennas, the Initial Solution is worse and the optimality gap is filled by the column generation procedure, which can improve the Initial Solution more than 45%. When the instance size increases, it is more and more difficult to find optimal admissible configurations heuristically, because the solution space increases exponentially; in this case the use of dual variables in the column generation procedure can greatly help to lead the generation towards configurations that improve the solution quality. The hardest of the considered problems is the Power and Rate Control scheme (see Table V) where both optimality gap and computational time increase. However, it can be observed that the frames length obtained with such transmission scheme is much shorter than that of the frames obtained with fixed power or power control only, so a difference of even a single slot with respect to the continuous relaxation solution has a great impact on the percentage gap.

Analyzing the column generation procedure with the OPT and FF methods, we can see how the FF method is less computational intensive than the OPT method, although the FF method generates more columns, that is, it executes more iterations. The reason is that the pricing problem involved in the configuration generation is very hard, and finding an admissible solution is much easier than finding and certifying the optimum. In addition, the Duality Theory guarantees that the Final Solution's optimality gap is not influenced by the column generation method.

Finally, comparing the computational results between scenarios with omnidirectional antennas and directional antennas, we can observe that the column generation takes more time with directional antennas than with omnidirectional ones. Directional antenna scenarios have more degrees of freedom than those with omnidirectional antennas, whereas there are many more admissible configurations. The column generation must look for all the configurations that can potentially improve the objective function value and this is a hard-working task with directional antennas.

VI. CONCLUSION

In this paper we addressed the joint routing and scheduling problem in Wireless Mesh Networks where nodes are equipped with directional antennas.

TABLE III
RESULTS FOR THE FP SCHEME: PERCENTAGE GAPS, SOLVING TIMES, PERCENTAGE IMPROVEMENT AND NUMBER OF ITERATIONS.

	Nodes	INITIAL SOLUTION				COLUMN GENERATION							FINAL SOLUTION			
		Gap		Time [s]		impr	OPT			FF			Gap		Time [s]	
		ave.	max	ave.	max	ave.	iters	T[s] ave.	T[s] max	iters	T[s] ave.	T[s] max	ave.	max	ave.	max
O	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
M	10	4.4	16.7	0.1	0.1	4.4	3.6	0.2	1.2	5.3	0.2	1.1	0.0	0.0	0.0	0.1
N	15	9.7	20.0	0.2	0.5	9.7	41.3	107.2	242.7	65.3	22.9	49.2	0.0	0.0	0.2	0.9
D	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I	10	11.7	35.5	0.1	0.4	11.7	5.7	0.4	1.6	8.1	0.3	1.3	0.0	0.0	0.3	0.2
R	15	17.3	40.0	1.7	7.8	17.0	25.9	174.0	2214.9	94.6	47.5	524.8	1.7	7.1	5.2	32.5

TABLE IV
RESULTS FOR THE PC SCHEME: PERCENTAGE GAPS, SOLVING TIMES, PERCENTAGE IMPROVEMENT AND NUMBER OF ITERATIONS.

	Nodes	INITIAL SOLUTION				COLUMN GENERATION							FINAL SOLUTION			
		Gap		Time [s]		impr	OPT			FF			Gap		Time [s]	
		ave.	max	ave.	max	ave.	iters	T[s] ave.	T[s] max	iters	T[s] ave.	T[s] max	ave.	max	ave.	max
O	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
M	10	6.5	25.0	0.1	0.1	6.5	7.7	1.6	7.2	10.9	0.3	1.2	0.0	0.0	0.0	0.1
N	15	13.9	23.5	0.5	1.8	13.9	85.9	884.9	4317.3	133.8	22.9	90.9	0.2	2.6	0.4	0.9
D	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
I	10	8.7	37.9	0.2	0.5	8.7	3.0	0.3	1.7	8.8	0.2	1.1	0.0	0.0	0.3	4.0
R	15	12.7	36.0	2.6	16.9	12.7	14.7	792.0	11829.5	50.3	87.0	1297.5	0.3	4.6	21.0	127.2

TABLE V
RESULTS FOR THE PRC SCHEME: PERCENTAGE GAPS, SOLVING TIMES, PERCENTAGE IMPROVEMENT AND NUMBER OF ITERATIONS.

	Nodes	INITIAL SOLUTION				COLUMN GENERATION							FINAL SOLUTION			
		Gap		Time [s]		impr	OPT			FF			Gap		Time [s]	
		ave.	max	ave.	max	ave.	iters	T[s] ave.	T[s] max	iters	T[s] ave.	T[s] max	ave.	max	ave.	max
O	5	23.0	100.0	0.2	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	23.0	100.0	0.1	0.7
M	10	39.9	75.0	1584.6	12310.4	1.6	4.9	4.2	25.1	8.4	0.5	1.5	38.2	75.0	1765.2	14400
N	15	44.3	66.7	6896.0	14400	7.2	54.3	6063.8	28800.0	86.7	34.6	118.7	36.8	55.6	7327.9	14400
D	5	16.3	50.0	0.3	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	16.3	50.0	0.1	0.3
I	10	41.3	100.0	482.4	2288.1	14.3	43.2	47.6	144.9	65.9	6.2	15.7	21.0	50.0	998.4	9271.0
R	15	79.7	150.0	4172.0	14400	46.3	182.1	16285.1	28800.0	419.7	1919.3	7185.3	36.7	100.0	8281.3	14400

Our proposed models are based on Mixed Integer Linear Programming formulations, and they are solved using a column generation approach. We modeled accurately the behavior of WMNs, considering the cumulative effect of interference at the receivers, power control and rate adaptation.

We solved our models in realistic network scenarios, evaluating the impact of all the technologies considered in the paper (i.e. the antenna type, directional or omnidirectional, power control, rate adaptation) on the network performance.

The numerical results we obtained show that directional antennas can greatly enhance the performance of WMNs, thus enabling high-throughput services, like, for example, video streaming broadcast.

As future research issues, we plan to consider WMNs with multi-radio, multi-channel nodes, and multicast services, that can further increase the network performance. In addition, we are currently developing more efficient heuristics that could help to solve rapidly large-size problem instances, thus allowing to track traffic variations and support network reconfiguration.

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