Asymptotic Analysis of Schoute's Estimate for Dynamic Frame Aloha

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***Abstract*—This paper represents a further attempt to provide definite results about the asymptotic performance of protocols of the dynamic frame Aloha (DFA) family for radio frequency identification (RFID) systems. Here we deal with a simple and popular backlog estimate known as Schoute's estimate, apt to the DFA version that do not make use of the Frame Restart capability. This estimate performs very well in multiple access systems, but presents some efficiency impairment in RFID. Recent studies have shown that, with a perfect backlog estimate, the asymptotic efficiency of DFA, with or without Frame Restart, equals e^{-1}. Here we prove that the asymptotic efficiency of Schoute's backlog estimate is 0.311 for any finite initial frame length. The analysis shows that the impairment is due to the slow convergence of the estimate to the true value, and opens the path to future work.**

***Index Terms*—RFID, Frame Aloha, Tag Identification, Tag Estimate, Collision Resolution, Anti-collision.**

I. Introduction

The reference architecture of Radio Frequency Identification (RFID) systems includes a reader which interrogates a set of tags in order to identify each of them [1]. Upon being interrogated, concurrent tags responses may collide and a collision resolution protocol (CRP) is needed to arbitrate collisions. Several CRPs have been proposed in the literature also leading to standard solutions [2]; among the different CRPs, the sub-family of Dynamic Frame Aloha (DFA) protocols has been largely discussed in the literature.

DFA originates in 1983 as a multiple access protocol proposed for satellite communications in a short paper by Schoute [3]. In brief, DFA operates as follows: an initial number N of users, also called *tags*, reply to a reader interrogation on a slotted time axis where slots are grouped into frames; a tag is allowed to transmit only one packet per frame in a randomly chosen slot. In the first frame all tags transmit, but only a part of them avoid collisions with other transmissions and get through. The remaining number of tags n, often referred to as the *backlog*, re-transmit in the following frames until all of them succeed. Outcomes of slots, *i.e.*, successfully used, not used, or collided, are continuously observed to derive an estimate of the backlog, $\hat{n}$, which is used to set the length r of the next frame till all tags have been identified. The problem arises to get at each frame a suitable estimate $\hat{n}$, and to determine the most favorable frame length r. The RFID standard also introduces an additional capability, called *Frame Restart* (FR), that allows to restart a new frame at any slot even though the present frame is not finished.

Schoute's estimate is very simple and is provided by $\hat{n} = \text{round}(2.39c)$, where c represents the number of collided slots in the observed frame, and $\text{round}(x)$ is the closest integer to x. Subsequently, the new frame length r is set equal to the estimate $\hat{n}$. Unfortunately, Schoute's procedure, which requires the observation of the entire frame, can not be fitted to DFA-FR, although adaptations have been recently proposed [4], [5]. So new procedures have been proposed, such as the simple, though not well understood yet, Q-algorithm [2], [6].

Although DFA has been largely discussed in the literature, a complete comprehension of the mechanism, as achieved for the alternative protocol family known as Tree Algorithms [7]–[11], is still lacking. Referring to RFID, the analysis of DFA and DFA-FR have mostly concerned estimates, in many cases variations of Schoute's, and problems such as the inefficiency arising from the mismatch between N and the initial frame length r_0 (see for example [12]–[18]). With both DFA versions, however, none appears to address the problem of an infinite range for N, nor to determine the asymptotic efficiency of their estimate. This has practical impact since in many RFID applications we expect a very large N.

In the last years, we have been active in trying to fill the knowledge gap cited above, analyzing the performance of some estimates and procedures proposed in the literature, especially referring to large N. In [19], we have analyzed many proposals both for DFA and DFA-FR assuming an initial population size which is Poisson distributed, as it happens in multiple access systems or when a large N is subdivided in small groups. In [20] we have shown that, using DFA, a mechanism exists that provides efficiency 0.469, very close to the best ever attained, 0.487, reached with a sophisticated algorithm [8], [9] of the Tree Protocols family. In [21] we have proved that, when N is known, the optimal frame setting is $r = n$, an the asymptotic efficiency is e^{-1}. In [5] we have shown that the FR procedure does not improve the DFA asymptotic efficiency, and have proposed a mechanism to reach it. Up to now no asymptotic analysis has appeared for a plain DFA practical estimate.

In continuing the work exposed above, in this paper we

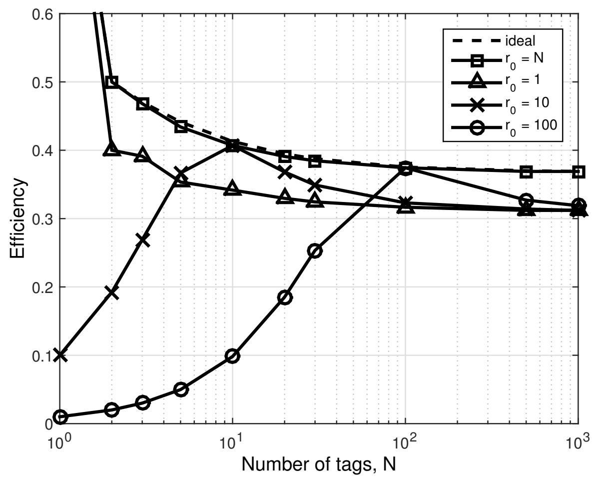

Figure 1. Efficiency versus the initial number of tags N of Schoute's DFA mechanism for different values of the initial frame length r_0.

analyze the asymptotic efficiency of the first and simpler of the plain DFA estimates, namely the Schoute's method. We prove that its asymptotic efficiency is 0.311, quite below the theoretical value e^{-1}, when the initial frame length is any finite value. On the other side, we prove that this simple mechanism reaches e^{-1} when starting with $r_0 = N$. The analysis is able to enlighten the reason of Schoute's estimate misbehavior and opens the path to future work.

The paper is organized as follows. In Section II we show some preliminary results about Schoute's protocol. In Section III we produce the asymptotic analysis and in Section IV draw the conclusions and the lines of future works.

II. Preliminaries

Since the throughput of a frame is maximized when $n = r$, Schoute's proposal is based on the idea that the mechanism should maintain the traffic, *i.e.*, the average number of tags per slot, equal to one. Therefore, it assumes that the number of tags transmitting in a slot can be approximated by a Poisson variate of average 1. Hence, the average number of terminals in a collided slot is

$$H = (1 - e^{-1})/(1 - 2e^{-1}) \approx 2.39,$$

and the estimate is $\hat{n} = \mathrm{round}(Hc)$.

In Fig. 1 the efficiency for different values of N with initial frame length $r_0 = N$, $r_0 = 1$, $r_0 = 10$ and $r_0 = 100$ is shown. Values up to $N = 30$ have been evaluated using the formula in [3], whereas values for $N = 500$ and $N = 1000$ have been obtained by simulating the algorithm. To allow comparisons we have also reported the performance with a perfect estimate $\hat{n} = n$ for each frame (dashed line), that represents a benchmark for all estimation mechanisms. We have also reported the case where only the estimate of the first frame is perfect, *i.e.*, when the first frame length is set to N. The comparison of the latter cases shows that Schoute's mechanism is able to track the backlog quite well if compared to the perfect estimate case, asymptotically approaching the best possible efficiency e^{-1}. In all the other cases, the Schoute's estimate suffers the mismatch between N and the initial frame length r_0, and the efficiency degrades monotonically when N increases beyond r_0, indicating the existence of a possible asymptote well below e^{-1}.

III. Asymptotic Analysis

The protocol analysis is subdivided into steps. In the remainder of the paper lowercase letters represent random variables, whereas calligraphic upper cases represent averages.

Step 1. Here we derive recursive formulas for the backlog. We initially assume that the $i-$th frame size r_i, and the backlog n_i, are so large that the number of transmissions in a slot can be approximated by a Poisson variate with average n_i/r_i. This allows to evaluate the probability of an empty, successful, and collided slot respectively as

$$p_e = e^{-n_i/r_i}; \quad p_s = \frac{n_i}{r_i} e^{-n_i/r_i}; \quad p_c = 1 - p_e - p_s.$$

We note that relations above also hold when starting with small r, because in this case, being $N - i$ always very large, every slot is collided with probability one. In Appendix A we show that, in the conditions assumed, the ratio $k_i = n_i/r_i$ can be safely replaced by the ratio of the respective averages $\mathcal{K}_i = \mathcal{N}_i/\mathcal{R}_i$, which is the traffic per slot. With this substitution the probabilities above are denoted by $\mathcal{P}_e, \mathcal{P}_s, \mathcal{P}_c$. This means that the average number of collisions and the average backlog size can be expressed as

$$\mathcal{C}_i = \mathcal{R}_i \mathcal{P}_c, \qquad \mathcal{N}_{i+1} = \mathcal{N}_i (1 - \mathcal{P}_s). \tag{1}$$

The frame length evolves with law $r_{i+1} = \mathrm{round}(Hc_i)$, so that

$$\mathcal{R}_{i+1} = \mathsf{E}\left\{\mathrm{round}(Hc_i)\right\}, \tag{2}$$

where $\mathsf{E}\{\cdot\}$ is the expectation operator. Equations (1) and (2) form a recursion that provides sequences $\{\mathcal{R}_i\}$ and $\{\mathcal{N}_i\}$ that determine the efficiency. Unfortunately, the rounding operation in (2) makes their analysis practically unfeasible.

Step 2. When c_i is large, by exploiting the limit $\lim_{x \to \infty} \mathrm{round}(x)/x = 1$, we can approximate the rounding operation $\mathrm{round}(Hc_i)$ in (2) with Hc_i, obtaining

$$\mathcal{R}_{i+1} = \mathsf{E}\left\{\mathrm{round}(Hc_i)\right\} \approx H\mathsf{E}\left\{c_i\right\} = H\mathcal{C}_i \triangleq R_{i+1}. \tag{3}$$

If we use (3) together with (1) we get the recursions, written with capital letters, that do not take into account the rounding operation:

$$R_{i+1} = HR_i \left(1 - K_i e^{-K_i} - e^{-K_i}\right), \tag{4}$$

$$N_{i+1} = N_i \left(1 - e^{-K_i}\right), \tag{5}$$

$$K_{i+1} = K_i \, \frac{1}{H} \frac{1 - e^{-K_i}}{1 - K_i e^{-K_i} - e^{-K_i}}. \tag{6}$$

Recursions (4)-(5) correspond to the actual sequences $\{\mathcal{R}_i\}$, $\{\mathcal{N}_i\}$, and $\{\mathcal{K}_i\}$, respectively. In Step 5 we show that replacing $(\{\mathcal{R}_i\}, \{\mathcal{N}_i\}, \{\mathcal{K}_i\})$ with $(\{R_i\}, \{N_i\}, \{K_i\})$ has no effect on the evaluation of the asymptotic performance. In Step 6 we

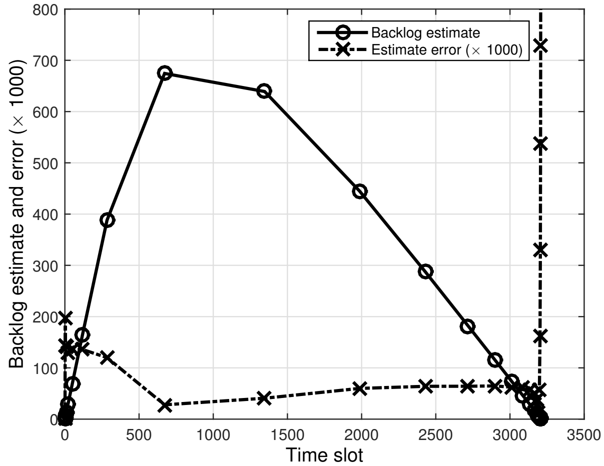

Figure 2. Average Schoute's backlog estimate $\hat{\mathcal{N}}_i$ at the end of the frames versus time slot ($N = 1000$, $r_0 = 1$). The dash-dotted line represents the relative error $\times 10^3$ with respect to the actual values N_i.

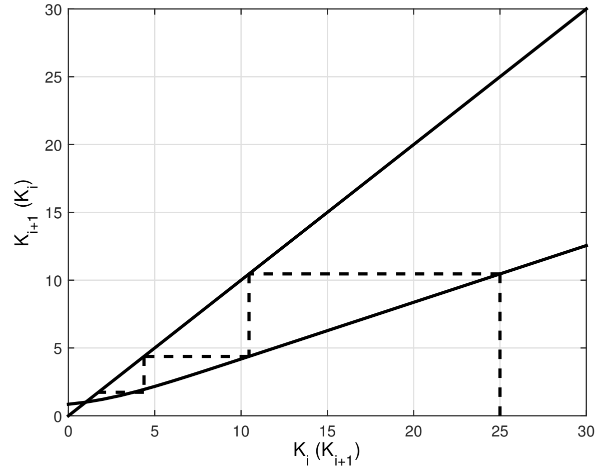

Figure 3. Representation of the trajectory of the sequence $\{K_i\}$. Solid lines: $K_{i+1} = K_i$ and Eq. (6).

show that this holds even for finite values of the initial frame size r_0. In practice, we find that sequence $\{R_i\}$ approximates fairly well sequence $\{\mathcal{R}_i\}$, even for moderate values of N, and this allows recurrence (6) to be used to evaluate the performance.

As an example, Fig. 2 shows sequence $\{\mathcal{N}_i\}$ derived by averaging 10^4 simulation samples in the case $N = 10^3$ and $r_0 = 1$. We can clearly see a first phase where the estimate increases in order to converge to the true value $N = 10^3$; actually the estimate reaches a maximum value that is lower than the the true value because in the meantime some packets have been correctly transmitted. In the second phase, optimal conditions are met, collisions are solved and the backlog decreases steadily to reach zero at about the 25-th iteration. We prove in the next step that in the descending phase the rate of descent is e^{-1}, showing that Schoute's algorithm is capable to correctly track the backlog and to solve contentions in the most efficient way. Figure 2 also shows the relative error sequence $\{(\mathcal{N}_i - N_i)/\mathcal{N}_i\}$ multiplied by 10^3 (dash-dotted line). The error is always very small except at the end of the process, where $\mathcal{N}_i$ becomes small and ignoring the rounding effect is no longer appropriate. However, this error has no effect on the efficiency since it occurs for a small period of time, negligible when compared to the entire collision resolution length.

Step 3. The evolution of the entire process is represented by recurrence (6) that depicts the evolution of the average traffic K_i. This is represented by the dashed trajectory in Fig. 3. This Figure also shows that the evolution of the process is asymptotically stable since recurrence (6) leads to the fixed point in $K_i = 1$. This point is also a point of optimality because in here we attain the optimal condition $r_i = n_i$ that provides maximum throughput.

When the starting point in (6) is $K_0 = 1$, the CRP proceeds with a correct backlog estimate, yielding $K_i = 1$ for all subsequent i, and

$$R_{i+1} = (1 - e^{-1})R_i, \qquad i \geq 0. \tag{7}$$

The solution of recurrence (7) is

$$R_i = (1 - e^{-1})^i N, \qquad i \geq 0,$$

which shows that at each round the backlog reduces by the fraction e^{-1}. The total number of slot in this resolution phase is

$$L(N) = \sum_{i=0}^{\infty} R_i = Ne,$$

yielding an asymptotic throughput $N/L(N) = e^{-1}$.

When $K_0 = N/r_0 > 1$, the length of the entire procedure can be evaluated as

$$L(K_0) = \sum_{i=0}^{\infty} R_i = r_0 \sum_{i=0}^{\infty} a_i,$$

where $r_0 = R_0$. The sequence $\{a_i = R_i/r_0\}$ just depends on K_0, whichever r_0 is, as it appears from (4). Therefore, the efficiency only depends on $K_0 = N/r_0$ and is evaluated as

$$\frac{N}{L} = \frac{K_0}{\sum_{i=0}^{\infty} a_i}.$$

Step 4. Here we show that for large values of the initial traffic K_0 the dependence of the efficiency on K_0 is negligible.

Since the protocol always starts with a finite r_0, large N means large K_0, so we attain practically the same efficiency whichever the initial frame length r_0 is.

As an example, in Fig. 4 we have reported the efficiency $N/L(N)$, evaluated through (4) and (6), for different values of traffic K_0. Starting from $K_0 = 1$, the optimal case, not reported in the Figure, the efficiency at first decreases as K_0 increases until about $K_0 = 500$ where it begins to oscillate without reaching an asymptote, around a mean value of 0.31125, with a period that increases geometrically with H.

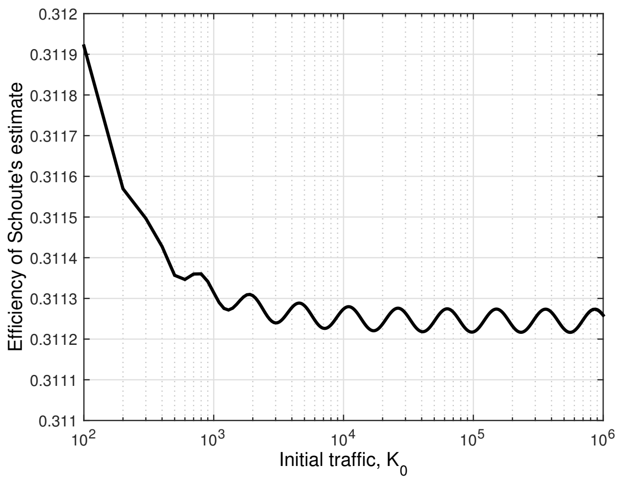

Figure 4. Efficiency of Schoute's backlog estimate versus initial traffic K_0.

To analyze the asymptotic behavior, during the solving process we consider three phases. The first phase, the *approaching phase*, starts at frame 0 with infinite traffic and ends at frame u, where u is chosen in such a way that the traffic K_u is finite and practically no successes occur up to frame u; as an example, we may arbitrarily assume u such as $K_u \geq 10$. Although in this way K_u and u appear arbitrarily defined, we show below that this has no effect on the evaluation of the efficiency, as, in fact, the initial traffic K_0 has no effect. The assumed definition for u assures that $u \to \infty$ as $N \to \infty$ and $R_u = N/K_u$.

The second phase, the *convergence phase*, starts at frame $u+1$ and ends at frame $u+v$ such that $K_{u+v} \approx 1$. At this point the third phase, the *tracking phase*, begins where tags are solved with efficiency e^{-1}. Denoting by L', L'', and L''' the lengths of the three phases, respectively, the efficiency is evaluated as

$$\frac{N}{L(N)} = \frac{N}{L' + L'' + L'''}.$$

With high values of $K_0 = N/r_0$, in the first phase the frame length increases deterministically with law $R_i = r_0 H^i$, for $i \geq 0$. The average number of slots up to frame u where the first phase ends is

$$L' = \sum_{i=0}^{u} R_i = r_0 \frac{H^{u+1} - 1}{H - 1} \approx \frac{H}{H-1} R_u.$$

Replacing $R_u = N/K_u$, the average length of the first phase becomes

$$L' = \frac{H}{H-1} \frac{N}{K_u} = N A(K_u),$$

where $A(K_u)$ is the proportionality constant, which expressly shows the dependence on K_u.

The second phase starts at frame $u+1$, when K_u is such that the collision probability is practically one, and ends at frame $u+v$ when $K_{u+v} \approx 1$. Equation (4) can be used to evaluate the length of phase two by the following sum over a finite number of terms:

$$L'' = \sum_{j=1}^{v} R_{u+j} = R_u \sum_{j=1}^{v} \alpha_j = N B(K_u),$$

where terms α_j are all finite and, again, where $B(K_u)$ is the proportionality constant expressing the explicit dependence on K_u. The average backlog size at the end of the second phase can be evaluated by (5) as

$$N'' = N_{u+v} = N_u \prod_{j=1}^{v} \left(1 - e^{-K_{u+j}}\right) = NC,$$

where we have exploited the fact that $N_u = N$. The coefficient C does not depend on K_u, since in frame $u+1$ we still observe all collisions ($e^{-K_{u+1}} \approx 0$).

The third phase presents efficiency e^{-1} and its average length is

$$L''' = N'' e = NCe.$$

The efficiency with very large N is then

$$\frac{N}{L(N)} = \frac{N}{L' + L'' + N''e} = \frac{1}{A + B + Ce}. \tag{8}$$

We note that (8) does not depend on the choice of v, once the condition $K_{u+v} \approx 1$ is assured. If we replace v by $v+1$, coefficient A is not affected, and also term $B + Ce$ is not affected. In fact, B is augmented by the term R_{u+v+1} which, by (4) with $K_{u+v+1} \approx 1$, is equal to

$$R_{u+v+1} = N_{u+v}(1 - e^{-1}). \tag{9}$$

On the other side, term Ce is diminished by

$$(N_{u+v} - N_{u+v+1})e = N_{u+v}(1 - e^{-1}),$$

that is equal to term (9). Nevertheless, efficiency (8) does depend on the choice of K_u, through coefficients A and B. However, if we replace K_u, chosen as suggested above, with $K_u \cdot H$, efficiency (8) does not change because this only implies the shifting of term R_u from term A to term B. Therefore, the efficiency is periodic in a logarithmic scale and all the asymptotic amplitudes of the oscillations in Fig. 4 can be obtained by replacing K_u with any value K' in the range (K_u, HK_u).

Table I shows the efficiency attained by (8) for different values of K_u chosen in the range $(20, 20H)$. As we can see, the values fit very well to those shown in Fig. 4. From what has been exposed above, we can conclude that the efficiency of Schoute's algorithm can be mathematically expressed as

$$\frac{N}{L(N)} = 0.311245 + \xi(\ln N) + \omega(N), \tag{10}$$

where $\xi(\ln N)$ is a periodic function of $\ln N$, such that $|\xi(\ln N)| < 0.0001$, and $\lim_{N\to\infty} \omega(N) = 0$. For all practical purposes, the asymptotic efficiency can be assumed equal to 0.311.

It is very interesting to note that expression (10) very closely resembles similar ones related to Tree Algorithms [10], which

Table I

ANALYTICAL VALUES OF THE ASYMPTOTIC EFFICIENCY OF DFA WITH SCHOUTE'S ESTIMATE FOR DIFFERENT VALUES OF THE PARAMETER K_u.

K_u	20	25	30	35	40	45	47.8
$N/L(N)$	0.31125	0.31127	0.31125	0.31122	0.31122	0.31123	0.31125

appears somehow originated by the geometric subdivision of the traffic operated by the protocol.

Step 5. Now we show that replacing L', L'', and L''', in the limit $r_0 \to \infty$, with $\mathcal{L}', \mathcal{L}''$, and $\mathcal{L}'''$, in which the rounding operation is taken into account, does not change the results provided that the initial frame length is still r_0. In Appendix B we show that

$$\lim_{r_0\to\infty} \frac{\mathcal{L}'(N)}{L'(N)} = \frac{\sum_{i=0}^{\infty} \mathcal{R}_i}{\sum_{i=0}^{\infty} R_i} = 1.$$

We also have

$$\lim_{r_0\to\infty} \mathcal{L}''(N)/L''(N) = 1,$$

because the second phase is composed of a finite number v of frames, each of them so large that the rounding effect is negligible. What shown also implies that at the end of the second phase we have $\lim_{r_0\to\infty} \mathcal{N}_i/N_i = 1$, and, therefore, since those tags are solved with efficiency e^{-1}, also for the length of the third phase we have

$$\lim_{r_0\to\infty} \mathcal{L}'''(N)/L'''(N) = 1.$$

Step 6. If r_0 is small and (3) can not be assumed, the first phase is split into two sub-phases in which the second sub-phase starts at frame-index x such that, from this frame onward, the rounding operation in (2) can be disregarded. Index x is finite and the length of the first sub-phase does not depend on N, whereas the length of the second sub-phase and of the other phases is proportional to N. Therefore, as $N \to \infty$, the length of the first sub-phase vanishes, with respect to the other phases, and the asymptotic efficiency remains approximately 0.311 even with small r_0.

IV. CONCLUSIONS

In this paper we have presented an asymptotic analysis of the DFA protocol for RFID systems when applying Schoute's backlog estimate. The analysis shows that when the initial frame length is set to match the tag number N, the estimate is able to track the real value, providing the theoretical efficiency $e^{-1} \approx 0.367$. When a mismatch exists, such as when r_0 is finite and N grows much larger, the analysis shows that the asymptotic efficiency drops to 0.311, far below the theoretical maximum. The analysis further shows that the low performance is due to the slow convergence phase, whose length is proportional to N. This is due to the geometric increase of the frame, which is not needed to refine to estimate of N. In other words, the Frame Restart property of the standard can be used to shorten the frame in the convergence phase, making the overhead vanish with respect to N, and reaching the theoretical efficiency e^{-1}. This improvement will be addressed in future work.

APPENDIX A

If n_i and r_i are both large, collided slots in frame i become distributed according to a binomial with average $r_i p_c$ and variance $r_i p_c(1-p_c) \le r_i$. Therefore, being, in the approaching phase and for large r_0, $r_{i+1} = Hc_i$ we have

$$\mathsf{Var}\{r_{i+1} \mid n_i, r_i\} = H^2\, \mathsf{Var}\{c_i \mid n_i, r_i\} \le H^2 r_i. \qquad (11)$$

Since the number of collisions can not be larger than $N/2$ it follows that

$$r_i \le H\frac{N}{2},\ \forall i. \qquad (12)$$

Substituting (12) into (11) yields

$$\mathsf{Var}\{r_{i+1} \mid n_i, r_i\} \le Nd,$$

where d is a constant value. Using this bound with Chebyshev's inequality yields

$$\mathsf{P}\left(|r_{i+1} - \mathcal{R}_{i+1}| \ge \epsilon N | n_i, r_i\right) \le \frac{d}{N\epsilon^2},$$

that can be reduced to

$$\mathsf{P}\left(|r_{i+1} - \mathcal{R}_{i+1}| \ge \epsilon N\right) \le \frac{d}{N\epsilon^2}. \qquad (13)$$

Relation (13) shows that, for $N \to \infty$, we have $r_i/N \to \mathcal{R}_i/N$, where the convergence is in probability.

Much in the same manner one can show that $n_i/N \to \mathcal{N}_i/N$, and, therefore, we have $n_i/r_i \to \mathcal{N}_i/\mathcal{R}_i$ in probability.

APPENDIX B

Here we consider sequence $\{\mathcal{R}_i\}$ during the first phase, where all the slots are collided, *i.e.*, $\mathcal{C}_i = \mathcal{R}_i$ and relation (2) becomes

$$\mathcal{R}_{i+1} = \mathsf{E}\{Hc_i + \xi_i\} = H\mathcal{C}_i + \Xi_i = H\mathcal{R}_i + \Xi_i, \quad i \ge 0,$$

where ξ_i is a random variable that accounts for the rounding operation, and is such that $|\xi_i| \le 1/2$. On the other side we have

$$R_{i+1} = HR_i, \qquad i \ge 0,$$

with $\mathcal{R}_0 = R_0 = r_0$. Solving the recursions we get

$$\mathcal{R}_i = r_0 H^i + \sum_{k=0}^{i-1} H^{i-1-k}\Xi_k, \qquad (14)$$

$$R_i = r_0 H^i, \qquad (15)$$

for $i \ge 0$. Relation (14) can be rewritten as

$$\mathcal{R}_i = R_i + \sum_{k=0}^{i-1} H^{i-1-k}\Xi_k.$$

Since $|\Xi_k| \leq 0.5 < 1$, and being

$$\sum_{k=0}^{i-1} H^k = (H^i - 1)/(H - 1),$$

we can write

$$R_i - \frac{H^i - 1}{H - 1} < \mathcal{R}_i < R_i + \frac{H^i - 1}{H - 1}, \qquad i \geq 0,$$

and

$$1 - \frac{f(H)}{r_0(H-1)} < \frac{\sum_{i=0}^{\infty} \mathcal{R}_i}{\sum_{i=0}^{\infty} R_i} < 1 + \frac{f(H)}{r_0(H-1)},$$

with

$$f(H) = \left(\sum_{i=0}^{\infty}(H^i - 1)\right) / \left(\sum_{i=0}^{\infty} H^i\right),$$

having exploited (15). Since it is

$$H^i - 1 < H^i, \qquad i \geq 0,$$

we also have $f(H) < 1$, and finally

$$\lim_{r_0 \to \infty} \left(\sum_{i=0}^{\infty} \mathcal{R}_i\right) / \left(\sum_{i=0}^{\infty} R_i\right) = 1.$$

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