Sample exam

<table>
<thead>
<tr>
<th>COGNOME</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NOME</td>
<td></td>
</tr>
<tr>
<td>MATRICOLA</td>
<td></td>
</tr>
</tbody>
</table>
Exercise 1.
In the network shown in the Figure, $N_1$ VoIP flows are multiplexed by a FIFO scheduler with transmission capacity equal to $C_1$. These flows are then multiplexed by another FIFO scheduler with other $N_2$ flussi VoIP; the capacity of the second FIFO scheduler is $C_2$.

The VoIP codec is the G.726 with Voice Activity Detection (VAD). The parameters are:
- $\lambda (s^{-1})$: transition rate from OFF state (silence) to ON state (voice);
- $\mu (s^{-1})$: transition rate from ON state to OFF state;
- $P$ (bit/s): coded speed, in the ON state (note that this is the application layer speed, it does not take into account protocol overheads);
- $d_f(s)$: frame duration;
- $d_a(s)$: algorithmic delay (only in the coding phase);
- $d_p(s)$: playout buffer depth.

The protocol stack is RTP/UDP/IP/PPP. The total protocol overhead per packet is $L$ (bit).
Calculate the probability density of the queuing delay in the FIFO buffer 1, and its average value $D_1$. $D_1$ does not include coding delay and the transmission time of VoIP packets.

**Soluzione.**

Length of VoIP packets:

$$L_{\text{VoIP}} = Pd_f + L \text{ (bit)}.$$ 

Physical-layer peak transmission speed:

$$P_{\text{phy}} = \frac{L_{\text{VoIP}}}{d_f} = \frac{Pd_f + L}{d_f} \text{ (bit/s)}.$$ 

Average transmission speed of one VoIP flow, at the physical layer:

$$r = \frac{\lambda}{\lambda + \mu} P_{\text{phy}} = \frac{\lambda}{\lambda + \mu} \frac{Pd_f + L}{d_f} \text{ (bit/s)}.$$ 

Burst length of one VoIP flow:

$$b = 2 \left( \frac{\mu}{\lambda + \mu} \right)^2 P_{\text{phy}} = 2 \left( \frac{\mu}{\lambda + \mu} \right)^2 \frac{Pd_f + L}{d_f} \text{ (bit)}.$$ 

For $N_1$ flows:

$$R_i = N_i \frac{\lambda}{\lambda + \mu} \frac{Pd_f + L}{d_f}.$$ 

$$B_i = N_i 2 \left( \frac{\mu}{\lambda + \mu} \right)^2 \frac{Pd_f + L}{d_f}.$$ 

Thus, the probability density of delay in the first scheduler is:

$$f_{D_1}(t) = 2 \frac{C_i(C_i - R_i)}{R_i B_i} \exp \left( -2 \frac{C_i(C_i - R_i)}{R_i B_i} t \right).$$ 

And its average value is:

$$E[D_1] = \frac{R_i B_i}{2C_i(C_i - R_i)}.$$
Now, calculate the probability density and average value of delay in the second scheduler.

**Solution.**

The available service in the first scheduler is $C_t t$, thus, the variance of the traffic at the output of the first scheduler is equal to the variance of traffic at its input. Thus, at the input of scheduler 2:

$$R_2 = (N_1 + N_2) \frac{\lambda}{\lambda + \mu} \frac{Pd_f + L}{d_f}.$$  

$$B_2 = (N_1 + N_2) 2 \frac{\mu}{(\lambda + \mu)^2} \frac{Pd_f + L}{d_f}.$$  

Thus:

$$f_{D_2}(t) = 2 \frac{C_2 (C_2 - R_2)}{R_2 B_2} \exp \left( -2 \frac{C_2 (C_2 - R_2)}{R_2 B_2} t \right).$$  

$$E(D_2) = \frac{R_2 B_2}{2C_2 (C_2 - R_2)}.$$
Now, calculate the end-to-end average delay of traffic, including coding/decoding and transmission delay

**Solution.**

\[
E(D_{\text{total}}) = d_f + d_a + E(D_1) + \frac{L_{\text{VoIP}}}{C_1} + E(D_2) + \frac{L_{\text{VoIP}}}{C_2} + d_p
\]
Si calcola la probabilità della durata di ritardo $D_1 + D_2$.

**Soluzione.**

$$f_{D_1}(t) = 2 \frac{C_1 (C_1 - R_1)}{R_1 B_1} \exp\left(-2 \frac{C_1 (C_1 - R_1)}{R_1 B_1} t\right) = k_1 e^{-k_1 t}$$

$$k_1 = 2 \frac{C_1 (C_1 - R_1)}{R_1 B_1}$$

$$f_{D_2}(t) = 2 \frac{C_2 (C_2 - R_2)}{R_2 B_2} \exp\left(-2 \frac{C_2 (C_2 - R_2)}{R_2 B_2} t\right) = k_2 e^{-k_2 t}$$

$$k_2 = 2 \frac{C_2 (C_2 - R_2)}{R_2 B_2}$$

Quindi:

$$f_{D_1+D_2}(t) = \frac{k_1 k_2}{-k_1 + k_2} e^{-k_1 t} + \frac{k_1 k_2}{k_1 - k_2} e^{-k_2 t}$$
Exercise 2.
Let us consider a series of two schedulers, with 5 service classes for each scheduler. The first scheduler is Strict Priority, and it serves the traffic flows X1, X2, X3, X4, X5, with priority 1, 2, 3, 4 and 5, respectively. Each flow has parameters $m$ (average rate), $b$ (burst length) and $H$ (Hurst parameter). The traffic flow X5, at the output of the first scheduler, is named Y5 and it is offered to a second scheduler, GPS, that serves also the fresh traffic flows Y1, Y2, Y3 and Y4. The weights of the traffic flows Y1, Y2, Y3, Y4 and Y5 are $w_1$, $w_2$, $w_3$, $w_4$ and $w_5$, respectively. The capacity of schedulers 1 and 2 is $C_1$ and $C_2$, respectively. Calculate the probability that the delay of the flow Y5 in the second scheduler is larger than $d$.

Parameters:
- $m_{x_1}=1 \times 10^6$ bit/s, $b_{x_1}=1 \times 10^6$ bit, $H_{x_1}=0.95$;
- $m_{x_2}=2 \times 10^6$ bit/s, $b_{x_2}=2 \times 10^6$ bit, $H_{x_2}=0.92$;
- $m_{x_3}=3 \times 10^6$ bit/s, $b_{x_3}=3 \times 10^6$ bit, $H_{x_3}=0.89$;
- $m_{x_4}=4 \times 10^6$ bit/s, $b_{x_4}=4 \times 10^6$ bit, $H_{x_4}=0.87$;
- $m_{x_5}=5 \times 10^6$ bit/s, $b_{x_5}=5 \times 10^6$ bit, $H_{x_5}=0.85$;
- $m_{y_1}=1 \times 10^6$ bit/s, $b_{y_1}=1 \times 10^6$ bit, $H_{y_1}=0.96$;
- $m_{y_2}=2 \times 10^6$ bit/s, $b_{y_2}=2 \times 10^6$ bit, $H_{y_2}=0.94$;
- $m_{y_3}=3 \times 10^6$ bit/s, $b_{y_3}=3 \times 10^6$ bit, $H_{y_3}=0.92$;
- $m_{y_4}=4 \times 10^6$ bit/s, $b_{y_4}=4 \times 10^6$ bit, $H_{y_4}=0.90$;
- $w_1=1/15$, $w_2=2/15$, $w_3=3/15$, $w_4=4/15$, $w_5=5/15$;
- $C_2=45$ Mbit/s
- $d=0.05$ s

Solution

\[
E\{X_1(t)\} = m_{x_1} t, \quad \text{var}\{X_1(t)\} = m_{x_1} b_{x_1} t^{2H_{x_1}} \quad E\{Y_1(t)\} = m_{y_1} t, \quad \text{var}\{Y_1(t)\} = m_{y_1} b_{y_1} t^{2H_{y_1}}
\]

\[
E\{X_2(t)\} = m_{x_2} t, \quad \text{var}\{X_2(t)\} = m_{x_2} b_{x_2} t^{2H_{x_2}} \quad E\{Y_2(t)\} = m_{y_2} t, \quad \text{var}\{Y_2(t)\} = m_{y_2} b_{y_2} t^{2H_{y_2}}
\]

\[
E\{X_3(t)\} = m_{x_3} t, \quad \text{var}\{X_3(t)\} = m_{x_3} b_{x_3} t^{2H_{x_3}} \quad E\{Y_3(t)\} = m_{y_3} t, \quad \text{var}\{Y_3(t)\} = m_{y_3} b_{y_3} t^{2H_{y_3}}
\]

\[
E\{X_4(t)\} = m_{x_4} t, \quad \text{var}\{X_4(t)\} = m_{x_4} b_{x_4} t^{2H_{x_4}} \quad E\{Y_4(t)\} = m_{y_4} t, \quad \text{var}\{Y_4(t)\} = m_{y_4} b_{y_4} t^{2H_{y_4}}
\]

\[
E\{X_5(t)\} = m_{x_5} t, \quad \text{var}\{X_5(t)\} = m_{x_5} b_{x_5} t^{2H_{x_5}}
\]

\[
\text{var}\{S_{x_5}(t)\} = \text{var}\{X_1(t)\} + \text{var}\{X_2(t)\} + \text{var}\{X_3(t)\} + \text{var}\{X_4(t)\}
\]

\[
\text{var}\{Y_5(t)\} = \max\{\text{var}\{X_5(t)\}, \text{var}\{S_{x_5}(t)\}\}
\]

\[
E\{Y_5(t)\} = E\{X_5(t)\}
\]

\[
E\{S_{y_5}(t)\} = w_5 C_2 t + \sum_{j=1}^{4} \sum_{k=L, k \neq j}^{5} \frac{w_j}{w_k} \left( w_j C_2 t - E\{X_j(t)\} \right)
\]
\[ \text{var} \left( S_{Y_j} (t) \right) = \sum_{j=1}^{4} \left( \frac{w_j}{\sum_{k \neq j} w_k} \right)^3 \text{var} \left( X_j (t) \right) \]

\[ \alpha(t) = -\frac{E \left( Y_5 (t) \right) - E \left( S_{Y_5} (t + d) \right)}{\sqrt{\text{var} \left( Y_5 (t) \right) + \text{var} \left( S_{Y_5} (t + d) \right)}} \]

Numerically

\[ p = \exp \left( -\frac{3.17^2}{2} \right). \]
Exercise 3.
Explain how applications can be mapped onto PHBs in a Diffserv network. Which kinds of PHBs are properly selected for each application?