A non linear multicommodity network design approach to solve a location-allocation problem in freight transportation

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Problem and data have been kindly provided by DANZAS
- activated international terminals

--- international lines

--- internal collection/distribution
PROBLEM DEFINITION

The company is reorganizing the international transportation:

- which terminals have to be closed and where to open new terminals
- size of the terminals
- assign international lines to terminals
- assign local customers to terminals
- evaluate the introduction of inter-terminal lines
COST ANALYSIS

collection/distribution carried out by third parties
  ⇒ the costs are linear in the transported volume

international transportation carried out by DANZAS
  ⇒ concave costs (economies of scale)
      different shapes depending on the lengths

internal flow in terminals
  ⇒ concave costs
International transportation costs

\[ y = 1.7325x^{0.6678} \]

\[ R^2 = 0.928 \]
FLOW MODEL

terminal 1

terminal 2

domestic customers

international destinations
FLOW MODEL

inter-terminal flow
FLOW MODEL

commodities jk (domestic origin j, international destination k)

\( d_{jk} = \text{volume of goods to be transported from j to k} \)

\( x_{jh}^{jk} = \text{amount of flow of commodity jk going to terminal h} \)

\( x_{hh'}^{jk} = \text{amount of flow of commodity jk inside terminal h} \)

\( x_{h'k}^{jk} = \text{amount of flow of commodity jk going to destination k from terminal h} \)

\( z_h = 1 \text{ if terminal h is activated, 0 otherwise} \)
FLOW MODEL

\[
\min \sum_j \sum_h f_c^{jh} \left( \sum_k x_{jk}^{jh} \right) + \sum_h f_m^h \left( \sum_{jk} x_{hh'}^{jk}, z_h \right) + \sum_k \sum_h f_i^{hk} \left( \sum_j x_{h'k}^{jk} \right)
\]

\[E_{jk} x_{jk} = \delta_{jk}\] for each commodity \(jk\) [flow conservation]

\[x_{hh'}^{jk} \leq u_h z_h\] for each commodity \(jk\) and terminal \(h\) [design]

\(E_{jk}\) is the node/arc incidence matrix related to commodity \(jk\)
\(\delta_{jk}\) is the demand vector
\(u_h\) is an upper bound of the flow passing by terminal \(h\)
LINEARIZATION OF SEPARABLE COST FUNCTIONS

\[ x = \sum_{l=0}^{k} a_{l} \xi_{l} ; \quad f(x) = \sum_{l=0}^{k} b_{l} \xi_{l} ; \quad \sum_{l=0}^{k} \xi_{l} = 1; \quad \sum_{l=0}^{k-1} \eta_{l} = 1; \]

\[ \sum_{j=0}^{l} \eta_{j} \geq \sum_{j=l+1}^{k} \xi_{j} \geq \sum_{j=l+1}^{k} \eta_{j} \quad \text{for } 1 \leq l \leq k-2 \]

\[ 0 \leq \xi_{0} \leq \eta_{0} ; \quad 0 \leq \xi_{k} \leq \eta_{k-1} ; \quad \eta_{l} \in \{0, 1\} \quad \text{for } l=1,...,k-1 \]

\( x \) is a convex combination of two consecutive interval endpoints

formulation "locally ideal" [Padberg]
"ASSIGNMENT" MODEL

In the previous model the flow of one commodity can **split** between two or more terminals (even though it is not convenient).

\[ x_{jhk} = \begin{cases} 
1 & \text{commodity } jk \text{ is assigned to terminal } h \\
0 & \text{otherwise.} 
\end{cases} \]

The flow collected by \( h \) from customer \( j \) is given by:

\[ \sum_k d_{jk} x_{jhk} \]

The flow inside terminal \( h \) is given by:

\[ \sum_{jk} d_{jk} x_{jhk} \]

The flow on international line \( hk \) is given by

\[ \sum_j d_{jk} x_{jhk} \]
"ASSIGNMENT" MODEL

\[
\begin{align*}
\min & \quad \sum_j \sum_h \left( f_c^{jh} \left( \sum_k d_{jk} x_{jhk} \right) + \sum_h f^h_m \left( \sum_{jk} d_{jk} x_{jhk}, z_h \right) \right) \\
& + \sum_k \sum_h f_i^{hk} \left( \sum_j d_{jk} x_{jhk} \right) \\
\sum_h x_{jhk} &= 1 \quad \text{for each commodity } jk \\
x_{jhk} &\leq z_h \quad \text{for each commodity } jk, \text{ for each terminal } h \\
x_{jhk}, z_h &\in \{0, 1\}
\end{align*}
\]
"ASSIGNMENT" MODEL

Inter-terminal flows

\[ x_{jhrk} = \begin{cases} 
1 & \text{commodity } jk \text{ uses terminals } h \text{ and } r \text{ in the order } \\
0 & \text{otherwise.} 
\end{cases} \]

the constraints are modified accordingly
LINEARIZATION OF SEPARABLE COST FUNCTIONS (2)

\[ x = a_0 + y_1 + \ldots + y_k \]

\[ f(x) = b_0 + \frac{b_1 - b_0}{a_1 - a_0} y_1 + \ldots + \frac{b_k - b_{k-1}}{a_k - a_{k-1}} y_k \]

\[ 0 \leq y_l \leq a_l - a_{l-1} \]

\[ y_l \geq (a_l - a_{l-1}) z^l, \quad y_{l+1} \leq (a_{l+1} - a_{l+1}) z^l, \text{ for } l=1, \ldots, k-1 \]
Also this formulation is "locally ideal" [Padberg]
VALID INEQUALITIES

let $S$ be a subset of commodities and $h$ a terminal such that
\[ \sum_{jk \in S} d_{jk} > a_i^h \]
if all the commodities in $S$ are routed through terminal $h$
\[ \Rightarrow z_i^h = 1 \]

\[ \sum_{jk \in S} x_{jhk} - |S| + 1 \leq z_i^h \]
separable by solving small knapsack problems
**COMPUTATIONAL RESULTS**

4 instances derived from real data provided by DANZAS

domestic areas / terminals / international destinations

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<th>&quot;assignment&quot; model</th>
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<th>&quot;flow&quot; model</th>
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## COMPUTATIONAL RESULTS

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ILP solved with CPLEX 6.6

*best integer after 1000 seconds (about 1-3% from optimum)
### COMPUTATIONAL RESULTS

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Comparison with the best solution obtained with 9 open terminals

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## Computational Times

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