Design, Routing and Wavelength assignment in Optical Networks

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Given: a network (nodes and links) and capacities
a traffic demand (# of dedicated O/D channels)
WDM technology

Route all the traffic assigning wavelengths (colors) to dedicated channels

Capacity = 2
1-3: 1 channel
2-1: 1 channel
3-2: 1 channel
Routing and wavelength assignment in optical networks

Given: a network (nodes and links) and capacities
a traffic demand (# of dedicated O/D channels)

Capacity = 2
1-3: 1 channel
2-1: 1 channel
3-2: 1 channel

In this network we cannot route all the traffic unless we introduce an **OPTICAL CROSS CONNECT (OXC)**
Approaches in the literature:

- OXC in each node (⇒ multicommodity flow)
- NO OXC

- OXC's are very expansive
  ⇒ selective placement of OXC
    (one of the most challenging problem in the field)

Are they really necessary or is it more worthy allocating more capacity in the links?

ILP formulation of the routing and selective OXC placement

some valid inequalities
Capacity allocation vs. OXC placement

- ILP model
  - routing + WL assignment + network design
- valid inequalities
- experiments on real networks (metro and geographical)

Assumption: all the traffic of each commodity is routed along the same path
NOTATION

Oriented graph \( G = (N, A) \)

\( p \in P^k : \) feasible path for commodity \( k \in K \) (O/D pair \((s^k, t^k)\))

\( P(u, v) : \) set of paths containing arc \((u, v)\)

\( d^k : \) demand of commodity \( k \)

\( H : \) set of available WL

\[
\begin{align*}
\chi_p &= \begin{cases} 
1 & \text{traffic of commodity } k \text{ is routed through } p \in P^k \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]

\[
\begin{align*}
\chi_{kh} &= \begin{cases} 
1 & \text{WL } h \text{ assigned to a unit of commodity } k \text{ on arc } (u, v) \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]

\[
\begin{align*}
\chi_{uv} &= \begin{cases} 
1 & \text{a WL occurs at node } v \text{ commodity } k \\
0 & \text{otherwise}
\end{cases} \\
\end{align*}
\]
Minimizing the number of switches

$$\min \sum_{v \in N} \max_k z_v^k$$

$$\sum_{p \in P^k} x_p = 1 \quad \forall k \in K$$

$$\sum_{k \in K} y_{uv}^{kh} \leq 1 \quad \forall h \in H, \forall (u,v) \in A$$

$$\sum_{h \in H} y_{uv}^{kh} = \sum_{p \in P^k \cap P(u,v)} d^k x_p \quad \forall k \in K, \forall (u,v) \in A$$

$$z_v^k \geq y_{wv}^{kh} - y_{vu}^{kh} - \left(1 - \sum_{p \in P^k \cap P(w,v) \cap P(v,u)} x_p\right) \quad \forall k \in K, \forall h \in H, \forall (w,v),(v,u) \in A$$

$$z_v^k \geq y_{vu}^{kh} - y_{wv}^{kh} - \left(1 - \sum_{p \in P^k \cap P(w,v) \cap P(v,u)} x_p\right) \quad \forall k \in K, \forall h \in H, \forall (w,v),(v,u) \in A$$
The constraints setting $z$ variables derive from:

$$z_v^k = |y_{wv}^kh - y_{vu}^kh|$$

if commodity $k$ is routed through arcs $(w,v)$ and $(v,u)$

there is a switch if and only if a lightpath leaves a node with a color different from the entering one
Including the capacity allocation into the model

\( c_e \): capacity of the link if WDM device \( e \) is installed
\( g_e \): cost of device \( e \)
\( f_v \): cost of OXC in node \( v \)

assume: \( c_e > c_{e-1} \), and \( g_e > g_{e-1} \) \((c_0 = g_0 = 0)\)

we allocate the capacity incrementally

\[
Y_{uv}^e = \begin{cases} 
1 & \text{capacity } (c_e > c_{e-1}) \text{ is added to arc } (u, v) \\
0 & \text{otherwise}
\end{cases}
\]
\[
\min \sum_{v \in N} f_v \max_k z_v^k + \sum_{e \in L} (g_e - g_{e-1}) \sum_{(u,v) \in A} Y_{uv}^e
\]

\[
\sum_{k \in K} \sum_{h \in H} Y_{uv}^{kh} \leq \sum_{e \in L} (c_e - c_{e-1})Y_{uv}^e \quad \forall (u,v) \in A
\]

\[
Y_{uv}^e \leq Y_{uv}^{e-1} \quad e = 2, \ldots, |L|, \quad \forall (u,v) \in A
\]

{the other constraints defined before}
Valid inequalities (1)

"Cover" inequalities on the arc capacities

Let $K(u,v)$ be the set of commodities which may use arc $(u,v)$ and let $e$ such that $c_{e-1} < \sum_{k \in K(u,v)} d^k \leq c_e$

The following inequalities are valid:

$$\sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} x_p - |K(u,v)| + 1 \leq Y^e_{uv}$$

They can be applied to any subset $K'(u,v) \subseteq K(u,v)$ including singletons
Valid inequalities (2)
Derived from the selective OXCC placement problem

Let $G^* = (V, E)$ be the conflict graph of paths and $C$ a clique in $G^*$ such that $\sum_{p \in C} d^{k(p)} > |H|$ where $H$ is a subset of $C$ and $k(p)$ is the commodity of path $p$.

The following inequalities are valid:

$$\sum_{p \in C} x_p - |C| + 1 \leq \sum_{p \in C} \sum_{v \in V(C)} z_v^{k(p)}$$

where $V(C)$ is the set of nodes belonging to pairs of paths in $C$.
COMPUTATIONAL EXPERIMENTS

Metropolitan network:
11 nodes, 42/43 arcs, 136/142 paths, 22/23 commodities

Scenario 1: normal costs (example provided by Cox associated)
Scenario 2: high OXC costs
Scenario 3: low OXC costs

In scenarios 2 and 3 the graph and the demand have been modified wrt to scenario 1 in order to have the alternative between introducing an OXC or a new fiber

Geographical Network (NFS Net):
14 nodes, 54 arcs, 35 commodities
we generated a subset of 110 paths
two scenarios depending on OXC costs
<table>
<thead>
<tr>
<th></th>
<th>metro network</th>
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<th>NFS net</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Scenario 1</td>
<td>Scenario 2</td>
<td>Scenario 3</td>
<td>Scenario 4</td>
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<td>52%</td>
<td>50%</td>
<td>42%</td>
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<td><strong>Best integer after</strong></td>
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<td>1278801</td>
<td>4400000</td>
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<td><strong>after simple cuts</strong></td>
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<tr>
<td><strong>(singletons)</strong></td>
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<td>11.46%</td>
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<td><strong>and &quot;placement&quot; cuts</strong></td>
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<td>1</td>
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FUTURE RESEARCH

- Column generation and Branch&Cut
- More sophisticated heuristics
- Study the case where the flow is allowed to split among different paths