Combination of low and high resolution images SAR for differential interferometry.

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Abstract

In this paper we propose a combination of low resolution (LR) and full resolution (FR) SAR images for differential SAR interferometry. The principal problem in such a combination is the volume scattering decorrelation that limits the useful baseline of the interferometric pair to very small, impractical values. A technique has been introduced to remove this decorrelation by exploiting a digital elevation model (DEM) of the area imaged. The resulting interferogram has the quality (in terms of coherence, or phase noise standard deviation) of a conventional FR interferogram, but reduced geometric resolution. The technique is suited to those differential interferometry applications where the resolution of the "differential" phase field to be monitored is much coarser than that of local topography. As a possible application, we propose a system that exploits the wide swath imaged by low-resolution ScanSAR modes for frequent monitoring hazardous areas.

1 Introduction

Applications of SAR interferometry (InSAR) are well known and described in the literature [1, 2, 3, 4, 5]. There is growing interest in differential interferometric surveys (DInSAR), i.e. an interferogram with zero baseline, or where the topography is compensated. DInSAR surveys are usually generated starting from two unwrapped interferograms, or an interferogram and a digital elevation model (DEM). The "residual" differential interferogram that is obtained can be used to detect small surface deformations (like glacier motions, earthquakes, landslides [2, 3, 6]), or even atmospheric changes [7].

Usually, DInSAR surveys are generated on the basis of a high resolution SAR system but, in most cases, the output differential interferogram has an intrinsic low geometric resolution (it is smooth); therefore, it can also be monitored by Low-Resolution SAR.

This paper introduces first a novel technique to combine a full resolution (FR) SAR image, a low resolution (LR) one and a DEM to synthesize a zero baseline low resolution differential interferogram. This technique is able to remove most of the volume scattering decorrelation, expected in such a combination, leading to the best quality obtainable from the data.

As a possible application of the proposed technique two frequent revisiting systems are discussed. They exploit the wide-swath accessed by the LR ScanSAR mode available on most of the newest and on future SAR sensors. One system provides the monitoring of unpredictable events by continuously acquiring images in ScanSAR mode. Each of these images could then be combined with an existing "data-base" of FR SAR images to compute the differential interferogram. The other system, proposed by Y. L. Desnos in [11], is obtained by exchanging the acquisition order. First a data base of LR ScanSAR images of the entire area of interest (eventually the whole earth surface) is built up in a short time (due to large swath coverage). Thereafter, each new FR acquisition would be interferometric, as it would then be combined with the proper one in the data-base. Such a system would be suitable to monitor large scale changes like surface deformation, meteorological effect, earth tides, according to user demand.

Finally, the last section gives examples achievable with the proposed technique, and discusses the results; of particular interest are those obtained by processing the Landers earthquake dataset [3, 8] in low-resolution mode.

2 ScanSAR - SAR interferometry

Reference is made here to a repeat pass interferometric system that involves a sensor with both FR and LR capabilities. The simplified system geometry we will refer to is sketched in Fig. 1a, that shows a terrain patch imaged in the two modes, FR and LR.

Figure 1: (a) Geometry of the ISAR system combining a FR and a LR mode. The focused beams have been evidenced by shading. (b) Geometry in the case of constant slope topography.

Low resolution is obtained in most satellite borne SAR (SIR-C, RADARSAT, ASAR, LightSAR) systems by operating in the ScanSAR mode [9]. ScanSAR achieves a wide swath coverage by periodically switching on the fly- the antenna pointing in several range "sub-swaths." The loss of data (compared to FR SAR) due to the non-acquired pulses causes a proportional loss in the azimuth resolution. This is what ScanSAR sacrifices for increased range coverage. Usually, each target is imaged several times throughout the synthetic aperture, to get "looks" that can be averaged (after focusing and amplitude detection) to reduce scalloping (e.g. the amplitude modulation due to non uniform antenna azimuth pattern). Thus, the azimuth resolution is reduced, with respect to FR SAR, to the ratio between the number of echoes in each "burst" (or "look") and the number of echoes in the synthetic aperture. For example, if a ScanSAR sensor operates by switching the antenna over 50 looks in a footprint, its azimuth resolution is degraded by a factor 50/150 = 1/3 sub-swaths and images the same target in up to 50 25 sub-swaths. On the other hand, the azimuth resolution is degraded by a factor 50/150 = 1/3 sub-swaths and images the same target in up to 50 looks in a footprint, its azimuth resolution is degraded by a factor 50/150 = 1/3 sub-swaths. In ScanSAR mode, since the system bandwidth is somewhat reduced to improve images SNR.

In the following we will assume a generic C band SAR sensor with a typical 6 × 20 m (azimuth, 1 look × ground range) spatial resolution in FR mode and 150 × 50 m in the LR mode with 5 azimuth looks. These modes are close to what can, or could, be achieved by RADARSAT and ASAR [10]; moreover they are easily simulated by means of ERS data.

The most important system parameters assumed here are defined in Tab. 1, together with their nominal values for the ASAR (or ERS) FR mode and for the typical LR mode just discussed.
a relative elevation $\varphi$ with respect to the flat earth surface can be expressed as follows:

$$\Delta \varphi = \varphi(P_2) - \varphi(P_1) = \frac{4\pi f \Delta \theta}{c} (r_2(P_2) - r_1(P_1)) - (r_2(P_1) - r_1(P_1))$$

$$= \frac{4\pi f_0}{c} \Delta \theta \frac{q}{\sin \theta} = \frac{4\pi f_0}{c} h_0 \frac{q}{\sin \theta}$$

where $P_1$ is a reference point, $r_2$ the average slant range, and $\theta$ the incidence angle with respect to a horizontal plane. If the scatterers are uniformly distributed within a layer of thickness $q_x$, measured with respect to a slant range reference plane:

$$q_x = \frac{c q_0}{2 f_0 h_0 \sin \theta}$$

Their total contribution to the interferogram is pure noise, since the phases are randomly distributed over $[0, 2\pi]$. For the ASAR like geometry defined in Tab. 1, and assuming a baseline $B_{nc} = 300$ m, we get $q_x = 290$ to $450$ m (depending on the incidence angle). Thus, the large resolution cell in the LR mode limits the availability of interferometry to flat surfaces with the same slope as the look angle. For example, for a distributed target over a flat, horizontal terrain surface, the size of the ground resolution cell is subject to the limit (derived from (3))

$$B_{nc} \leq 110 \text{ m.}$$

The occurrence of such baselines will be so marginal, (< 10% of the interferometric acquisitions for the ERS case, according to [15]), to exclude the use of ScanSAR - ScanSAR interferometry for the frequent monitoring application proposed in section 3.

2.2 Zero Baseline Steering

A more feasible DInSAR system that exploits low-resolution imagery is proposed in this section [11]. High quality DInSAR images can be obtained, provided that a proper pre-processing is applied to the DInSAR image to steer the FR/LR image pair to an equivalent zero-baseline. The only requirement is a DEM of that area, a common pre-requisite for DInSAR.

Let us refer to the geometry drawn in Fig. 1a. The figure represents a distributed scatterer on the terrain surface, as imaged by the FR sensor, $S_1$ and by the LR sensor, $S_2$. The 1D case has been assumed for simplicity. The data are assumed to be already focused.

Let $r$ be the slant-range coordinate of satellite $S_1$. Let us define $v_1(r)$ the complex backscatter (or “reflectivity”) of the distributed target, observed by the high resolution sensor, $S_1$. Neglecting spherical divergence effects, antenna gains, etc [12] the Fourier transform of $v_1$ can be expressed as follows:

$$V_1(f) \cong \left( \int_{r_{rm}}^{r_{TM}} v_1(r) \exp \left( \frac{4\pi f_0}{c} r \right) dr \right) \cdot H_1(f)$$

where $H_1(f)$ is the bandpass filter of the target slant range reflectivity, $v_1(r)$, that would correspond to a bandwidth much greater than $B_{nc}$. The same target observed by sensor $S_2$ results in the image reflectivity $v_2(r_2)$, that can be expressed in the frequency domain:

$$V_2(f) \cong \left( \int_{r_{rm}}^{r_{TM}} v_2(r_2) \exp \left( \frac{4\pi f_0}{c} r_2 \right) dr \right) \cdot H_2(f)$$

where $H_2(f)$ is the bandpass filtered version of the target slant range reflectivity, $v_2(r)$.

Let us refer to the interferometric geometry of Fig. 1b. The interferometric phase contribution of a target with

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>FR</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>platform altitude</td>
<td>800 km</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>incidence angle (flat earth)</td>
<td>$17^\circ \sim 45^\circ$</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>interferometric phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_0$</td>
<td>interferometric baseline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>slant range axis</td>
<td>830 km</td>
<td>1000 km</td>
</tr>
<tr>
<td>$r_0$</td>
<td>sensor-target mean distance</td>
<td>8 m</td>
<td>7 m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>slant range resolution</td>
<td>25 m</td>
<td>50 m</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>azimuth resolution</td>
<td>6 m (1 look)</td>
<td>150 m (5 looks)</td>
</tr>
<tr>
<td>$x$</td>
<td>azimuth axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0$</td>
<td>range central frequency</td>
<td>5.3 GHz</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>light speed</td>
<td>$300 \cdot 10^3$ m/s</td>
<td></td>
</tr>
<tr>
<td>$B_r$</td>
<td>range bandwidth</td>
<td>$8 \sim 16$ MHz</td>
<td>3 $\sim 7$ MHz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>range sampling frequency</td>
<td>19 MHz</td>
<td></td>
</tr>
<tr>
<td>$f_{PR}$</td>
<td>azimuth sampling frequency</td>
<td>3.7 kHz</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: List of symbols and parameters. Values are given for the two assumed modes: Full Resolution (FR), and Low Resolution (LR).
being the distance between the sensor and a target placed in \( r_1 \), i.e. the slant range of sensor \( S_1 \) (see Fig. 1). Here again, the focused signal from sensor \( S_2 \) is a bandpass filtered reflectivity, but with a much smaller bandwidth: \( B_{S2} \ll B_{S1} \). The term \( r_2[r_1] \) in (5) represents the influence of the different geometry, due to the change in view from the two targets.

The geometry-dependent contribution can be isolated by rearranging (5) as follows:

\[
V_2(f) = \int_{r_{min}}^{r_{max}} v(r) \exp \left( \frac{4\pi f}{c} \Delta R(r) \right) \exp \left( \frac{j4\pi f}{c} r \right) dr \cdot H_2(f)
\]

where we have defined \( \Delta R = r_2(r_1) - r_1 \), i.e. the difference in the two sensor-target travel paths.

Moreover, we can apply the monochromatic approximation (see Appendix 1) for \( f \approx f_{0} \):

\[
V_2(f) \approx \int_{r_{min}}^{r_{max}} v(r) \exp \left( \frac{4\pi f_{0}}{c} \Delta R(r) \right) \exp \left( \frac{j4\pi f_{0}}{c} r \right) dr \cdot H_2(f)
\]

For zero-baseline, \( \Delta R = 0 \), the LR reflectivity, \( V_2(f) \), can simply be derived by filtering FR reflectivity, \( V_1(f) \) in (4). For non zero-baseline, \( V_2(f) \) could theoretically be synthesized according to (7), once both \( v(r) \) and the geometry dependent terms are known:

\[
s(r) = \exp \left( \frac{j4\pi f_{0}}{c} \Delta R(r_1) \right)
\]

Note that the phase \( s(r) \) is a sort of "synthetic fringe". In the actual case, the complex reflectivity \( v(r) \) is not known, only its bandpass filtered version \( v_{1}(r_{1}) \), in (4); hence the idea is to use \( v_{1} \) in place of \( v \) for the actual synthesis of LR reflectivity in (7):

\[
\tilde{V}_2(f) = \int v_{1}(r) \exp \left( \frac{4\pi f_{0}}{c} \Delta R(r) \right) \exp \left( \frac{j4\pi f_{0}}{c} r \right) dr \cdot H_2(f) \approx V_2(f)
\]

The space domain counterparts of (7,9) can be obtained by inverse Fourier transform and by exploiting (4,8):

\[
v_2(r_2) \approx \left( v_1(r_1) \cdot s(r_1) \right) \ast h_2(r_2)
\]

\[
\tilde{V}_2(r) = \left( v_{1}(r_{1}) \cdot s(r_{1}) \right) \ast h_2(r_1) = \left( \left( v(r_1) \cdot h_1(r_1) \right) \cdot s(r_1) \right) \ast h_2(r_1)
\]

where \( * \) means convolution and \( h_1, h_2 \) are the space domain filters, inverse transforms of \( H_1, H_2 \).

Expression (11) actually performs the "zero baseline steering" (zbs), that converts the FR reflectivity, \( v_1 \), into the reflectivity \( V_2 \) (or, better, its approximation \( \tilde{V}_2 \)) that would be imaged by a LR sensor in a displaced orbit.

### 2.3 Limitations

The LR reflectivity \( \tilde{V}_2(r) \), synthesized from FR by means of (11), can be combined with the actual LR acquisition to get an equivalent zero-baseline interferogram, provided that the following conditions hold:

1. there are no changes in the scattering properties between the two images;
2. the synthetic fringe component, \( s(r) \) in (11) is known with sufficient accuracy. This, in turn, implies some constraints both in the space resolution and in the accuracy of the DEM used to compute \( s(r) \) according to (8);
3. the FR reflectivity, \( v_1 \), is known within a very fine resolution, much finer than the LR one.

The first item refers mainly to temporal changes. This is one of the major sources of decorrelation in repeat pass SAR interferometry, and one of the most restrictive limits to the availability of the frequent revisiting system we are proposing. This limit, that is still valid in the case of FR/FR SAR interferometry, is unavoidable and should to be carefully considered when selecting the areas and the subject of interest.

The second and third items are more closely linked to the technique we present and will be investigated in the present and the following sections.

The error due to the limited bandwidth of FR reflectivity, \( v_1(r_1) \), and the limited DEM resolution can be estimated in the frequency domain, by comparing the Fourier Transform of (10,11):

\[
V_2(f) \approx (V(f) \ast S(f)) \cdot H_2(f)
\]

\[
\tilde{V}_2(f) = \left( \left( V(f) \cdot H_1(f) \right) \cdot s(r_1) \right) \cdot H_2(f)
\]

where \( S(f) \) is the transform of the "synthetic" fringes, \( s(r_1) \) in (8). If we assume that \( H_2(f) \) is an ideal band-pass filter with bandwidth \( B_{S2} \), then (13) is a very good approximation of (12) if the bandwidth of \( s(r) \) is \( \leq B_{S1} \approx B_{S2} \). It means that the fringes term \( s(r) \) should be approximately linear phase within the FR resolution cell. The bandwidth of \( s(r) \) can be estimated by (8), assuming that \( s(r) \) is a frequency modulated (FM) signal where the local terrain slopes act as modulating term. With this assumption, common in SAR interferometry [19], the instantaneous (slant range) frequency, \( f_s \), can be expressed (in Hz) as follows:

\[
f_s = \frac{f_0}{2\pi} \arg \left( \frac{r_1 - r_2}{r_1} \right)
\]

It can be related to the local terrain slope, \( \alpha \) in (15), by exploiting the geometry of Fig. 3

\[
r_1 = r_0 + l \sin(\theta - \alpha)
\]

\[
r_2 = r_0 + l \sin(\theta - \alpha - \Delta \theta) \pm l \cos(\theta - \alpha - \Delta \theta) \Delta \theta = r_2 - r_1 = \Delta \theta = -\frac{\Delta \theta}{\sin(\theta - \alpha)}
\]

Finally, by applying (15) to (14) we get the same expression of the frequency shift derived in [4] for constant slopes:

\[
f_s = \frac{-f_0}{2\pi} \frac{\Delta \theta}{\sin(\theta - \alpha)} = -f_0 \frac{\Delta \theta}{\tan(\theta - \alpha)}
\]

The bandwidth of \( s(r) \) can now be found as for conventional frequency modulation processes. Let us first assume

![Figure 3: Interferometric geometry for constant slope topography.](image-url)
The zbs technique thus comes down to a spectrum shift and a bandpass filtering (this is the case assumed in [4]). The LR reflectivity is reconstructed with no decorrelation until the spectral shift falls within the limit: $|f| < B_0/2 - B_{FR}/2 \approx B_0/2$. This limit is half the “critical baseline” for FR/FR interferometry derived in (3), yet larger than the practical baseline limit that is usually assumed in that case. Note that the DEM is not really required since the topography is assumed to vary so slowly that slopes (or the spectral shifts they cause) can be estimated as the peak of the interferogram power spectrum [4, 19].

Let us then assume that the terrain surface is flat within the FR resolution cell, but not in the LR resolution cell. Here the fringes term can be regarded a stationary FM signal, e.g. a space variant modulation, where the frequency shift, $f_i(r)$ changes slightly from pixel to pixel in the FR image. (18) becomes:

$$
\hat{V}_2(f) \approx V_1(f - f_i(r))H_2(f)
$$

The zbs is now a space variant demodulation and a filtering (that is just a summation of all the contributions in each LR pixel) of the FR image. The synthesis is exact, subject to the above mentioned limitation: $|f| < B_{FR}/2$, but assuming $Q_i$ to be the maximum value in the LR cell. Here topography should be known within the finer resolution (FR), therefore the DEM is required (it cannot be retrieved by data). In this case, a more accurate study of the error introduced by the proposed technique, that accounts for the proper power spectrum of synthetic fringes $s(r_j)$, is developed in the Appendix. It is shown there that the same baseline range for FR/FR interferometry can be exploited, achieving comparable decorrelation.

Finally, we introduce that the terrain surface is rugged, not smooth, i.e. the fringes resolution is much finer than the FR cell. In this case the band–limiting of $s(r_j)$ implied in (13) introduces a decorrelation noise that is equivalent to the volume scattering decorrelation [4, 12] i.e. it is not a true zero-baseline steering. This effect has been characterized in the Appendix: it is shown that - for realistic white in the FR bandwidth - the decorrelation is roughly proportional to the ratio between the power of $s(r_j)$ within the high resolution bandwidth, $H_{2iz}$, and its total power. This is the result that we would expect in the presence of “volume scattering” for FR/FR interferometry. In this case the DEM is useful only to find the average slope, therefore is of no interest to have a DEM resolution finer than the FR one.

### 2.4 DEM accuracy

The role of DEM errors in the synthesis of the ScanSAR reflectivity just proposed depends much on the error statistics, on its space resolution and on the type of DEM used. Smooth errors, whose correlation extent is much larger than the LR space resolution (e.g. hundred of meters or more), would not influence the result since the “residual” slopes that they leave can be estimated by means of spectrum analysis techniques [19], and compensated for on the data themselves. Errors coming from incorrect inverse geocoding of DEM on the SAR reference (like systematic errors due to approximate knowledge of satellite orbits or lack of ground control points), and/or space coregistration of the two images, fall in this category.

Therefore, we will now focus on those errors that cannot be compensated, e.g. when their autocorrelation has an extent comparable or smaller than the LR resolution cell.

#### 2.4.1 Conventional DEM

We can hypothesize a normal distribution for DEM vertical errors. The resulting coherence can be estimated as shown in Appendix 1, the following expression holds:

$$
|\gamma_b| = \frac{|E[|v_{g,b}|]|}{\sqrt{E[|v_{g,b}|^2]E[|v_{g,b}|^2]}} = \sqrt{\frac{\int_{-\pi/2}^{\pi/2} \exp(j\gamma_b v_g) dv_g}{\int_{-\pi/2}^{\pi/2} \exp(j\gamma_b v_g) dv_g}} = \sqrt{E[|\exp(j\gamma_b v_g)|]}
$$

(19)

where $\gamma_b$ is the phase error due to incorrect DEM. Let us assume that the local topography was compensated by a constant slope $\alpha$. Then, the phase contribution of a vertical elevation error $q_d$ can be computed by (1), and referring to Fig. 1.b:

$$
\phi_d = \phi(F_i) - \phi(F) = \frac{4\pi q_d}{c} B_{0,\text{cos} \alpha} r \sin(\theta - \alpha)\gamma_{\phi_d}
$$

The probability density of the interferometric phase when DEM errors are normally distributed $N(0, \sigma_d)$:

$$
\gamma_{\phi_d} = \frac{1}{\sqrt{2\pi \sigma_{\phi_d}}} \exp \left(\frac{-\sigma_{\phi_d}^2}{2\sigma_{\phi_d}^2}\right)\sigma_{\phi_d} = \frac{4\pi q_d}{c} B_{0,\text{cos} \alpha} r \sin(\theta - \alpha)\gamma_{\phi_d}
$$

(21)

The following expression for the contribution of DEM error to the coherence is:

$$
|\gamma_b| = \sqrt{\frac{\int_{-\pi/2}^{\pi/2} \exp(j\gamma_b v_g) dv_g}{\int_{-\pi/2}^{\pi/2} \exp(j\gamma_b v_g) dv_g}} = \sqrt{E[|\exp(j\gamma_b v_g)|]}
$$

Let us accept a coherence $|\gamma_b| \geq 0.7$, then, by inverting (21), we get $\sigma_{\phi_d} = \sqrt{-2 \log (|\gamma_b|)} = 0.85 \text{ rad}$, for a baseline $B_{FR} = 250 \text{ m}$, the required DEM accuracy $\sigma_d$ is $\sim 8 \text{ m}$ for flat terrain. An overview of the requested DEM accuracy to perform topography compensation is shown in Fig. 4, that plots the maximum $\sigma_d$ for different combinations of baselines and slopes. Values of $\sigma_d$ are in the range from $5 \text{ m}$ to $10 \text{ m}$ for baselines $< 300 \text{ m}$. This accuracy can be provided by currently available DEM ($\sigma_d \approx 7 \text{ m}$, for USGS level 1 DEM [18]).

![Figure 4: Maximum tolerated DEM std deviation, as a function of the baseline, and the local slopes required for coherence ≥ 0.7](image-url)
Let $\Delta \phi_D$ be the phase of a FR/FR “reference” interferogram (with baseline $B_{ref}$), thus related to the topography by exploiting (1):

$$\Delta \phi_D(P) = \frac{4\pi f_b B_{ref} \sin \theta}{c B_{ref}} q(P) + n_0 \Delta \phi(D)$$  \hspace{1cm} (22)

where $q(P)$ is the elevation of the target in P, $\theta$ the local slope (slowly varying with respect to FR resolution), and $n_0 \Delta \phi(D)$ is a phase noise contribution, mainly due to temporal decorrelation.

The fringe signal $s(r)$ can be derived by extracting the target elevation $q(P)$ from (22), but this elevation is reconstructed with errors due to the noise term $n_0 \Delta \phi(D)$ (provided that no phase unwrapping error occurs):

$$\hat{q}(P) = q(P) + n_0 \Delta \phi(D) \frac{c_B \cos \theta}{4 B_{ref}} \Rightarrow \hat{s}(r) = \exp \left( - j \frac{4\pi f_b B_{ref} \cos \theta}{c B_{ref}} \hat{q}(P) \right) = s(r) \exp \left( - j \frac{B_{ref} \Delta \phi(D)}{B_{ref} \Delta \phi} \right)$$  \hspace{1cm} (23)

$B_{ref}$, $B_{fr}$ being the baseline of the FR/LR pair. The zero baseline steered reflectivity, $s_0$, is obtained by substituting the approximate fringes, $\hat{s}$ in (23), in place of the exact one, $s$, in (11), and approximating it for $B_{ref} \gg B_{fr}$

$$s_0 = (\nu + n_0) \cdot \hat{s}$$  \hspace{1cm} (24)

In (24), a decorrelation noise $n_\nu$ has been added to the FR acquisition. The acquired LR reflectivity can be expressed as follows:

$$s_\nu = \nu \cdot s + h_2 + n_L$$  \hspace{1cm} (25)

$n_L$ being a decorrelation noise term in the LR image. Coherence can then be found by combining (19) and (23–25), and assuming white reflectivity, $v$

$$|\chi| = \left| \frac{E[\nu \cdot s + h_2 + n_\nu \cdot s + h_2 + n_\nu \cdot s + h_2 + n_L]}{E[|\nu|^2]} \right| \approx P_{d_2} E \left[ \exp \left( - j \frac{B_{fr} \Delta \phi(D)}{B_{fr} \Delta \phi} n_0 \right) \right] \hat{s}_2$$  \hspace{1cm} (26)

$P_{d_2}$ being the power of reflectivity after filtering in the LR bandwidth, $s_2$, $P_{d_2}$ the power of noise $n_\nu$ filtered in the LR bandwidth, and $P_{n_\nu}$ the power of noise $n_\nu$. If we assume same noise power for $n_L$ and $n_\nu$, then (26) becomes:

$$|\chi| = \left| \frac{E \left[ \exp \left( - j \frac{B_{fr} \Delta \phi(D)}{B_{fr} \Delta \phi} n_0 \right) \right] \hat{s}_2}{\sqrt{P_{d_2} + P_{n_\nu}} \sqrt{P_{d_2} + P_{n_\nu}}} \right| \approx \exp \left( - j \frac{\sigma^2_n}{2} \right)$$  \hspace{1cm} (27)

where

$$\sigma^2_n \approx E \left[ \frac{h_2 B_{ref} \Delta \phi(D)}{B_{ref} \Delta \phi^2} \right]^2$$

The decorrelation noise in the final InSAR is the result of the three noise contributions in (27):

(1) the term at denominator is the noise in the “reference” interferograms, scaled by the ratio of the baselines and by the ratio between LR and FR bandwidth;

(2) the noise in the FR acquisition, again scaled by the ratio between LR and FR bandwidth;

(3) the noise in the LR acquisition.

Therefore, though a constraint on the quality of the reference interferogram could be derived from (27), no appreciable loss of quality – compared with that of LR ScanSAR acquisition – is expected, provided the interferometric pair is properly chosen (short repeat time) and unwrapped with no errors.

2.4.3 Results from simulations

Fig. 5 shows an example of the performance that can be achieved by compensating topography with an exact / approximate DEM. Here the zbs technique has been tested on a simulated dataset by assuming a very low resolution mode, with $\nu_0 \approx 800$ m, similar to ASAR Global Monitoring Mode. A distributed target was simulated by randomly placing point scatterers on the 1D topography shown in Fig. 5a. A pair of InSAR acquisitions were obtained by varying baselines in the range 20–300 m and terrain slopes within 30°. Fig. 5b shows the result of combined LR/FR interferometry, where the FR image was compensated only for flat earth before low-pass filtering (in the LR band) and computing the interferogram. Decorrelation is strong, except for the cases of flat earth or small baseline ($B_{ref} = 20$ m). Fig. 5c shows the result achieved with the zbs technique and the exact DEM: decorrelation was cancelled everywhere, up to the coherence level that would be achieved by processing two FR images. Finally, an error surface was superposed on the DEM by adding a gaussian, white noise ($\nu_0 = 10$ m) every 100 m, and then interpolating to the finer SAR grid: as Fig. 5c shows, and in agreement with (21), decorrelation could be removed only for baselines up to 200 m.

![Figure 5](image)

Figure 5: Simulations of combined FR / LR interferometry, 1D case. A very low resolution, $\nu_0 \approx 800$ m was assumed for LR. (a) DEM; (b) coherence (%) achieved without DEM compensation (but for a constant shift); (c), (d) coherence achieved by compensating topography by means of zbs technique and assuming respectively exact DEM and a DEM std error of 10 m.

2.5 Implementation

The generation of combined LR/FR differential interferograms is detailed in the block diagram of Fig. 6.

First of all, the LR ScanSAR raw data should be phase preserving complex focused, e.g. by using any algorithm in literature ([13, 14, 16]). Then azimuth looks should be separated: a single look is the contribution of one burst. In the focused image, looks are identified by the diagonal shapes in the time-varying power spectrum density contour plot of Fig. 7. As the figure shows, a scatter at a certain azimuth contributes to several looks: each look corresponds to a different portion of the target spectrum. Looks separation can be made by an azimuth variant filtering, as described in [14]. One burst (or look) gives a strip as wide as the synthetic aperture. Strips from different looks should be processed separately and averaged only after interferogram formation, otherwise, intermediate modulation decorrelation could arise in the Hermitian multiplication.
The focused image should then be processed to extract looks that match, in space and frequency, the LR ScanSAR ones. Frequency domain matching between the two images in achieved by steering the FR acquisition to a zero baseline (zbs). Space co-registration is then performed as for conventional interferometry [1, 17] by registering it to the LR reference (and with the same sampling). Nominal attitude information is supposed to be adequate (for the current and forthcoming system) for a first guess, refinement could then be performed by iterative techniques that maximize coherence as a figure of merit.

The zero baseline steering is performed according to (11), whose 2D extension is straightforward:

$$v_2(r) = \left[ v_1(r, x) \cdot \exp \left( \frac{4\pi f_0}{c} (R_2(r, x) - R_1(r, x)) \right) + \hat{h}_2(r, x) \right] + \hat{h}_2(r, x)$$

$$\Delta R(r, x) = R_2(r, x) - R_1(r, x)$$

being the DEM-derived difference in the two sensor-target travel paths. In (28) the azimuth varying bandpass filter, $\hat{h}_2$ accounts for the narrow range and azimuth bandwidth of the ScanSAR acquisition:

$$\hat{h}_2(f_r, f_x; l, x) = \text{rect} \left( \frac{f_r}{B} - f_0 \right) \text{rect} \left( \frac{f_x}{B} - f_0 \right)$$

The central azimuth frequency $f_0(l, x)$ depends on the look, $l$, and shifts with azimuth, as shown in Fig. 7. It can be computed on the basis of the Doppler rate and the acquisition timeline. For the actual implementation of azimuth varying filtering see reference [14]. Note that azimuth variant filtering applied to FR focused data should be synchronized with the ScanSAR pattern, to achieve a perfect match between corresponding looks in the two acquisitions. In practice, the same $f_0(l, x)$ should be used for the space-variant filtering the two data-sets (see Fig. 6). This can be done by estimating the value of $f_0$ at a certain azimuth on the ScanSAR focused image, by means of power spectrum techniques, and then using that value to process the FR image.

Once matching looks have been extracted from both data-sets, looks interferograms can be easily generated by Hermitian multiplication, and then averaged to get the final (differential) multi-look interferogram. If coherence estimate is required, scalloping should be compensated for. In fact, a possible mismatch in the Doppler Centroid of the two acquisitions would introduce a different azimuth varying amplitude modulation that would strongly bias the coherence estimate.

Finally, if an efficient implementation is requested, the proposed technique can be improved: (a) by subsampling (in azimuth) FR data after space variant filtering, according to the output resolution, and (b) by focusing, space variant filtering and subsampling, all at the same time, the LR ScanSAR data i.e. by implementing one of the algorithms in literature[14, 16].

### 3 Frequent monitoring system performance

The DInSAR technique just proposed could usefully exploit the wide swath-low-resolution ASAR modes for a frequent monitoring system. The system will first make a reflective data-base of the area to be monitored. Then, each time a new image is acquired, it is combined with a proper one in the data-base and with an existing DEM to provide the requested differential interferogram.

The large swath coverage of ScanSAR system ensures frequent monitoring, with revisiting time:

$$\Delta t = \frac{40^{10} \cos(\theta)}{2 \Delta R \eta_p} \text{ days}$$

where $\eta_p$ being the number of revolutions per day, $\Delta R$ the total swath that can be accessed within one pass and $\theta$ the latitude of the area to be observed. Notice that both ascending and descending orbit coverage is assumed in equation (30). For example, for ASAR sensor $\eta_p = 501/35$ and $\Delta R = 400$ km, the revisiting interval is distributed with latitude as follows:
If the system is intended to monitor unpredictable events over large areas, by using the LR mode, then the reflectivity data-base should contain full resolution images, acquired by different view angles, so that for each new acquisition, there is at least one image in the data-base with a suitable baseline to perform interferometry. If the *zbs* technique is exploited, we can assume the same “useful” baseline limit of FR interferometry, derived from (3). The complexity of the system lies mainly in the number of SAR images required to build up the minimal “data-base”. This depends on the orbit span (i.e. its drift due to air drag and gravitational effect [15]) and the number of sub-swaths considered, i.e. the desired revisit time. For an orbit span \( \leq L_{\text{max}} \), and a useful baseline \( B_u \), then at least

\[
N = \frac{L_{\text{max}}}{2B_u}
\]

(31)

images for each sub-swath should be gathered to guarantee useful interferogram generation. For the ASAR case, where an orbit span of \( \pm 1000 \) m can be assumed\(^2\), six SAR images (3 ascending orbits, + 3 descending) are necessary to cover a 120 km swath and the corresponding revisit time is about 5 days. For larger orbit spans or shorter revisiting times, the number of images increases. One of the critical points is the time required to get the (equal orbit spaced) images of the database. In the case of random orbit sampling the largest interval has an average value \( \Delta L \):

\[
\Delta L = \frac{L_{\text{max}}}{M} \left( \log(M) + C \right)
\]

(32)

\(L_{\text{max}}\) being the orbit span, \(M\) the number of images and \(C = 0.577\) the Euler constant [20]. The number of images that should be acquired to build the data-base has been computed for different combinations of revisiting time and orbit span, assuming randomly spaced orbits, and is shown in the bar-plot of figure 8. Note that for the combinations corresponding to the larger orbit span and the shorter revisiting time, the acquisition time for the whole data-base would require years.

As a different strategy, one can create a LR data base and revisit, *when necessary*, the location by using the FR mode, interfering with the proper LR image in the data base [11]. In this way there is the advantage of a smaller

\(^2\)For ERS case, that has a similar orbit control accuracy, 1 km orbit span covers 70%–98% of the interferometric acquisitions, [15].
Figure 9: Multi-look detected values of test site Etna II, obtained: (a) FR mode, (b) LR mode, resolution $150 \times 50$ m, (c) LR mode, resolution $800 \times 800$ m.

Figure 10: Coherence maps referred to test site I, Mt. Etna. (a) FR full-resolution interferometry; (b) FR-LR interferometry by compensating topography for flat earth and (c) by applying the proposed zs technique.

Fringes of the earthquake’s area are shown in either high resolution and low-resolution modes in Fig. 12. It must be noted that - if measured on a pixel-by-pixel basis, the quality of the two interferograms is comparable. However, on averaging the FR/LR interferogram to obtain the same resolution as the FR/FR one, the SNR improves approximately in proportion to the square root of the resolution ratio (according to (27)); an effect that is quite evident on comparing Figs. 12.a and 12.b.

A comparison of the FR/LR interferograms achieved in the two different ways just discussed can be seen in Fig. 13.a, that shows a map of the coherence differences. As expected, the zs technique gives a better SNR in non-flat areas, and this is quite obvious when comparing the coherence differences with the local slopes map in Fig. 13.b. The advantage of using zs over sloped terrain is highlighted also in Fig. 14, that plots the average coherence measured for each local slope (and by excluding the most decorrelated areas).

As expected, the low-resolution interferogram obtained with the zs technique gives roughly the same quality as the full-resolution one. Yet for the interferogram achieved by simply compensating for flat earth, the decorrelation increases with slope, according to the expression: $\gamma = \gamma_s \cdot \sin(f_s)$, where $\gamma_s$ is the scene coherence and $f_s$ the spectral shift due to slope [4, 14].

Finally, a very low resolution differential interferometry was achieved by simulating the $800 \times 800$ m resolution mode already discussed in this paper. Fig. 15 shows the fringes achieved with and without zs, in this second case the terrain slopes caused total decorrelation whenever the area in the large resolution cell was not flat. However, once these effects were removed, an adequate detection of the coseismic motion fringes was possible.

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Figure 12: Differential interferometry of Landers earthquake, obtained (a) by two full resolution acquisitions and, (b), by combining a full-resolution and a low resolution acquisition with the proposed technique.

Figure 13: (a) Difference between the coherence maps obtained by the proposed $z_f s$ technique and that obtained by performing flat earth compensation only. (b) Local slope map (measured in the steepest direction and then scaled by the sign of range slopes).

Figure 14: (a) Average coherence as a function of local slope, measured for the cases: full resolution, low resolution + $z_b$s, low resolution obtained by compensating for flat earth only; (b) number of samples found for each slope.

Figure 15: LR/FR combined interferogram, obtained by exploiting an acquisition with a resolution of 800 x 800 m: (a) by compensating for flat earth and (b) by exploiting $z_b$s technique.
6 Conclusions

A technique has been presented to combine LR/FR acquisitions and a FR DEM to achieve high quality differential interferograms. This technique removes completely the decorrelation due to topography and volume scattering effects up to the limit implied in the FR interferometry. The availability of LR interferometry is thus extended to wider baselines (the same baseline range as FR interferometry), provided that DEM local accuracy is in the order of a few meters.

The FR/LR combination has led to the design of two frequent revisiting monitoring systems. The first one, suited for detecting unpredictable hazards, requires a large data base (~10 FR images for each site) and a significant time to build it. The other, suitable for routinely monitoring a specific site, requires a very compact data base (a single LR image of each site) that can be built in a relatively short time (some months for ASAR).

The major limits to these systems are in the detail of the differential phenomena to be monitored (that should be very smooth), and in the presence of temporal decorrelation, that limits availability to long term stable targets. Validation of the technique by means of ERS data has shown significant results, particularly in the case of the Landers earthquake.

1 Approximations in the synthesis of zero baseline

The basic expression of the zero baseline synthesis, (9), has been derived within two main approximations: the monochromatic approximation, in (7) and the use of the bandlimited reflectivity \(\gamma\) in place of the exact one, \(\nu\) in (9). The errors estimated here are in terms of coherence [21] between the approximated reflectivity and the exact one:

\[
\gamma = \frac{\mathcal{E} [\tilde{v}_2 \tilde{v}_2^*]}{\sqrt{\mathcal{E} [\tilde{v}_2^*] \mathcal{E} [\tilde{v}_2]}}
\]  

(33)

The coherence can be expressed in the frequency domain, by applying Parseval identity, and by exploiting the definition of \(\tilde{v}_2\), in (5):

\[
\gamma = \frac{\mathcal{E} \left[ \int \tilde{v}_2 (f) \tilde{V}_2 (f) df \right]}{\sqrt{\mathcal{E} \left[ \tilde{v}_2^*(f) \tilde{V}_2 (f) df \right] \mathcal{E} \left[ \tilde{V}_2 (f) df \right]}} = \frac{\mathcal{E} \left[ \int \tilde{v}_2 (f) \tilde{V}_2 (f) df \right]}{\sqrt{\mathcal{E} \left[ \tilde{v}_2^*(f) \tilde{V}_2 (f) df \right] \mathcal{E} \left[ \tilde{V}_2 (f) df \right]}}
\]  

(34)

In the case of monochromatic approximation, we apply (34) by assuming \(\tilde{v}_2\) as in (7):

\[
\gamma = \frac{\mathcal{E} \left[ \int \int v(r) \exp (jKf r) \exp (jKf \Delta R(r)) dr \int v^*(s) \exp (-jKfrs) ds \right]}{\sqrt{\mathcal{E} \left[ \int \int v(r) \exp (jKf r) dr \int v^*(s) ds \right] \mathcal{E} \left[ \int \int \tilde{v}_2 (f) df \right]}} = \frac{\mathcal{E} \left[ \int \int v(r) v^*(s) \exp (jKf \Delta R (r)) dr ds \right]}{\sqrt{\mathcal{E} \left[ \int \int v(r) v^*(s) dr ds \right] \mathcal{E} \left[ \int \int \tilde{v}_2 (f) df \right]}}
\]

In (35), yielding

\[
\frac{\Delta \phi}{4 \pi f_0 / c} = \Delta \theta \frac{q \sin \theta}{R_0 = 300 \text{ m}} = \frac{q}{6000}
\]

The displacement \(\Delta R\) is in the order of a few centimeters, and negligible when compared to the slant range resolution, \(\rho\) (25 m for LR) in (35), yielding \(\gamma \approx 1\). This justifies the monochromatic approximation for the order of resolution we are currently assuming.

The other approximation, involved in the synthesis of zero baseline, is the use of the FR, bandlimited reflectivity (within the bandwidth \(B_{\nu}\)) instead of the “true” reflectivity, \(\nu\) in (9). We start from (34), and substitute the terms in (12,13):
\[ \gamma = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f) \, df}{\int \hat{S}(f) \, df} = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f - \eta) \, df}{\int \hat{S}(f - \eta) \, df} \]

Eventually, we assume that reflectivity components at different frequencies are not correlated (as occurs for a wide scatterer), and we define as \( P_V, P_S \) the power spectrum densities of the reflectivity \( v(r) \) and the “fringes” term \( s(r) \):

\[ \gamma = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f) \, df}{\int \hat{S}(f) \, df} = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f - \eta) \, df}{\int \hat{S}(f - \eta) \, df} \]

Note that the difference between the two terms in the fraction (36) is only in the integration intervals. The role of the power spectrum of the synthetic fringes, \( P_S \), is that of spreading the contribution of the convolution \( P_V \ast P_S \) outside the FR bandwidth, and thus to decorrelate. In particular, for white slant-range reflectivity, (36) becomes

\[ \gamma = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f) \, df}{\int \hat{S}(f) \, df} = \frac{\int [V(f) - \bar{V}] H(f) \hat{S}(f - \eta) \, df}{\int \hat{S}(f - \eta) \, df} \]

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\( s^2_\delta \) being the power of \( s(r) \) and \( P_{sm} \), its "smoothed" power spectrum, where smoothing is actually a frequency domain averaging with a window of size \( B_{2, \delta} \). As a simple result, (37) predicts that the total decorrelation due to band-limiting \( s(r) \) depends on the ratio between its power in the FR band \( B_{2, \delta} \) and its total power.

The power spectrum \( P_S \) can be estimated once a statistic of the terrain slopes is given: in fact for the stationary FM modulation assumption \( P_S \) is proportional to the probability distribution of its instantaneous frequency (e.g. slope). The slope statistics have been measured on several DEM available (mostly USGS DEM [18] from many different sites in US), these were transformed into slant range and the power spectrum \( P_S \) was derived by exploiting (16). As an example, for a rather large baseline of 300 m, 80% of the range power spectrum was found (in all the topographies exploited) within the bandwidth \( f = f_0 \leq 6000 \mathrm{Hz} \), centered on the spectral shift for flat terrain \( f_0 = \pi / 2 \mathrm{MHz} \), whereas 80% of the azimuth power spectrum was within \( f_0 / 13 \) in range and only 1/50 in azimuth. This justifies the 1D assumption made in this paper.

- The decorrelation due to azimuth slopes is negligible with respect to range (once \( s^2_\delta \) is used) since the ratio between the bandwidth of \( P_S \) and that of full resolution is 1/13 in range and only 1/50 in azimuth. This justifies the 1D assumption made in this paper.

- The volume scattering decorrelation found with \( s^2_\delta \) is close to that measured from a FR/FR interferogram, in fact the effective interferogram bandwidth does not increase too much (2.6 MHz with respect of 2 MHz for the larger baselines value).

Finally, we may assume that the terrain is rugged, so that the quasi-stationary assumption does not apply. Here we can use the expression of the Carson bandwidth for frequency modulated processes, i.e. by adding the bandwidth of the local ruggedness (or “volumetric effects” [12]). The obvious result of (37) is that, when the autocorrelation extent of volumetric scattering is lower than the FR resolution, decorrelation is found to be proportional to the ratio of the power scattered by volume and the total power scattered.