Adaptive Removal of Azimuth Ambiguities in SAR Images

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Abstract—We introduce an innovative algorithm that is capable of suppression of strong azimuth ambiguities in Single Look Complex (SLC) SAR images, to be used both for detected products and for interferometric surveys. The algorithm exploits a band-pass filter to select that portion of the azimuth spectrum that is less influenced by aliasing, the one that corresponds to the nulls in the replicated azimuth antenna pattern. The selectivity of this filter adapts locally depending on the estimated ambiguity intensity: the filter is more selective in presence of strong ambiguities, and becomes all-pass in absence of ambiguities. The design of such filter frames in the Wiener approach with two different normalization options, depending on the use of the SLC image, either for getting multi-look averaged, detected products or for interferometric applications. Preliminary results achieved by processing ENVISAT ASAR data are provided.

I. INTRODUCTION

In Synthetic Aperture Radar, azimuth ambiguities are caused by the aliasing of the Doppler phase history of each target, that is sampled according to the sensor azimuth sampling frequency, $f_{sa}$, which equals the Pulse Repetition Frequency (PRF) [1]. A list of the acronyms adopted in the paper is in Tab. I. Usually, such ambiguities are kept below a reasonable level by exploiting the Azimuth Antenna Pattern (AAP), that acts as a sort of anti-alias pre-filter, by illuminating ground patches in the direction of the desired contributions and blocking returns from “interfering” angles. However, the large dynamic of SAR reflectivity, imposes strong constraints on the azimuth ambiguity ratio (both peak and distributed), and this turns into the design of very “long” antennas, usually up to 10 m, a cost driver for spaceborne sensors. The suppression of azimuth ambiguities is then fundamental in the role of current and future space-borne sensors, both for detected images and for interferometric applications [2]. In the paper we propose a post-processing that can be applied to any Single Look Complex (SLC) image to adaptively suppress azimuth ambiguities. In the literature, two different processing approaches have been proposed, based either on power nulling or on in-phase cancellation. The first case represents the most classical approach, it uses a band-pass filter to select, from the whole azimuth spectrum, those narrow-band looks that are less influenced by ambiguities. This approach, that we will refer to as ”Selective Filtering” (SF), is quite effective in the sense that it provides almost a complete removal of aliasing, at the cost of a strong loss in the azimuth bandwidth and resolution [1]. The second approach, the in-phase cancellation, tries to reconstruct the sources that generated the alias in a sort of “blind deconvolution”, and exploits this knowledge and the Signal-to-Alias transfer function to provide the optimal in-phase subtraction of the ambiguities [3], [4]. Such technique is quite effective as long as the sources can be estimated from the data, like for point scatterers [3]. It is however subject to failure in a general context, when the spectrum of SAR reflectivity becomes uncorrelated, preventing a proper reconstruction of the ambiguity structure.

In this paper, we present a general-purpose scheme, that takes the robustness of the Selective Filtering approach, but implements the filtering in a space-adaptive way. The design of the filter is based on Wiener Minimum Mean Squared Error (MMSE) solution, slightly modified to retain the radiometric calibration. The technique developed exploits as a first step the Selective Filter approach whose goal is to find the areas populated by ambiguities (the ”ambiguity map”). A second step is then implemented to provide an estimate of both signal and ghost backscatter: this step is the most critical, as Wiener filters are quite sensitive to SNR. The last step is the design of the adaptive selective filter and its implementation for removing the ambiguities. Finally, results achieved by the proposed techniques are shown in some experimental cases, referred to ENVISAT-ASAR images.

II. AZIMUTH AMBIGUITIES

Let us refer to the geometry drawn in Fig. 1, where the SAR sensor flying on a straight path (with velocity $v$) is represented together with two point targets on the ground $P$ and $Q$. The assumption of straight path will be removed in the appendix. The figure shows the contributions coming from the target $Q$, that is displaced by an azimuth angle $\Delta \psi = \psi_Q - \psi_P$ respect to that of $P$. For small bandwidth SARs, we get the usual linear relation between azimuth frequency and angle, at wavelength $\lambda$:

$$f_a = -\frac{2v}{\lambda} \sin \psi \simeq -\frac{2v}{\lambda} \psi$$

(1)

The two contributions shown in the figure become indistinguishable when the azimuth frequency corresponding to the angle $\Delta \psi$ is equal to a multiple of the sampling frequency, $f_{sa} = PRF$:

$$\frac{2\Delta \psi}{\lambda} = -\frac{f_{sa}}{v} \Rightarrow \Delta \psi = -\frac{\lambda}{2v} f_{sa}$$

(2)
These contributions add up coherently, with weights proportional to the two way Azimuth Antenna Pattern (AAP): $G(\Delta \psi)$. The AAP in thus acting as a sort of anti-alias prefilter.

A peculiar feature of the quadratic phase history associated with (1) is that these aliased contributions also have quadratic phase and therefore they are also focused, but have ghosts in the wrong position. That is, ghosts are displaced in slow azimuth time by a footprint (or integer multiple):

$$\Delta \tau = f_{sa} / f_R$$  \hspace{1cm} (3)

where $f_R$ is the Doppler rate (Hz/s).

### A. Ambiguity Analysis

Let us analyze with more detail the generation of ghosts in SAR focusing. We can take advantage by the focusing expression in the Range-Doppler domain, that is quite good for small squint angles (the case we are interested to). The SAR acquisition transfer function can be written as follows (see for example [5]):

$$G(\psi (f_{sa})) \cdot \exp \left( -j \frac{\pi c r_0}{2 v_0 f_{sa}} f^2 \right) \exp \left( j \frac{\pi}{f_R f_{sa}} f^2 \right).$$  \hspace{1cm} (4)

where $G(\psi (f_{sa}))$ is the AAP at the angle $\psi$, that in turn depend on the azimuth frequency $f_{sa}$, according to (1), $r_0$ is the closest approach distance, $f_R$ the Doppler rate and $f$ the range frequency, conjugate to the range, fast-time domain. The two exponential in (4) represent range migration and Doppler history respectively. For the azimuth antenna pattern, we may well exploit the usual sinc-squared approximation:

$$G(f_{sa}) = \text{sinc}^2 \left( \frac{L}{2v} (f_{sa} - f_{DC}) \right).$$  \hspace{1cm} (5)

where $f_{DC}$ is the Doppler centroid and $L$ the antenna length, and $\text{sinc}(x) = \sin(\pi x) / (\pi x)$. Let us now assume focus is obtained by exploiting a filter with phase\(^1\) matched to (4):

$$\exp\left(-j \frac{\pi c r_0}{2 v_0 f_{sa}^2 f} \right) \exp\left( j \frac{\pi f_{sa}^2 f}{f_R} \right) \text{rect}\left( \frac{f_{sa}}{f_{sa}} \right).$$  \hspace{1cm} (6)

where * denotes complex conjugate, and we have used the notation $f_{sa}$ for the azimuth sampling frequency: $f_{sa} = P/R$. The rect function:

$$\text{rect}(x) = \begin{cases} 1 & |x| < 0.5 \\ 0 & \text{elsewhere} \end{cases}$$

has been introduced to bound the focusing operator in the azimuth spectrum. Let us apply the operator to the signal coming from a single target: although its spectra extends indefinitely, we limit its contribution to the first left and right folded segments in the frequency domain:

$$X(f, f_{sa}) = \sum_{k=-1}^{1} P(f_{sa} + k f_{sa}) G(\psi (f_{sa} + k f_{sa})) \cdot \exp\left(-j \frac{\pi c r_0}{2 v_0^2 f_{sa}^2 f} (f_{sa} + k f_{sa})^2 f \right) \cdot \exp\left( j \frac{\pi}{f_R} (f_{sa} + k f_{sa}) \right).$$

where $P(f)$ is the target spectrum. We cross-correlate with the focusing operator, getting three terms:

1. the focused contribution that is proportional to:
   $$X_0(f, f_{sa}) = P(f_{sa}) G_0(f_{sa})$$  \hspace{1cm} (7)
   where $G_0(f_{sa}) = G(\psi (f_{sa})) \text{rect}\left( \frac{f_{sa}}{f_{sa}} \right)$

   that is the baseband contribution of the spectrum of the target (we assume that the Doppler centroid is in the baseband, e.g. yaw-steering mode);

2. two terms responsible for left and right ghosts. The ghost on the right can be obtained by assuming $k = +1$ (similar derivation can be made for the ghost on the left):
   $$X_L(f, f_{sa}) = P(f_{sa} + 2 f_{sa} f) G_L(f_{sa})$$  \hspace{1cm} (8)
   where $G_L(f_{sa}) = G(\psi (f_{sa} + 2 f_{sa}))$\(^2\)\exp\left(-j \frac{\pi c r_0}{2 v_0^2 f_{sa}^2 f} (f_{sa} + 2 f_{sa}) f \right) \cdot \exp\left( j \frac{\pi}{f_R} (f_{sa} + 2 f_{sa}) \right) \text{rect}\left( \frac{f_{sa}}{f_{sa}} \right)$

The four factors in (8) can be interpreted as follows:

- $P(f_{sa} + 2 f_{sa})$ is the high frequency components of the target spectrum, hence its reflectivity,

- $G(\psi (f_{sa} + 2 f_{sa}))$ is the weight provided by the AAP for the ambiguous contribution, it should be zero for an ideal antenna (no sidelobes),

- the first exponential is a residual, azimuth-frequency dependent migration. Its effect is mainly a migration, and a very slight defocusing,

- the second exponential can be decomposed in a constant term and a linear phase that is responsible for the azimuth (right) shift of the azimuth ghost already in (3).

As an example Fig. 2 plots the impulse response achieved by simulating the focusing of a point target in the ENVISAT-ASAR like geometry. The figure draws the target with its first left and right ghosts, and two further echoes hardly noticeable (due to the antenna pattern attenuation); the major effects are indeed the azimuth and range shifts of the echoes, whereas the defocusing is irrelevant.

### II. AZIMUTH AMBIGUITY REDUCTION

Two classes of techniques have been proposed in literature to reduce ambiguities basing on data processing: the “matched filter approach” and “look shaping”.

The “matched filter approach” removes ambiguities by exploiting the signal-to-ambiguity transfer function, just the ratio between (7) and (8), and performing in-phase cancellation (see [3] and [4]). Although this approach cannot work in a general case, where the high frequency component of the target’s reflectivity, $P(f_{sa} \pm f_{sa})$ are different from the baseband contributions, $P(f_{sa})$, it is well suited to specific cases, like point scatterers, for which $P(f_{sa}) \approx$ constant. In practice, the capability of the matched filter approach in removing ambiguities diminishes as long as the target behave as a complex structure, with several scattering centers, and fails completely in the case of fully developed speckle, that is frequency-to-frequency uncorrelated. An example of such target is in Fig. 3,
compared with its ghost (ENVISAT-ASAR datasets). The two images come form different spectral contributions, let us say \( P_L \) and \( P_0 \), and are clearly different in many details, making in-phase cancellation impossible in this case.

On the other hand, the technique that we propose represents a complementary approach, based on the "look-shaping" technique, that consists basically in a frequency domain nulling of ambiguities energy [1]. The rationale of this technique is explained in the plots of Fig. 4, that shows the AAP of a typical spaceborne sensor (ENVISAT-ASAR), converted into the frequency domain by the linear angle-frequency mapping (1):

\[
G_\psi \propto \sin^2(\psi L/\lambda)
\]

(9)

In the figure, the first folded replica is superposed. This replica is responsible for the ambiguity (together with other replicas). The baseband domain is shaded for a quick evaluation of the ambiguity energy \( x^2 \). Notice, that the maximum ambiguity energy is located close to \( f_{sa}/2 \), e.g. Nyquist frequency, as expected (here zero Doppler centroid is assumed). However, there is a specific spectral region where the folded antenna is responsible for the ambiguity (together with other replicas). In the figure, the first folded replica is superposed. This replica is responsible for the ambiguity (together with other replicas). The baseband domain is shaded for a quick evaluation of the ambiguity energy \( x^2 \). Notice, that the maximum ambiguity energy is located close to \( f_{sa}/2 \), e.g. Nyquist frequency, as expected (here zero Doppler centroid is assumed). However, there is a specific spectral region where the folded antenna has a notch, hence no ambiguity energy is located there. This notch corresponds to the null of the AAP, shifted of \( f_{sa} \):

\[
\psi_{null} \simeq -\lambda/L \Rightarrow f_{null} = f_{sa} - 2\psi
\]

The null frequency is 200 Hz for ERS and ENVISAT-ASAR looking at 23° (\( f_{sa}=1700 \) Hz), and shifts to 500 Hz for ENVISAT-ASAR looking at 33° (subswath IS5, \( f_{sa}=2 \) kHz). A simple ambiguity nulling can be achieved by band-pass filtering the focused SLC for \( \pm f_{null} \), where sign "\( \pm \)" accounts for the both the first left and right replica. It is worthy to note that this is approximately the baseband for ERS and ENVISAT-IS2 subswath, in fact it is known that ambiguity is reduced just by averaging the complex product, but not for ENVISAT-IS5.

An example of ambiguity "notching" is provided in Fig. 5 that refers to the same area subject to ambiguities as in Fig. 3, before and after applying the selective filter whose spectrum is in Fig. 6 (the details of the filter design will be explained in the next section).

It is quite clear that the ambiguities have been cancelled, however the band-pass is reduced to approximately 20% of the original. Such reduction in the complex image bandwidth results either in a loss in the geometric resolution, or in a loss in the radiometric resolution, if the image is detected and multi-look averaged. This is the case shown in Fig. 5, where the ambiguity is removed at the cost of an evident increase in the speckle noise.

III. ADAPTIVE AMBIGUITY CANCELLATION

The approach proposed here exploits the band-pass filtering just discussed, however implementing it adaptively, i.e. to change form pixel to pixel the notch bandwidth, according to the expected ambiguity energy. Wiener filters are well suited for such purpose, having the advantage of being computed efficiently and providing the optimal estimate (in the Bayesian sense) for the case of Gaussian sources, as for SAR speckle.

The scheme of the proposed Wiener-based filtering is detailed in Fig. 7: the figure draws the block model of the forward model, responsible of ghosts, and the Wiener filter that removes those ghosts. In all the blocks capital letters refer to transfer functions in the azimuth frequency domain, \( f_a \). Let us assume to remove the ambiguity at the generic azimuth pixel \( n \), and to exploit \( 2N+1 \) samples of the SAR image. We do it by means of the kernel \( h_w \):

\[
\hat{p}_0(n) = \sum_{k=-N}^{+N} x(n-k) h_w(k)
\]

(10)

\( x \) being the SAR complex reflectivity, affected by alias, and \( \hat{p}_0 \) the cleaned signal. Eventually, we introduce the vector notation \( h_w \) (size \( [2N+1] \)) for the kernel \( h_w \), and the column vector \( \hat{p}_0(n) \) is defined as follows:

\[
\hat{p}_0(n) = [\hat{p}_0(n-N) \ldots \hat{p}_0(n) \ldots \hat{p}_0(n+N)]^T
\]

(11)

where suffix \( T \) stands for matrix transposition and the dependence upon local index \( (n) \) has been introduced to remark its non-stationary behavior. In the same way we define the vector of the ambiguity-free signal, \( p_0(n) \) (see Fig. 7). Although the actual value of \( p_0 \) is unknown, we assume knowledge of its statistics: a multivariate Gaussian with the covariance matrix:

\[
R_0(n) = E[p_0(n)p_0^*(n)]
\]

where \( E[\ ] \) stands for expectation and \( * \) stands for Hermitian transposition.

Eventually, we define the contributions of the right and left ghosts (also shown in the figure) as the column vectors \( p_L(n) \) and \( p_R(n) \), each of them of size \( [2N+1,1] \) exactly like \( p_0(n) \). These contributions are also unknown, but we assume to know their covariance matrices:

\[
R_L(n) = E[p_L(n)p_L^*(n)]; \ R_R(n) = E[p_L(n)p_L^*(n)]
\]

We furthermore need the cross-correlation between the data and the unambiguous input, the column vector \( r_0(n) \) of size \( [2N+1,1] \), whose elements are defined as follows:

\[
r_0(i) = E[p_0(n+i)x^*(n)] \quad -N \leq i \leq N
\]

At each azimuth sample, \( n \), a Wiener filter \( h_w \) can be computed in order to minimize the distance (in L2 norm) between the reconstructed reflectivity and the "true", unambiguous, one:

\[
\arg \min_{h_w(n)} E[(\hat{p}_0(n) - p_0(n))^*(\hat{p}_0(n) - p_0(n))] \]

(12)

The filter is made adaptive to account for changes in both the energy of signal and ambiguity noise (that varies quickly), and in the AAP \( G(f_a) \), that varies slowly, depending on the Doppler parameters (5).

The MMSE problem (12,10) leads to the autocovariance equation (see [6]):

\[
(R_0 + R_L + R_R + \sigma_w^2 I) h_w = r_0
\]

(13)

\[
h_w = (R_0 + R_L + R_R + \sigma_w^2 I)^{-1} r_0
\]

(14)

\( I \) being the identity matrix.
A. Implementation

The implementation of the Wiener filter would require the knowledge of $R_L$, $R_R$, $R_0$ and $r_0$, that can be computed, for each azimuth bin, basing on the forward model defined in frequency domain in Fig. 7. The filter itself can be computed in the frequency domain and back-transformed, provided that we approximate the covariance matrixes in (13) as circulant. In that case, we can derive the Wiener filter by imposing uncorrelation between error and data directly in the frequency domain:

$$E[(H_w X - P_0) X^*] = 0$$

$$H_w = \frac{E[P_0 X^*]}{E[XX^*]}$$

where we have used capitals letter for the Fourier Transforms, and we have omitted the obvious dependency on the azimuth frequency, $f_a$. Expression (15) is the frequency domain counterpart of (13), involving the cross-spectra between the data and the unambiguous sources, $X_0(f_a)$, and the power spectrum of the data themselves. We should emphasize that the model here exploited, shown in Fig. 7, is an equivalent monodimensional one. The sources for right and left ghosts, $F_L$ and $F_R$ in the figure, are assumed as complex Normal processes, white and uncorrelated with data (however, we know that these sources are actually taken from the data with a proper azimuth and range shift). The frequency domain formulation in (15) implicitly requires that both $\sigma^2_L$, $\sigma^2_R$, and the variance of the source, $\sigma^2_a = E[y^2_0]$, are stationary within the filter extent, 2N+1. Although this may seem too approximated, we did not experience, in practical cases, relevant differences to support the non-stationary design made by (13), that requires a cost-effective matrix inversion for each new sample.

Let us provide a close form expression for the filter by exploiting the model in Fig. 7. We derive from (15) the following expression:

$$H_w = \frac{\sigma_a^2 G_0(f_a) \ast \sigma_x^2}{\sigma_x^2 |G_0(f_a)|^2 + \sigma_L^2 |G_L(f_a)|^2 + \sigma_R^2 |G_R(f_a)|^2 + \sigma_w^2}$$

$$= \left( G_0(f_a) + \frac{\sigma_L^2 |G_L(f_a)|^2}{\sigma_x^2 |G_0(f_a)|^2} + \frac{\sigma_R^2 |G_R(f_a)|^2}{\sigma_x^2 |G_0(f_a)|^2} + \frac{\sigma_w^2}{\sigma_x^2} \right)^{-1}$$

where $G_0$, $G_R$ and $G_L$ have been already computed in (7) and (8). The noise power $\sigma_w^2$ is required to avoid singularities in the inversion involved in (16). In our implementation we have assumed $SNR = \sigma_x^2/\sigma_w^2$ up to 30 dB. The filter was computed by (1) evaluating (16) on a fine grid in the frequency domain; (2) back transforming into time domain; (3) wrapping around the sequence to get a real representation (with time 0 at the center of the sequence); and, (4), by truncating the impulse response by means of a 30-40 samples Hann window. The filter length was derived in the worst case, as the most selective one, whereas practical filters were quite smaller that extent.

1) Selective filtering: A particular case of interest is the filter that provides the maximum removal of ambiguities see Figs. 5, 6. We derive this filter as a worst case of the Wiener filter, when alias noise is quite large compared to the signal. We then assume:

$$SNR = \frac{\sigma_w^2}{\sigma_L^2 + \sigma_R^2} \ll 1$$

$$\sigma_L^2 = \sigma_R^2 = \sigma_w^2$$

The first term in (16) can be ignored, leading to:

$$H_w \approx SNR^{-1} \left( \frac{|G_L(f_a)|^2 + |G_R(f_a)|^2}{|G_0(f_a)|^2} + \frac{\sigma_w^2}{\sigma_x^2} \right)^{-1}$$

We furthermore ignore the constant scale term $SNR^{-1}$, getting the space invariant$^2$ filter, that corresponds to the look shaping approach shown in Fig. 5: the removal of alias is derived at the cost of the poorest radiometric resolution.

2) Space Adaptive Filtering: The case of interest in this paper is the space-adaptive formulation of the filter. In principle we should recompute the filter at every azimuth pixel, $n$, iterating on all the azimuth lines, in practice we will show that filtering can be restricted to the sole area of interest. The space variation of the filter is provided by the local ASR: $\sigma_L^2(n)/\sigma_a^2(n)$, and $\sigma_R^2(n)/\sigma_a^2(n)$ involved in (16). The effect of these terms is shown in Fig. 8, that plots frequency domain response of filters designed with different levels of ambiguity suppression. Notice that the filters adapts to the increase in ambiguity energy becoming much more selective.

The most tricky aspect in such filters is just the accurate estimate of signal and alias noise powers. Both are quickly varying with space, demanding for a very local estimate. We do it according to the scheme in the block diagram of Fig. 9.

We first estimate the total backscatter and ambiguity energy on a local scale by ML averaging the given (SLC) image (we use 30 samples, azimuth $\times$ 15 samples, range). As the local reflectivity and the ambiguities are uncorrelated, we get:

$$\sigma_w^2 = \sigma_R^2 + \sigma_L^2 + \sigma_x^2$$

where $\sigma_x^2$ is the value just estimated.

We then assume that only one of the ghosts dominates at one time: either $\sigma_R^2$ or $\sigma_L^2$, thus all what we need is an estimate of $\sigma_x^2$, the ambiguity-free backscatter. We compute an image that is virtually ambiguity-free, by exploiting the filter (17), and we again perform the ML average, giving the result shown in Fig. 5. This is clearly a coarse estimate of $\sigma_x^2$, let us call it $\hat{\sigma_x^2}$; its resolution is too coarse for exploiting into (18), but it is sufficient to identify the areas where ambiguity predominates. For each pixel $P$ we select the location of the interfering targets (those whose ambiguities are placed close to $P$) by accounting for the proper range and azimuth shifts as shown in Fig. 10. We first evaluate the right and left ghost energy as $\hat{\sigma}_R^2(P) = g_r \cdot \hat{\sigma}_a^2(P_R)$, where $P_R$ is the possible interfering location, shown in Fig. 9, and the scale factor $g_r$ is derived from the forward model (7,8):

$$g_r = \left( \int \frac{|G_R(f_a)|^2}{|G_0(f_a)|^2} df_a \right)^{1/2}$$

$^2$Actually the filter changes smoothly according to the Doppler parameters, but can be assumed stationary in small images.
and similarly for \( q_t \).

Prior to performing any filtering, we estimate if \( P \) is severely affected by ambiguity, that happens when \( \sigma_R^2(P) \geq \sigma_z^2(P) \). In practice, it is possible to derive a-priori this ambiguity mask by a quick lookup of the ambiguity-free image: this speed up strongly the processing as it allows to filter only the areas of interest. An example is shown in Fig. 11, that plots the azimuth varying estimates of \( \sigma_z^2 \) (from the ambiguity-free image) and \( \sigma_R^2 \) (from the SLC) on a single azimuth line. In the figure, the area subject to ambiguity are clearly identified by the ambiguity mask.

Unfortunately, the estimate \( \sigma_R^2 \) so derived is not reliable enough to be exploited for computing the Wiener filter. We overcome the problem by assuming that the true backscatter is uniform in the area affected by ambiguity, therefore we interpolate the values of the SLC at the border of that area into the inner part, getting an estimate of \( \sigma_z^2 \), and then deriving \( \sigma_R^2 \) (or \( \sigma_R^2 \)) form (18). Thereafter, all the elements for computing the filter in (16) are known.

3) Filter normalization: A different filter normalization needs to be implemented depending on the use: either for interferometry or for detected amplitude. In the interferometric case, we need the most faithful reconstruction of the source reflectivity, with the minimal signal-to-noise ratio, thus (12) fits the goal, and the filter is exactly the one that comes out (13,16).

In the case of detected application, we need to preserve of the average backscatter, e.g. the radiometric calibration. In that case we need to normalize the filter for constant energy:

\[
h_v(n) = \frac{h(n)}{\sum |h_n|^2}
\]

An example of the results of Wiener filtering achieved by different normalization in presence of a strong ambiguity provided in Fig. 12.

IV. EXPERIMENTAL RESULTS

The generation of an adaptive Wiener filtered image according to the proposed technique is detailed in Fig. 13, that shows the images generated at various steps in the block diagram of Fig. 9. Notice in the figure the presence of targets in the sea (ships and similar bright targets), that are not cancelled even by the strongest Wiener filter (second image from top), but are quite more evident in the final, adaptive filtered image.

Another example, taken from the same datasets, is shown in Fig. 14, where a strong ambiguity of a bright target appears on a very dark background. Even in that case, the Wiener filter was able to reduce the ambiguity of more than 15 dB bringing the Signal-to-Ambiguity ratio from 32 to 50 dB.

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VI. CONCLUSIONS

A refined approach to remove ambiguities from a complex SAR image has been discussed. The approach exploits Wiener filters to perform a space adaptive estimate of the non-aliased contributions in the MMSE sense. This approach has been shown effective, demonstrating capabilities of removing alias and at the same time avoiding the loss in equivalent number of looks that would result from a look selection technique.

APPENDIX

The implementation of the Wiener filter requires a most faithful model for the signal and ambiguity ACF, and this ends up in the need for estimating accurately the AAP in the frequency domain. Let us model the geometry of a spaceborne SAR acquisition according to Fig 15. We assume the usual \( \sin^2 \) pattern for the AAP:

\[
G(\beta) \simeq \sin^2 \left( \frac{L_{az}}{\lambda} \sin \psi \right)
\]

where \( \psi \) is the generic azimuth angle between the satellite and the target. The problem is deriving the relation between \( \psi \) and the azimuth frequency. We assume the sensor on a circular orbit, and the target on a spherical earth, observed at time \( \tau = 0 \) by a squint angle \( \psi_0 \) (shown in the figure). We describe the sensor and target motion as a function of the slow time, \( \tau \):

\[
S = \begin{bmatrix}
R \sin (v \tau / R) \\
0
\end{bmatrix}
\]

\[
P = \begin{bmatrix}
-r \tan \psi_0 \\
-r \sin \theta \\
-r \cos \theta
\end{bmatrix}
\]

\( \theta \) being the off-nadir angle and \( r \) the closest approach distance. Eventually:

\[
v(\tau) = \frac{\partial S}{\partial \tau} = \begin{bmatrix}
v \cos \left( \frac{v \tau}{R} \right) \\
0 \\
-v \sin \left( \frac{v \tau}{R} \right)
\end{bmatrix}
\]

\[
P - S(\tau) = \begin{bmatrix}
-r \tan \psi_0 - R \sin \left( \frac{v \tau}{R} \right) \\
-r \sin \theta \\
-r \cos \theta + R \left( 1 - \cos \left( \frac{v \tau}{R} \right) \right)
\end{bmatrix}
\]

We can now compute the antenna angle in the plane containing the orbit and the sensor-target vector:

\[
\sin \psi(\tau) = \frac{v(\tau) \cdot (P - S(\tau))}{|v||P-S(\tau)|}
\]

\[
\simeq \sin \psi_0 - \frac{\tau}{r} v \left( 1 - \frac{r \cos \theta}{R} \right) \cos \psi + O(\tau^2)
\]

that has been approximated to the first term of its power series expansion. The expression found can be inserted into (19) to provide the antenna pattern illumination as a function of the target slow-time \( \tau \); we acknowledge the same expression that we would get for a rectilinear geometry, once that we introduce the definition of the “equivalent” velocity:

\[
v_{eq} = v \left( 1 - \frac{r \cos \theta}{R} \right)
\]

It can be noted that, although the equivalent velocity should be updated for range (due both to the off-nadir angle and the closest approach), its variation is indeed minor over a full frame, and can be kept constant.
REFERENCES


Azimuth ambiguity in SAR geometry. The target Q is aliased and its contribution will be superimposed to the one of the target P if the angular displacement is \( \Delta \psi = \frac{\lambda}{(2v) \cdot f_{sa}} \).

Table I

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<thead>
<tr>
<th>Acronym</th>
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<tr>
<td>AAP</td>
<td>Azimuth Antenna Pattern</td>
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<td>ACF</td>
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<td>MMSE</td>
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<td>PRF, f_{sa}</td>
<td>Pulse Repetition Frequency</td>
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<td>SAR</td>
<td>Synthetic Aperture RADAR</td>
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<td>SLC</td>
<td>Single Look Complex</td>
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<td>SNR</td>
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Fig. 2. Perspective view of the log-amplitude of a focused image, achieved by simulating a single scatterer and assuming ENVISAT geometry (swath: IS5). Notice the focused target, in the central part of the image, and its four replicas, that are also focused, but displaced in azimuth and range.

Fig. 3. Details of a ghost (on the left) properly scaled, compared with the target responsible of that ghost, (on the right). Although the shape on the left is represented on the right, the fine details of the two images are quite different.
Fig. 4. The azimuth antenna pattern and its first replica (responsible for aliasing) is shown for $f_{sa} = 1700$ Hz and $f_{sa} = 2000$ Hz, i.e., ENVISAT-ASAR IS2 and IS5. The ambiguous pattern has a notch at frequency $f_{NULL}$, that is close to zero only in the first case.

Fig. 5. Left: a multi-look averaged, focused SAR image near Flevoland, ENVISAT ASAR mission, swath: IS5 (orbit: 3354, October 21, 2002). The azimuth ambiguities are quite visible. Right: ambiguities have been removed by means of a band-pass filter, at the cost of an increased speckle noise.

Fig. 6. Band-pass filter used for canceling ambiguities in the dataset shown in Fig. 3 (for swath SS5, $f_{sa}=2$ kHz): impulse response (above), and its Fourier Transform (below).

Fig. 7. Block diagram of the proposed adaptive selective filter. The forward model (on the left) is exploited to derive the Wiener filter (middle) and then estimate the ambiguity-free source reflectivity, $\hat{P}_0$. $W$ is an additive noise term accounting for thermal and quantization effects.

Fig. 8. Frequency transforms of ambiguity-nulling filters for different level of ASR.

Fig. 9. Block diagram for the estimate of the local intensity of the ambiguity-free image.
Fig. 10. Above: geometry for computing the left and right range shift of the ghosts. In the scheme below, the gridded areas are those exploited to estimate ambiguity energy when filtering target at position P.

Fig. 11. Azimuth cut of a line in presence of ambiguity. $\sigma_y$ is the intensity estimated by ML averaging from the given SLC, $\hat{\sigma}_x$ is obtained by exploiting the selective filtering (complete ambiguity removal). The bottom line identifies the ambiguity mask, for which $\sigma_y > \hat{\sigma}_x$.

Fig. 12. Removal of ambiguity from a strong scatterer with (a) "normalization" for overall constant backscatter (e.g. to preserve radiometric calibration), (b) Wiener approach (to provide minimal noise reconstruction).

Fig. 13. From top to bottom: (1) the original multi-look averaged amplitude image with ambiguities, (2) the result of selective filtering, (3) the result of the proposed technique, (4) masked areas where ambiguities have been identified.
Fig. 14. Removal of ambiguity in the case of a very bright target over a dark background, the signal to ambiguity ratio is improved from 32 dB to more than 50 dB.

Fig. 15. Geometry for deriving the equivalent velocity.