High resolution spaceborne SAR focusing by SVD-STOLT

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Abstract—In space borne SAR, the orbit curvature may prevent the use of the ωk processor, causing artifacts that depend on both the extent of the orbit arc and the slant range interval. A viable solution has been derived by extending the SVD-STOLT approach proposed for geophysical applications to microwave SAR. The resulting processor has the same simple scheme as the ωk approach, but a different (numerical) computation of both the reference and the Stolt interpolation.

I. INTRODUCTION

Current and future technologies in space-borne Synthetic Aperture Radar (SAR) demand accurate phase-preserving processing at high resolution. In fact, the new SAR generation will retain the same significant swath depth of the past, but will exploit a larger orbital arc due to smaller antenna (TerraSAR-X, COSMO), longer wavelengths (ALOS, SAOCOM), availability of SPOT modes, and the eventual combination of these features. The phase preserving focusing of data gathered by large apertures and bandwidths, motivate the use of the ωk processor, imported from geophysics [1] and widely adopted by the SAR community [2], [3], [4]. However the processor was designed for straight orbit, and has known limitations for large swaths due to the variation of the Doppler Centroid (DC) with range [3].

In the following, we will assume the typical geometry of the next ESA sensor, GMES-Sentinel-1. The parameters are listed in Tab I and are well representative of future SAR, where the orbit will be sun-synchronous and provide short revisit time (12 days) at reasonable ground swath coverage (250 km).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis</td>
<td>707980 m</td>
<td>Inclination</td>
<td>98.2°</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.00118</td>
<td>Arg. perigee</td>
<td>90°</td>
</tr>
<tr>
<td>Cycle length</td>
<td>175 orbits</td>
<td>Repeat cycle</td>
<td>12 days</td>
</tr>
</tbody>
</table>

TABLE I

Mean-keplerian and other parameters for the GS1 orbit

A way to handle the spaceborne case is to assume an earth fixed reference, so that the targets’ motion is accounted for by a non-planar orbit. The resulting motion compensation problem can then be approached by one of the many known literature techniques. Modifications of the ωk schemes have been proposed, and are based on suitable pre or post processors [5], [6] or by interleaving several processing steps [4]. The complexity of these methods is justified by the irregularity of the track in the airborne case. Instead, spacecraft velocity is quite uniform, and a simple focusing scheme can be designed for that. This leads us to investigate the capability of the SVD-Stolt (SVDS) method, proposed in geophysics more than two decades ago [7], to handle the curved orbit.

II. IMPULSE RESPONSES AND HODOGRAPHS

In this section we evaluate the actual spaceborne SAR impulse response and the impact of focusing implemented by the Straight Orbit Approximation (SOA), the classical ωk approach. The geometry referred can be seen in Fig. 1: the notations are those assumed by Bamler in [2]. Let us define the reference system in the data (or signal) space by the slow time, fast time coordinates, (τ, t), and in the model space (the focused data) by the zero-Doppler time, zero-Doppler distance (τ0, r). The orbit eccentricity is so small it can be assumed that the sensor is moving at a constant velocity, vs; therefore the space variable x = vsτ0 is used for azimuth.

The impulse response function (IRF) of the SAR acquisition, for a point-like target δ(τ0, t − 2r/c) is expressed as follows:

$$h_s(\tau, t; \tau_0, r) = p\left(t - \frac{2R}{c}\right) \cdot \exp\left(-j\frac{4\pi}{\lambda} R\right),$$  (1)

λ being the carrier wavelength and p(t) the transmitted pulse, assumed range-compressed. R(τ; t0, τ0) is the target-to-satellite distance, the hodograph:

$$R(\tau; r, \tau_0) = |S(\tau) - P(\tau_0, r)|.$$  (2)

Expression (1) highlights the double role of the hodograph that causes a bending of the IRF, known as range migration, and a similar variation of its phase.

III. FOCUSING BY STRAIGHT ORBIT APPROXIMATION

In order to assess the focusing effect by the classical ωk method, let us express the hodographs of, say, Np targets at constant range spacing, Δr, acquired by a sensor moving with constant velocity, vso, on a straight orbit:

$$R_{so}(x, r_n) = \sqrt{r_n^2 + v_{so}^2 v_n^2} = \sqrt{(r_m + n\Delta r)^2 + v_{so}^2 v_n^2},$$  (3)
$\gamma_n = \frac{1}{T_o} \left| \int_{T_o}^{(\xi \cdot \Delta R_{E}(r, r_0 - \Delta r_{DC}, r_n))} dt \right|$, (6)

where $\tau_{DC}$ is the zero azimuth resolution and $\Delta R$ is the swath depths $\Delta R$. This azimuth resolution was assumed as input to derive the observation time as follows:

$$T_o = \frac{L_s}{v_s} = \frac{r_0 \lambda}{2v_s \rho_{az}}$$

Fig. 2.d shows a marked defocusing for the L band, at the finer resolution, $\rho_{az} = 1$ m, regardless of the swath width, while Fig. 2.e shows a similar effect in C band, for $\Delta R > 30$ km whatever the azimuth resolution. However, in this last case, DC plays an important role and very small artefacts would be expected for perfect yaw steering.

Finally, a quantitative evaluation of the impact of azimuth non-stationarity is evident in Fig. 2.c, that shows the differential error for a slow-time shift of $\Delta \tau = 5$ s. This error is always less than 1 mm, thus negligible even for the X band.

**IV. SVDS FOCUSING**

Let us derive SVDS focusing by a first Fourier Transform (FT) along azimuth of the time-domain convolution (4):

$$\hat{A}^x(k_x, r) = \int D^x(k_x, t - t_s)H_s^x(k_x, t; r) dt$$

where $\hat{A}^x(k_x, r)$ is the normalized amplitude of the target, focused by SOA. This value can be estimated by combining (1) and (4) and ignoring the residual migration (far less than the resolution):

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where we use capitals in the FT domain, and the apex for the dimension in which we transform. By exploiting Parseval identity, we get [2]:

$$\hat{A}'(k_x, r) = \int D^{x,t}(k_x, \omega) \exp\left(-j\left(\omega + \omega_0\right)t_s\right) \frac{H_s^{x,t}(k_x, \omega; r) d\omega}{H_s^{x,t}(k_x, \omega; r)}$$

(8)

We need to estimate the 2D IRF spectrum of the reference

$$H_s^{x,t}(k_x, \omega; r)$$

matched to (1). The FT along time results in:

$$H_s^{x,t}(\omega, r; r) = P(\omega + \omega_0) \cdot \exp\left(j\Omega R(x; r)\right)$$

(9)

where $$\Omega = \frac{2(\omega + \omega_0)}{c}$$

The FT of the range compressed pulse, $$P(\omega + \omega_0)$$, is almost constant in the system bandwidth, so it is omitted. The azimuth FT of (9) can be carried out by the Method of Stationary Phase (MSP) [2]. The MSP evaluates the integral:

$$H_s^{x,t}(k_x, \omega; r) = \int \exp\left(j\Omega R(x; r)\right) \exp\left(-jk_x x\right) dx$$

(10)

by assuming that the relevant contributions come from the stationary points, $$x_f$$, those that null the phase derivative:

$$\Omega \left. \frac{\partial R(x; r)}{\partial x} \right|_{x=x_f} - k_x = 0$$

(11)

the result is the following expression:

$$H_s^{x,t}(k_x, \omega) \simeq \exp\left(-j\left(k_x x_f - \Omega R(x_f; r)\right)\right)$$

(12)

where amplitude factors have been ignored. This expression, applied to the SOA in (3), leads to the known solution [2]:

$$H(\omega, k_x; r_n) \simeq \exp\left(jr_n \sqrt{\Omega^2 - k_x^2}\right).$$

(13)

The curved orbit case is naturally suited for a numerical evaluation that is both simple and efficient. This step can be done as proposed in [8] for the bistatic case, by exploiting a 4th order polynomial approximation of the hodograph. The resulting numeric reference provides perfect focusing at a specific reference range, but is not suited to the entire image. This is shown in Fig. 3, that draws the residual error surface $$\Delta R_{\xi}(\tau, r_n)$$ before and after compensation for the error at mid-swath. Such compensation is effective only for a small range interval; outside such interval the results would be even worse than the classical SOA.

A. Generalized Stolt interpolation

The solution is provided by finding the proper range update of the focusing operator, the Generalized Stolt Interpolation (GSI). By combining (8) and (12):

$$\hat{A}'(k_x, r) = \int D^{x,t}(k_x, \omega) \exp\left(-j\left(\omega + \omega_0\right)t_s\right) \cdot \exp\left(j\psi(k_x, \omega; r)\right) d\omega$$

where $$\psi(k_x, \omega; r) = \Omega R(x_f; r) - k_x x_f$$

(14)

a 2D focusing results that is exact but inefficient, as it is range variant.

Let us assume that for each wavenumber $$k_x$$ we can identify a quite narrow angular interval, so that the delay, $$t$$, becomes proportional to the zero-Doppler range, $$r$$. In the SOA we have:

$$r = \frac{ct}{2} \cos \theta; \sin \theta = \frac{k_x}{\Omega + \Omega_0}$$

(15)

$$\theta$$ being the squint angle. For quasi-monochromatic systems, $$\omega \ll \omega_0$$, a similar proportionality $$r \propto t$$ is expected for the curved orbit, as long as it is dependent on $$k_x$$. This makes sense as the orbit is supposed to be straight, in the extent of the resolution resulting from a synthetic antenna long as the full azimuth scene, in the order of meters. However, even if the azimuth wavenumber is fixed, a large bandwidth $$\Delta \omega$$, would introduce a squint angle variation $$\Delta \theta$$:

$$\frac{\partial}{\partial \omega} \frac{ck_x}{2(\omega + \omega_0)} \mid_{\omega=0} \Delta \omega \approx \left. \frac{\partial}{\partial \theta} \sin \theta \right|_{\theta=\theta_x} \Delta \theta$$

$$\theta_x$$ being the squint angle for $$(k_x, \omega_0)$$. In a C band system, with 300 MHz bandwidth, and a squint of 0.02 rad, for $$f_{DC}=5$$ kHz, we get $$\Delta \theta \approx 1$$ km. If the orbit can be assumed straight within such length, the expression (14) can be reduced to a 2D focusing by a proper, $$k_x$$-dependent resampling, i.e., the GSI. First in (14) isolate the range invariant contributions, then let $$r_{ref}$$ be a reference range. Then split the phase $$\psi$$ into a range variant and a range invariant term:

$$\exp\left(j\psi(k_x, \omega; r)\right) = \exp\left(j\psi_{ref}(k_x, \omega; r_{ref})\right) \cdot \exp\left(j\psi(k_x, \omega; r)\right)$$

(17)

$$H_{ref}(k_x, \omega) = \exp\left(-j\Omega \frac{ct}{2}\right) \exp\left(j\psi_{ref}(k_x, \omega; r_{ref})\right)$$

$$H_{ref}$$ being the FT of the reference computed at range $$r_{ref}$$. The following focusing expression results:

$$\hat{A}'(k_x, r) = \int D^{x,t}(k_x, \omega) H_{ref}(k_x, \omega) \exp\left(j\psi(k_x, \omega; r)\right) d\omega$$

For each wavenumber, $$k_x$$, it is assumed that there is separable decomposition:

$$\psi(k_x, \omega; r) = \psi(\omega; k_x) \cdot \psi(r; k_x)$$

(18)

Eventually we make a change of variable $$r \rightarrow v$$, and transform
along the stretched range, \( v \):

\[
\hat{A}^{x,v}_x(k_x,v) = \int D^{x,t}(k_x,\omega) H_{ref}(k_x,\omega) \cdot \left\{ \int \exp(ju(\omega;k_x)v) \exp(-jk_xv) dv \right\} d\omega
\]  

(19) \( \hat{A}^{x,v}_x \) being the transform of the focused data, after \( r \rightarrow v \) remapping. The inner integral in (19) leads to the GSI kernel:

\[
\frac{1}{2\pi} \int \exp(-ju(\omega;k_x) \cdot v) \exp(-jk_xv) dv = \delta(k_x-u(\omega;k_x)) = \delta(\omega-g(k_x;k_x)) \cdot \left| \frac{dv}{d\omega} \right|
\]  

(20) where \( \omega = g(k_x) \) reverses the mapping: \( k_x = u(\omega) \). Once again ignore slow varying amplitudes, i.e. the Jacobean, and combine (19) and (20) getting:

\[
\hat{A}^{x,v}_x(k_x,v) = \int D^{x,t}(k_x,\omega) H_{ref}(k_x,\omega) \delta(\omega-g(k_x;k_x))d\omega = S_g \left\{ D^{x,t}(k_x,\omega) \right\}
\]  

(21) where \( S_g \) is the GSI mapping [7]. Fig. 4 shows the algorithm implementation. The scheme is identical to \( \omega k \), except for the inclusion of the \( v \rightarrow r \) mapping (the shaded block). However, both the focusing kernel \( H_{ref}(k_x,\omega) \) and GSI require a proper numerical computation. SOA is a particular case of (18):

\[
v_n = r_n - r_{ref}; \ u_n = \sqrt{12 - k_x^2}
\]  

(22)

As an example, the extreme case \( \Delta R = 50 \text{ km} \) referred to in Fig. 2 has been approached in the case of a C band system with a bandwidth of 300 MHz. The azimuth bandwidth of 30 kHz allows an observation time \( T_0=12 \text{ s} \), thus handling any feasible DC. The residual phase error with respect to the exact transfer function, has been evaluated as a function of the 2D frequencies and range. The maximum error is represented in the 2D surfaces of Fig. 5.a,b. The peak error is always less than 0.1 radians, leaving a negligible defocusing (we measured \( \gamma > 0.99 \), whatever the range). The amplitudes of the first 4 eigenvalues are represented in dB scale in Fig. 5.c as a function of the azimuth wavenumber (hence the squint). For very small wavenumbers, i.e. \( k_x \rightarrow 0 \), the decomposition (18) is exact, as results from (15) and (16). In fact, all the eigenvalues but the first are at the numeric noise floor with the 17 decades of dynamics due to double precision maths. As the squint increases, the impact of the orbit curvature becomes relevant, and this appears in the trend of the 2nd eigenvalue that, however, remains at 9 decades below the 1st. This is indeed a dynamics; confirmed by the negligible decorrelation.

Furthermore, to check the quality of the numerical implementation, we applied SVDS in the straight orbit case, and compared the result with the known closed form solution (13), (22). The phase error was in the order of milliradians (peak), and the first four eigenvalues are plotted in Fig. 5.d. The SVDS achieved the exact decomposition in (22), with a dynamic better than 15 decades.

As a suboptimal approach, we can assume the same one-to-one mapping \( v_n = r_n - r_{ref} \) as for SOA, and model the range dependent phase as a 2\text{nd} order polynomial:

\[
\psi_r(k_x,\omega; r_n) \simeq (r_n - r_{ref}) \cdot \left[ \theta_1 + \theta_2 \Omega' + \theta_3 (\Omega')^2 \right]
\]  

(24) In matrix formulation:

\[
\frac{\psi_r(k_x,\omega; r_n)}{r_n - r_{ref}} = \left[ \begin{array}{ccc} 1 & \Omega & \Omega_2 \end{array} \right] \left[ \begin{array}{ccc} \theta_1 & \theta_2 & \theta_3 \end{array} \right]^T
\]

\[
\Phi = G \Theta
\]  

(25)

where the first vector, \( u_1 \), provides the GSI, and the second vector, \( v_1 \), the range mapping.

**B. SVDS, LSQ and numerical results**

Singular Value Decomposition is proposed in reference [7] as the best (in L2 norm) numerical approximation for decomposition (18). For each \( k_x \) we form the matrix \( \Psi_r \) of size \([N_\omega, N_P] \) by numerically evaluating the phase \( \psi_r(\omega, r; k_x) \) in (17). As this phase is a smooth function, like the hodograph from which it originated, we can sample it quite coarsely, compared to the size of the data matrix. We retain only the leading term in the SVD, the one associated to the largest eigenvalue:

\[
H_{k[N_\omega, N_P]} = U_{[N_\omega, N_P]} S_{[N_\omega, N_P]} V^*_{[N_P, N_P]}
\]  

\[
\simeq u_1[N_\omega,1] \lambda_1 v_1^*[1, N_P]
\]  

(23) where the first vector, \( u_1 \), provides the GSI, and the second vector, \( v_1 \), the range mapping.
block. Thus, it is formally identical to the $\omega k$; instead it has a different quadratic Stolt Interpolation kernel. This is an efficient solution that holds for reasonable bandwidth and swath depths.

An example of processing a set of simulated data can be seen in Fig.6, where the amplitudes of three point scatterers, focused by both the proposed SVDS and the conventional $\omega k$ approach, are plotted. An ENVISAT-IS2 like acquisition has been simulated, a C band system with 50 MHz bandwidth, $\Delta R =12$ km. We assumed $f_{DC} = 2$ kHz and we doubled the orbit curvature to enhance the distortion. The impact of the orbit curvature in this case is quite small: we estimated $\Delta R=12$ km, $\Delta x=4$ km. We assumed $f_{DC}=2$ kHz, and a doubled orbit curvature. The defocusing can be appreciated despite the almost "straight" geometry. $\gamma > 0.92$.

V. ACKNOWLEDGMENTS

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VI. CONCLUSIONS

The impact of orbit in high resolution space-borne SAR has been analyzed and characterized in terms of reduction in focusing quality, $\gamma$. That quality factor has been evaluated in different, extreme conditions, resulting in noticeable artefacts for the wider swaths and the finest azimuth resolution, depending upon the squint, the bandwidth and the frequency. A generalized $\omega k$ approach that uses a numerically calculated phase reference and Stolt interpolation kernel has been proposed. The SVDS method takes advantage of two degrees of freedom: (1) the capability of tuning the Stolt Interpolation independently on each wavenumber, and (2) the inclusion of an additional wavenumber dependent range resampling. A suboptimal LSQ-based inversion has been proposed, that makes use of a quadratic Stolt Interpolation and linear range stretch, both wavenumber dependent. Its full-numerical implementation retains the same quality of the $\omega k$ (up to numeric roundoff) for straight orbits, but it also cope with curved orbits with negligible defocusing.

REFERENCES