Multiuser space-time channel estimation for CDMA under reduced-rank constraint

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Abstract - **This paper addresses the problem of estimating the multiuser channels in CDMA systems when using a linear array of antennas. The training sequences to be used for multi-user channel estimation have limited lengths. In order to control the errors of the (unstructured) estimate of space-time (S-T) channel here we propose to reduce the complexity when parameterizing the channel. This is achieved by constraining *each* matrix that models the S-T channel of *each* user to have a low-rank. The rank of the matrix accounts for the degrees of space and/or time diversity in S-T channel. Here we propose to estimate the reduced-rank channel simultaneously for all the users so as to control the multiaccess interference (i.e., all the users are considered signals for each other). The covariance matrix of intercell interference is constrained accordingly. Simulation results for realistic propagation/interference environments confirm the expected benefits.**

I. INTRODUCTION

Wideband code division multiple access (CDMA) is the preferred modulation scheme for third generation cellular system. Standards of future cellular systems account for the possibility to reduce the interferences by multi-user detection and array processing. Both techniques are considered powerful for increasing the capacity, the quality and the coverage. When separately performed, the multi-user detection accounts for multiaccess interference while the array processing is called mainly to reduce intercell interference. However, the integration of space-time (S-T) into multi-user detection is known to provide a significant improvement in performance over the scalar case [1] even though it is computationally more expensive.

The estimation of the S-T channels is a very challenging task. Classically the S-T channel can be modelled by (few) temporal channels each associated to a direction of arrival (DOA). First the DOAs are estimated by any known method and then, for each DOA, the temporal channel is estimated separately after an appropriate beamforming. The interference rejection capability depends on the beamforming [2].

The training sequences have limited length, therefore a parsimonious parameterization of the S-T channel seems mandatory in order to estimate the minimum number of unknowns of the channel matrix. The reduced-complexity model proposed here is based on the trade-off between model distortion due to the under-parameterization and the variance of the channel estimate due to the limited training length [3]. In GSM system it was proposed the maximum likelihood (ML) estimate of the S-T channel under the reduced-rank (RR) constraint of the matrix that models the S-T channel (for short this is referred RR estimate). This was proved to be effective in reducing intercell interference and to be computationally attractive for the inner parallelism of the algorithm [4], [5]. The field tests with the prototype of a base-station equipped with array-processing capability have shown that the receiver based on the RR estimate outperforms the one based on the DOA estimate in all practical cases [6]. This motivated the extension of the RR estimate to multi-user communication.

The single-user RR estimation method is re-proposed in Section 2 for CDMA system with interferences from other users and from time-dispersive channel. Thus, it is extended to the multi-user RR case. Differently form the single-user approach, the channels for the overall users are estimated *jointly* with the constraint that the S-T channel for *each* user is low-rank.

Numerical studies (section 4) on third generation cellular systems (TDD mode of UTRA proposal for IMT-2000) indicate that: i) the S-T channels can be approximated by reduced-rank matrices with moderate distortion due to the implicit under-parameterization; ii) the multi-user channel estimation with reduced-rank constraint outperforms the unconstrained estimate by at least 5dB in SNR; iii) when using a MMSE multi-user detection scheme the channel distortion due to the reduced-rank constraints can be considered as negligible if the rank is properly chosen, according to the SNR.

Notation: Lowercase (uppercase) bold denotes vector (matrix), $(.)^T$ is matrix transpose, $(.)^H$ is the Hermitian transposition, $||.||_F$ is the Frobenius norm.

II. SPACE-TIME CHANNEL ESTIMATION

The equivalent baseband model of the uplink for DS/CDMA system with a uniform linear array (ULA) receiver is considered. The discrete-time model is obtained by sampling at the chip rate $1/T_c$ the received signals after the chip matched filter. The ULA at the base station is composed of K_a antennas half-wavelength spaced apart. K users are simultaneously active in the same cell, within the same block of data and in the same frequency band.

For the k-th user, the S-T channel can be described by the matrix $\mathbf{H}^{(k)} = [\mathbf{h}^{(k,1)}, \dots, \mathbf{h}^{(k,K_a)}]^T$ that consists of K_a vectors, each representing the discrete-time channel impulse response $\mathbf{h}^{(k,k_a)}$ of length W for the link between the k-th user and the k_a-th antenna (here $1 \leq k \leq K$ and $1 \leq k_a \leq K_a$). Within a (short) time interval the S-T channel is time-invariant and can be modelled as a superposition of L_k multipaths each characterized by the direction of arrival ($\vartheta_k^{(\ell)}$), the delay ($\tau_k^{(\ell)}$),

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and the complex amplitude ($\alpha_k^{(\ell)}$) that accounts for power fluctuations of each ray: $\mathbf{H}^{(k)} = \sum_{\ell=1}^{L_k} \alpha_k^{(\ell)} \mathbf{a}(\vartheta_k^{(\ell)}) \mathbf{g}_k^{(\ell)}(\tau_k^{(\ell)})^T$. Here the vector $\mathbf{g}_k^{(\ell)}(\tau_k^{(\ell)})$ is the temporal channel (length W) and $\mathbf{a}(\vartheta_k^{(l)})$ is the array gain for $\vartheta_k^{(l)}$.

The estimate of these K radio channel is performed on the base of K different training codes $\{x_m^{(k)}\}_{m=1}^M$ of length M. These are arranged into a $W \times K$ Toeplitz matrix $\mathbf{X}^{(k)} = [\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_M^{(k)}]$ with $\mathbf{x}_m^{(k)} = [x_m^{(k)}, \ldots, x_{m-W+1}^{(k)}]^T$. Let $\mathbf{Y} = [\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(K_a)}]^T$ be the $K_a \times M$ matrix containing the signals received by the array, the multi-user data model can be written by using the standard notation:

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}^{(k)}\mathbf{X}^{(k)} + \mathbf{N} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1}$$

$\mathbf{H} = [\mathbf{H}^{(1)}, \ldots, \mathbf{H}^{(K)}]$ and $\mathbf{X} = [\mathbf{X}^{(1)T}, \ldots, \mathbf{X}^{(K)T}]^T$ are matrices collecting the K channels and the training sequences. The additive noise $\mathbf{N}$ is assumed to be Gaussian, temporally uncorrelated and spatially correlated, the spatial covariance matrix is $\mathbf{R}_{ns} = E\left[\mathbf{N}\mathbf{N}^H\right]/M$. The ML estimate of the compound channel $\widehat{\mathbf{H}}$ and of the covariance matrix $\hat{\mathbf{R}}_{ns}$ is described below either by estimating a finite length impulse response channel for each antenna and each user (full-rank ML estimate) or by constraining each matrix $\mathbf{H}^{(k)}$ to have $\text{rank}\{\mathbf{H}^{(k)}\} = r^{(k)} < \min\{W, K_a\}$ (RR estimate). The rank of $\mathbf{H}^{(k)}$ reflects the minimum number of orthogonal space and time channels that can be used to describe parsimoniously (i.e., with minimum distortion) the S-T channel $\{\mathbf{H}^{(k)}\}_{k=1}^K$. The rank depends on the number of wavefronts that sampled by the array can be considered as independent. Therefore in a multipath propagation the rank-order depends on the angle of arrival and delay spreads compared to the resolution of the array and of the signature waveforms (or their bandwidth). In a RR approach the multiaccess interference is implicitly taken into account by estimating the K channels $\{\mathbf{H}^{(k)}\}_{k=1}^K$ jointly.

A. Maximum likelihood estimate

The ML estimate for multi-user channel and noise covariance is given by (see e.g., [7]):

$$\begin{aligned} \hat{\mathbf{H}}_{MU} &= \hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}, && \text{(2a)} \\ \hat{\mathbf{R}}_{ns} &= \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{xy}, && \text{(2b)} \end{aligned}$$

where $\hat{\mathbf{R}}_{yx} = \mathbf{Y}\mathbf{X}^{\mathbf{H}}/\mathbf{M}$ is the sample covariance matrix, $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{yy}$ are similarly defined. The multi-user channel estimate for the k-th user ($\hat{\mathbf{H}}_{MU}^{(k)}$) is obtained from (2a) by selecting the k-th block: $\hat{\mathbf{H}}_{MU}^{(k)} = \hat{\mathbf{H}}_{MU}\mathbf{J}^{(k)}$, $\mathbf{J}^{(k)}$ is the $KW \times W$ selection matrix.

If the training codes are (almost) orthogonal, the simultaneous estimation of the K channels can be decomposed into the S-T channel estimation in a single-user fashion according to the model

$$\mathbf{Y} = \mathbf{H}^{(k)}\mathbf{X}^{(k)} + \mathbf{N}^{(k)}, \tag{3}$$

$\mathbf{N}^{(k)} = \sum_{j \neq k} \mathbf{H}^{(j)}\mathbf{X}^{(j)} + \mathbf{N}$ includes intracell and intercell interference modelled as Gaussian noise with unknown covariance matrix $\mathbf{R}_{ns}^{(k)}$. The single-user estimate for the k–th user is

$$\hat{\mathbf{H}}_{SU}^{(k)} = \hat{\mathbf{R}}_{yx}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,-1}, \tag{4}$$

where $\hat{\mathbf{R}}_{xx}^{(k)} = \mathbf{X}^{(k)}\mathbf{X}^{(k)\,H}/M$ and $\hat{\mathbf{R}}_{yx}^{(k)} = \mathbf{Y}\mathbf{X}^{(k)\,H}/M$. The covariance matrix estimate $\hat{\mathbf{R}}_{ns}^{(k)}$ is obtained similarly to (2b).

B. Reduced-rank estimate: single-user

The single-user ML estimate under RR constraint is considered first. Let the rank of the k-th user be $r^{(k)} = \bar{r}$ and define the following covariance matrix:

$$\begin{aligned} \hat{\mathbf{R}}_{SU}^{(k)} &= \hat{\mathbf{W}}_{SU}^{(k)\,-H/2}\hat{\mathbf{R}}_{yx}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,-1}\hat{\mathbf{R}}_{xy}^{(k)}\hat{\mathbf{W}}_{SU}^{(k)\,-1/2}, && (5) \\ \hat{\mathbf{W}}_{SU}^{(k)} &= \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yx}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,-1}\hat{\mathbf{R}}_{xy}^{(k)} = \\ &= \hat{\mathbf{W}}_{SU}^{(k)\,H/2}\hat{\mathbf{W}}_{SU}^{(k)\,1/2}, && (6) \end{aligned}$$

$\hat{\mathbf{W}}_{SU}^{(k)} \to \hat{\mathbf{R}}_{ns}^{(k)}$ as $M \to \infty$ [8]. Let $\mathbf{\Pi}_{SU}^{(k)}$ be the projector onto the subspace spanned by eigenvectors associated to the r largest eigenvalues of $\hat{\mathbf{R}}_{SU}^{(k)}$ (or equivalently the projector onto the leading subspace of the symmetric positive definite pencil $(\hat{\mathbf{H}}_{SU}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)}\hat{\mathbf{H}}_{SU}^{(k)H}, \hat{\mathbf{W}}_{SU}^{(k)})$ as shown in [4]), the RR estimate $\hat{\mathbf{H}}_{RR-SU}^{(k)}$ and the estimate of the covariance matrix of noise $\mathbf{N}^{(k)}$ can be shown to be (the proof is not presented here, the RR estimate fully equivalent to the one used here is in [8]):

$$\begin{aligned} \hat{\mathbf{H}}_{RR-SU}^{(k)} &= \hat{\mathbf{W}}_{SU}^{(k)\,H/2}\mathbf{\Pi}_{SU}^{(k)}\hat{\mathbf{W}}_{SU}^{(k)\,-H/2}\hat{\mathbf{H}}_{SU}^{(k)}, && \text{(7a)} \\ \hat{\mathbf{R}}_{ns}^{(k)} &= \hat{\mathbf{R}}_{yy} - \hat{\mathbf{H}}_{RR-SU}^{(k)}\hat{\mathbf{R}}_{xy}^{(k)}. && \text{(7b)} \end{aligned}$$

The relationship (7a) that is used to derive the RR estimate can be interpreted as the projection of the pre-whitened full-rank channel estimate $\hat{\mathbf{H}}_{SU}^{(k)}$ onto the column space of $\hat{\mathbf{W}}_{SU}^{(k)\,-H/2}\hat{\mathbf{R}}_{yx}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,-1/2} = \hat{\mathbf{W}}_{SU}^{(k)\,-H/2}\hat{\mathbf{H}}_{SU}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,H/2}$, the multiplication by $\hat{\mathbf{W}}_{SU}^{(k)H/2}$ in (7a) compensates the spatial whitening after projection. If $\hat{\mathbf{R}}_{xx}^{(k)} = \mathbf{I}$ the RR-estimate reduces to the truncated SVD of the pre-whitened full-rank channel estimate. For the MMSE multi-user detector (section 3) it is needed the estimate of the covariance matrix of noise. The interference due to intercell users is $\hat{\mathbf{N}} = \mathbf{Y} - \sum_{k=1}^{K}\hat{\mathbf{H}}_{RR-SU}^{(k)}\mathbf{X}^{(k)}$, the sample covariance matrix $\hat{\mathbf{R}}_n = \hat{\mathbf{N}}\hat{\mathbf{N}}^H/M$.

C. Reduced-rank estimate: multi-user

The multi-user estimate under the RR constraint can be obtained similarly to the single-user estimate. Let re-define the covariance matrices for the ensemble of K users

$$\hat{\mathbf{R}}_{MU}^{(k)} = \hat{\mathbf{W}}_{MU}^{-H/2}\hat{\mathbf{H}}_{MU}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)}\hat{\mathbf{H}}_{MU}^{(k)H}\hat{\mathbf{W}}_{MU}^{-1/2}, \qquad (8)$$

the spatial pre-whitening $\hat{\mathbf{W}}_{MU}^{-1/2} = \hat{\mathbf{R}}_{ns}^{-1/2}$ is obtained from the full-rank estimate (2b). The RR estimate is obtained by projecting the multi-user estimate $\hat{\mathbf{H}}_{MU}^{(k)} = \hat{\mathbf{H}}_{MU}\mathbf{J}^{(k)}$ onto the subspace spanned by the leading (left) eigenvectors of $\hat{\mathbf{W}}_{MU}^{-H/2}\hat{\mathbf{H}}_{MU}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,H/2}$:

$$\hat{\mathbf{H}}_{RR-MU}^{(k)} = \hat{\mathbf{W}}_{MU}^{H/2}\,\mathbf{\Pi}_{MU}^{(k)}\hat{\mathbf{W}}_{MU}^{-H/2}\hat{\mathbf{H}}_{MU}^{(k)}, \qquad (9)$$

here $\mathbf{\Pi}_{MU}^{(k)}$ denotes the corresponding projection matrix onto the subspace spanned by the first $r^{(k)}$ eigenvectors of $\mathbf{R}_{MU}^{(k)}$. When compared to the single-user RR estimate (7a) some differences can be appreciated: i) the pre-whitening $\hat{\mathbf{W}}_{MU}$ is obtained from multi-user estimation so that it is dominated by intercell interference; ii) multiaccess interference is considered in the multi-user approach by selecting the corresponding S-T channel $\hat{\mathbf{H}}_{MU}^{(k)}$. Similarly to single-user approach [5], the implementation can benefit of the algorithm parallelism.

Remark: The best rank-r approximation (in the Frobenius norm metric) of the unconstrained ML estimate $\hat{\mathbf{H}}_{SU}^{(k)}$, or $\hat{\mathbf{H}}_{MU}^{(k)}$ (i.e., the truncation of the SVD of $\hat{\mathbf{H}}_{SU}^{(k)}$, or $\hat{\mathbf{H}}_{MU}^{(k)}$, to the leading r singular values), has a lower performance when compared with the RR estimate as there is no attempt to take into account for the intercell interference when evaluating the projection matrix (except for the trivial case $\mathbf{R}_{ns} = \sigma^2\mathbf{I}$). This is confirmed by the numerical analysis in Section 4.

III. MMSE Linear Multi-user Detection

Consider a K user synchronous CDMA uplink where each user transmits a block of N quadrature phase shift keyed (QPSK) symbols, with symbol duration T_s. Let $b_n^{(k)}$ denote the n-th symbol of k-th user, and $\mathbf{b}_n = [b_n^{(1)}, \ldots, b_n^{(K)}]^T$ the vector containing the n-th symbol for all K users. The k-th user is assigned a spreading sequence $\mathbf{c}^{(k)} = [c_1^{(k)}, \ldots, c_Q^{(k)}]^T$ and the spreading factor $Q = T_s/T_c$. The complex baseband representation of the output of the k_a-th sensor after the chip-matched filtering and with chip-spaced sampling is [9]

$$\mathbf{y}^{(k_a)} = [y_1^{(k_a)}, \ldots, y_{NQ+W-1}^{(k_a)}]^T = \mathbf{S}^{(k_a)}\mathbf{b} + \mathbf{n}^{(k_a)}, \quad (10)$$

where $\mathbf{S}^{(k_a)}$ is the $(NQ + W - 1) \times KN$ matrix with the signature sequences for all the K users (i.e., the convolution of the spreading codes $\{\mathbf{c}^{(k)}\}_{k=1}^{K}$ with the temporal channels $\{\mathbf{h}^{(k,k_a)}\}_{k=1}^{K}$), $\mathbf{b} = [\mathbf{b}_1^T, \ldots, \mathbf{b}_N^T]^T$ is the data vector of length NK. Symbols are independent $E\left[\mathbf{b}\mathbf{b}^H\right] = \mathbf{I}_{NK}$ and Gaussian noise is temporally uncorrelated $\mathbf{R}_{nt} = E\left[\mathbf{n}^{(k_a)}\mathbf{n}^{(k_a)H}\right] = \mathbf{I}_{NQ+W-1}$. Model (10) represents the equivalent KN-user synchronous system that describes the interference due to the other users and to self-interference caused by the frequency selective fading [10], $\mathbf{S}^{(k_a)}$ is the matrix containing the $(NQ + W - 1)$-dimensional signature vectors associated to the KN equivalent users. For the ensemble of K_a sensors the received signals can be arranged into a vector $\mathbf{y}$

$$\mathbf{y} = [\mathbf{y}^{(1)T}, \ldots, \mathbf{y}^{(K_a)T}]^T = \mathbf{S}\mathbf{b} + \mathbf{n}, \qquad (11)$$

$\mathbf{S} = [\mathbf{S}^{(1)T}, \ldots, \mathbf{S}^{(K_a)T}]^T$ is a $K_a(NQ + W - 1) \times NK$ matrix, $\mathbf{n} = [\mathbf{n}^{(1)T}, \ldots, \mathbf{n}^{(K_a)T}]^T$. The covariance matrix of the noise is $\mathbf{R}_n = E\left[\mathbf{n}\,\mathbf{n}^H\right] = \mathbf{R}_{ns} \otimes \mathbf{R}_{nt} = \mathbf{R}_{ns} \otimes \mathbf{I}_{NQ+W-1}$, $\otimes$ is the Kronecker product of the spatial ($\mathbf{R}_{ns}$) and temporal ($\mathbf{R}_{nt}$) term. According to the model (11), the minimum mean square error (MMSE) estimate of $\mathbf{b}$ can be shown to be [10]:

$$\widehat{\mathbf{b}}_c = (\mathbf{M} + \mathbf{I}_{NK})^{-1}\mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{y}, \qquad (12)$$

$\mathbf{M} = \mathbf{S}^H\mathbf{R}_n^{-1}\mathbf{S}$. The estimate $\widehat{\mathbf{b}}_c$ maximizes the output signal to interference-noise ratio. The computational complexity of $(\mathbf{M} + \mathbf{I}_{NK})^{-1}$ can be significantly reduced up to $O[K^2(N + KV^3)]$ by exploiting the block Toeplitz structure of the matrix [11], where $V = \lceil 1 + (W - 1)/Q \rceil$. Other linear multi-user detector schemes are not considered in this paper, see [10] for a complete discussion.

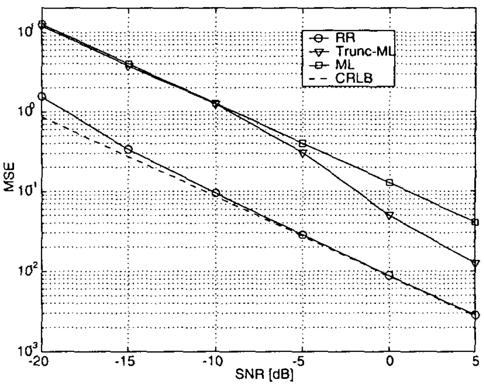

Fig. 1. Single-user channel estimation for $r = 2$.

IV. Performance studies

The performances of channel estimation techniques are obtained by simulating the uplink of TDD-UTRA proposal for IMT-2000, details on the system parameters assumed here are in [12]; numerical results are for $K_a = 8$ omnidirectional antennas.

Figure 1 compares the RR estimate with (full-rank) ML estimate and truncated SVD for single-user and $r = 2$. The rank-2 channel matrix $\mathbf{H}$ is randomly generated, channel-length is $W = 57$, the training sequence is $M = 456$

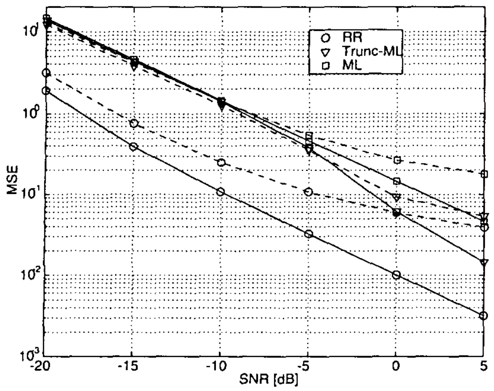

Fig. 2. Channel estimation for multi-user (solid line) and single-user (dashed line), with $r = 2$ and $K = 8$.

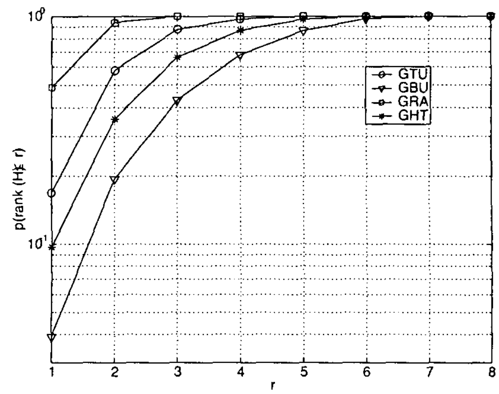

Fig. 3. Analysis of the rank of COST-259 channel models: cdf of rank for GBU, GTU, GRA, GHT channels.

[12], the Gaussian noise is spatially correlated as $[\mathbf{R}_{ns}]_{m,l} = \sigma^2\{0.9\exp(-i\pi\sin(\pi/6))\}^{l-m}$ (the DOA of the interference is 30deg). The channel is scaled to $||\mathbf{H}||_F^2 = K_a$, the signal to noise ratio (SNR) per antenna is $SNR = 10\log_{10}(1/\sigma^2)$. Figure 1 shows the mean square error (MSE) of the estimate $||\mathbf{H} - \widehat{\mathbf{H}}||_F^2 / ||\mathbf{H}||_F^2$ vs. SNR (each value is obtained from 500 independent runs of channel and noise). Rank $r = 2$ is assumed known (i.e., rank-order is not estimated here). The RR estimate outperforms the full-rank ML by approx. 10dB in SNR for the whole range of SNR considered here. The truncation of the SVD to the $r = 2$ leading eigenvectors of $\hat{\mathbf{H}}_{SU}$ (Trunc-ML) becomes effective for high SNR's (SNR>-5dB), the MSE reduces considerably compared to the ML estimate. For large SNR the RR estimate attains the lower bound (CRLB) derived in [8]. In summary, when the channel is low-rank and the rank-order is correctly estimated, the RR estimate has the lower MSE as the model is congruent with measured data.

Figure 2 compares the single-user and multi-user estimates when $K = 8$ users are active simultaneously (same parameters as in Figure 1). Single-user algorithms (dashed lines) are biased by the multiaccess interference. The "effective" SNR experienced by the single-user approach when the intercell interference vanishes is $1/(K-1)$ (or equivalently -8.5dB), the corresponding MSE from Figure 1 is 5×10^{-2}; this is the floor of single-user methods in Figure 2. Multi-user estimate outperforms single-user methods mainly for high SNR as multi-user methods handle properly the multi-access interference. According to system specification [12] the matrix $\mathbf{X}$ is full-rank right circulant square matrix. The estimation of the covariance matrix of intercell noise needs a reduction of the number of channel parameters that can be obtained by using, for each user, the effective channel length. If the number of active users is $K < 8$ then one (or more) empty slots can be used to improve the estimate of the noise covariance matrix. When $\widehat{\mathbf{R}}_{ns}$ is consistently estimated, the rank-reduction of $\widehat{\mathbf{H}}_{MU}^{(k)}$ performed for each user according to equation (9) yields to the MSE shown in Figure 2 (solid circle line).

We consider the feasibility of the RR approach in realistic propagation environment. First it needs to be established the rank-order of realistic S-T channels. The propagation channel considered here is the COST-259 Directional Channel Model (an evolution of COST-207, see [13]). COST-259 is a stochastic channel model for S-T system that includes both azimuthal and temporal dispersions (see [14] for a detailed description of the physical model). The channel response depends on N independent local scatterers (clusters), each described by multipaths with associated amplitude, delay and azimuth. All the parameters in the model are independent random variables; the probability density functions are assigned according to the propagation environment. In addition to the line-of-sight cluster there are other additional clusters, their numbers are Poisson distributed. Four COST-259 radio environments have been used: generalized typical urban (GTU), generalized bad urban (GBU), generalized rural area (GRA), generalized hill terrain (GHT). These propagation scenarios are specified by different values of the parameters described in [13]. For each of the propagation scenarios we have evaluated the rank-r approximation of the S-T channel, the approximation is within -20dB similarly to [5]. Figure 3 shows the cumulative distribution function (cdf) of the rank for the four channel models when the TDD-UTRA parameters and $K_a = 8$ are considered. GTU and GRA models have low-rank as they are characterized by a low space (GTU) and/or time (GRA) diversity.

The trade-off between distortion and variance of the RR estimate needs to be evaluated by considering the minimum value of the MSE vs. SNR due to an interfering user (e.g., with a DOA of 30deg). Table I reports the optimized rank-order for COST-259 environments and varying SNR. In a severe interfering environment (SNR$< -15dB$) the rank-1 approximation is the preferred solution (i.e., the least number of unknowns to be estimated). However, different environments call for different rank-order depending on the interference level (e.g., $r = 2$ for GRA holds up to SNR$\simeq 5dB$).

SNR [dB]	-15	-10	-5	0	5	10	15	20
rank-GTU	2	2	3	3	4	4	5	5
rank-GBU	2	3	4	4	5	6	6	6
rank-GRA	1	2	2	2	2	3	3	3
rank-GHT	2	2	3	4	5	5	6	6

TABLE I
OPTIMIZED VALUES OF RANK-ORDER VS. SNR FOR S-T CHANNELS OF FIG. 3 (COST-259 MODELS).

Figure 4 compares the performance in terms of BER for uncoded bits vs. SNR when the detection is performed by using MMSE multi-user detection (Section 3). According to [12] OVSF codes have $Q = 16$, raised cosine pulse with roll-off 0.22 and chip rate 4.096 Mchips/s, QPSK modulated symbols have period $T_s = QT_c = 3.906\mu s$. Block has $N = 61$ symbols (976 chips), with a periodic training sequence ($M = 456$ chips). The fading of the GTU propagation model is without Doppler effect. The spatially correlated noise is generated by 6 intercell interferers with log-normal shadowing, the dB spread is 13dB (i.e., this value is increased by a factor $\sqrt{2}$ with respect to 9dB as it is assumed a perfect power control within each cell). Background white Gaussian noise is set to 20dB. Roughly speaking, because of slow fading there is at least one powerful interferer that makes the noise spatially correlated, this favors the spatial filtering $\hat{\mathbf{W}}_{MU}^{-H/2}$ in (9). For $r = 2$ the RR estimate (RR-MU) outperforms the full-rank ML (ML) and the truncated SVD (Trunc-ML) estimates. Similarly to the MSE simulations (Figure 2) the multi-user RR estimate have better performances with respect to single-user approach (RR-SU), the gain is approx. 3-4dB in SNR. Performances can be optimized by choosing the rank according to the SNR (see Table I). Performances of fixed low-rank ($r = 2$) degrade when considering SNR $> 0dB$, a rank $r = 3$ would be more appropriate in this case. If needed, the performances for high SNR can be improved by progressively relaxing the rank-order according to the MDL (Minimum Description Length) criterium [15]; this topic is not covered here.

V. CONCLUSION

A space-time channel estimation for multiuser communication has been presented. The proposed method is based on an under-parameterization of the channel that constraints the ML estimate to have a reduced-rank. The RR estimation algorithm has been extended from single-user [4]-[5] to multi-user. Numerical simulations confirm the feasibility of a RR approach in realistic propagation environments and the better performances of the RR estimate compared to full-rank ML estimate.

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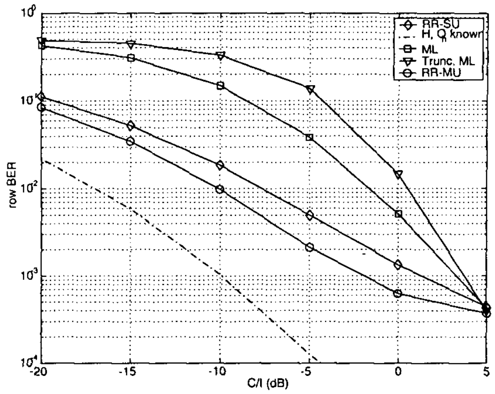

Fig. 4. Performance of MMSE multiuser detector with single-user (RR-SU) and multi-user (RR-MU) RR estimates, multi-user (ML) and truncated (Trunc-ML) ML estimates [GTU channel, rank r=2, $K = 8$ intracell users, 6 intercell interferers].

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