Reduced rank channel estimation and rank order selection for CDMA systems

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Abstract - **This paper investigates the problem of channel estimation in the uplink of CDMA systems with base station antenna array. The estimation is based on the transmission of training sequences with limited length. In order to improve multi-user receiver performance it is proposed to reduce the number of the unknowns in the channel estimation by constraining the space-time channel matrix of each user to be low rank. The rank-order is estimated according to the MDL criterion as this method provides the best trade-off between distortion (due to under-parametrization) and variance (due to the limited training length).**

I. Introduction

In the past few years the number of users of wireless communication systems has been growing exponentially. For this reason increasing system capacity is a critical issue, especially for next generation cellular systems. Wideband code division multiple access (CDMA) is the preferred multiple access technique for third generation systems. As CDMA capacity is interference limited, any reduction of co-channel interference from own cell (MAI, Multiple Access Interference) and neighboring cells (inter-cell interference) improves the system performance. A promising approach to suppress interference and multipath channel distortion is the integration of array signal processing and multi-user detection [1]-[2].

In order to detect transmitted symbols, the space-time (S-T) features of the propagation channels have to be estimated for all the active users. Conventionally channel estimation is based on the transmission (in every data packet) of training sequences. Due to the limited length of the training sequence, a reduction of the number of the unknowns of the channel matrix seems mandatory in order to improve the estimation performance. The reduced-complexity model proposed here is based on a parsimonious but effective parameterization of the channel as trade-off between model distortion (due to the under-parameterization) and the variance of the channel estimate (due to the limited training sequence length) [3].

In GSM system it was proved that the maximum likelihood (ML) estimate of the S-T channel under the reduced-rank (RR) constraint is effective in reducing intercell interference [4], [5] (performances of the receiver have been evaluated also by field tests with the prototype of a base-station equipped with array-processing capability [6]). This result motivated the extension in [7] of the RR estimation method to the multi-user case. In this paper it is shown how the channels for the overall users can be estimated *jointly* with the constraint that the S-T channel for *each* user is low-rank. The rank-order is estimated according to the MDL criterion (Minimum Description Length) [8] as the one that minimizes the mean square error of channel estimate.

The paper is organized as follows. The discrete-time system model for the uplink is presented in Section 2. In Section 3 the multi-user channel estimation under the reduced rank constraint is described. The rank-order selection according to MDL criterion is considered in Section 4. Simulations results are given in Section 5 for the uplink TDD-UTRA standard [9] and concluding remarks are finally discussed.

Notation: Lowercase (uppercase) bold denotes vectors (matrices), $(.)^T$ is matrix transpose, $(.)^H$ is the Hermitian transposition, $||.||^2$ is the Frobenius norm, $\mathbf{A}^{1/2}$ is the Cholesky factor of positive definite matrix $\mathbf{A}$: $\mathbf{A} = \mathbf{A}^{H/2}\mathbf{A}^{1/2}$.

II. System model

The uplink channel estimate in a symbol synchronous DS/CDMA mobile radio system with a uniform linear array (ULA) receiver is considered. The equivalent discrete-time model is obtained by sampling at the chip rate $1/T_c$ the received signals after the chip matched filter. At the base station the array is composed of M half-wavelength spaced apart antennas. K users are simultaneously active in the same cell, in the same time and in the same frequency band.

For the k-th user, the S-T channel can be described by the matrix $\mathbf{H}^{(k)} = [\mathbf{h}^{(k,1)}, \ldots, \mathbf{h}^{(k,M)}]^T$ that consists of M vectors, each represents the discrete-time channel impulse response $\mathbf{h}^{(k,m)}$ of length W for the link between the k-th user and the m-th antenna (here $1 \leq k \leq K$ and $1 \leq m \leq M$). Within a (short) time interval, the S-T channel is time-invariant.

The estimate of $\mathbf{H}^{(k)}$ $\forall k$ is based on the transmission of K different training sequences $\{x_n^{(k)}\}_{n=1}^{N+W-1}$ known at the receiver. The n-th time sample $\mathbf{y}_n$ of the signal received by the array is

$$\mathbf{y}_n = \sum_{k=1}^{K} \mathbf{H}^{(k)}\mathbf{x}_n^{(k)} + \mathbf{n}_n, \tag{1}$$

where the vector $\mathbf{x}_n^{(k)} = [x_n^{(k)}, \ldots, x_{n-W+1}^{(k)}]^T$ depends on the training sequence assigned to the k-th user. The noise is temporally uncorrelated and spatially correlated, $\mathbf{n}_n \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{ns})$, $\mathbf{R}_{ns}$ is the space covariance matrix with $[\mathbf{R}_{ns}]_{n,n} = \sigma^2$. By arranging N time samples into the matrix $\mathbf{Y}$, the multi-user data model can be written using the standard notation:

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}^{(k)}\mathbf{X}^{(k)} + \mathbf{N} = \mathbf{HX} + \mathbf{N}, \tag{2}$$

where $\mathbf{N} = [\mathbf{n}_1, \ldots, \mathbf{n}_N]$ and $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_N]$ collect the N time samples of the noise and the received signal respectively, $\mathbf{X}^{(k)} = [\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_N^{(k)}]$ is a $W \times N$ Toeplitz matrix composed of the k-th training sequence. $\mathbf{H} = [\mathbf{H}^{(1)}, \ldots, \mathbf{H}^{(K)}]$ and $\mathbf{X} = [\mathbf{X}^{(1)T}, \ldots, \mathbf{X}^{(K)T}]^T$ are multi-user matrices obtained by concatenating the K channels and the K training sequences.

III. Space-time channel estimation

According to the model (2), the ML estimate $(\hat{\mathbf{H}}, \hat{\mathbf{R}}_{ns})$ of the compound channel $\mathbf{H}$ and of the covariance matrix $\mathbf{R}_{ns}$ is described below, first by deriving the unconstrained full-rank solution (FR estimate) and then by deriving the ML estimate where each matrix $\mathbf{H}^{(k)}$ is constrained to have rank $r^{(k)} < \min\{W, M\}$ (RR estimate). The rank of $\mathbf{H}^{(k)}$ reflects the minimum number of orthogonal S-T channels that can be used to describe parsimoniously (i.e., with moderate distortion) the matrix $\mathbf{H}^{(k)}$. In a multipath propagation the rank-order depends on the angle of arrival and delay spreads compared to the

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resolution of the array and of the signature waveforms (or their bandwidth). In both methods (FR and RR) the multiple access interference is implicitly taken into account by estimating the K channels $\{\mathbf{H}^{(k)}\}_{k=1}^{K}$ jointly.

A. Full-rank ML estimate

The full-rank approach estimates the S-T matrix $\mathbf{H}$ without taking into account the multipath structure of the channel. According to the model (2), the negative log-likelihood function is (apart from uninteresting constants):

$$\mathcal{L}(\mathbf{H}, \mathbf{R}_{ns}) = \ln|\mathbf{R}_{ns}| + \frac{1}{N}||\mathbf{Y} - \mathbf{H}\mathbf{X}||^2_{\mathbf{R}_{ns}^{-1}}, \quad (3)$$

where $||\mathbf{Y} - \mathbf{H}\mathbf{X}||^2_{\mathbf{R}_{ns}^{-1}} = tr\{\mathbf{R}_{ns}^{-1}(\mathbf{Y} - \mathbf{H}\mathbf{X})(\mathbf{Y} - \mathbf{H}\mathbf{X})^H\}$. The ML estimate of $(\mathbf{H},\mathbf{R}_{ns})$ is obtained by minimizing (3)

$$\hat{\mathbf{H}}_{FR} = \hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}, \quad (4a)$$

$$\hat{\mathbf{R}}_{ns} = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{xy}, \quad (4b)$$

where $\hat{\mathbf{H}}_{FR} = [\mathbf{H}_{FR}^{(1)}, \ldots, \mathbf{H}_{FR}^{(K)}]$, $\hat{\mathbf{R}}_{yx} = \mathbf{Y}\mathbf{X}^H/N$, $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{yy}$ are similarly defined.

The FR channel estimate can be written as

$$\hat{\mathbf{H}}_{FR} = \mathbf{H} + \mathbf{N}_H, \quad (5)$$

where $\mathbf{N}_H$ is the Gaussian estimation error with temporal covariance $\mathbf{R}_{nt} = (\mathbf{X}\mathbf{X}^H)^{-1}$ and spatial covariance $\mathbf{R}_{ns}$. Indeed, by stacking matrix columns into the vectors $\hat{\mathbf{h}}_{FR} = vec\{\hat{\mathbf{H}}_{FR}\}$, $\mathbf{h} = vec\{\mathbf{H}\}$ and $\mathbf{n}_H = vec\{\mathbf{N}_H\}$, the equation (5) can be written as $\hat{\mathbf{h}}_{FR} = \mathbf{h} + \mathbf{n}_H$, where $\mathbf{n}_H$ is the noise term with covariance $\mathbf{R}_{n_H} = E[\mathbf{n}_H\mathbf{n}_H^H] = \mathbf{R}_{n_t} \otimes \mathbf{R}_{ns}$.

The unstructured estimate $\hat{\mathbf{H}}_{FR}$ is unbiased, but the price to be paid for the unbiasedness is the high variance of the estimate. If $|x_n^{(k)}| = 1$ and the training codes have ideal correlation properties, $\mathbf{X}\mathbf{X}^H = N \cdot \mathbf{I}_{KW}$, the mean square error (MSE) of the estimate reduces to

$$E[(||\mathbf{N}_H||^2)] = M\sigma^2 trace[(\mathbf{X}\mathbf{X}^H)^{-1}] = \sigma^2 KMW/N. \quad (6)$$

Equation (6) shows that the error increases with the total number of unknown coefficients (KMW) and decreases with training length $N \longrightarrow \infty$. Performances in terms of MSE can be improved by reducing the number of the parameters that describe the channel matrix (KMW) or equivalently by reducing the complexity of the channel model as described below.

B. Reduced-rank ML estimate: single-user

Let us first consider $K = 1$ (single-user) and assume that $\mathbf{H}$ has known rank $r \le \min\{M, W\}$. The ML estimates of $\mathbf{H}$ and $\mathbf{R}_{ns}$ are obtained by minimizing $\mathcal{L}(\mathbf{H}, \mathbf{R}_{ns})$ (see (3)) under the constraint that $rank\{\mathbf{H}\} = r$.

Let us define the following matrices:

$$\hat{\mathbf{W}} = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{xy}, \quad (7)$$

$$\hat{\mathbf{R}} = \hat{\mathbf{W}}^{-H/2}\hat{\mathbf{R}}_{yx}\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{R}}_{xy}\hat{\mathbf{W}}^{-1/2} = \hat{\mathbf{H}}_w\hat{\mathbf{H}}_w^H, \quad (8)$$

$\hat{\mathbf{W}}$ is an estimate of the noise covariance matrix ($\hat{\mathbf{W}} \rightarrow \hat{\mathbf{R}}_{ns}$ as $N \rightarrow \infty$), $\hat{\mathbf{H}}_w = \hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}\hat{\mathbf{R}}_{xx}^{H/2}$ is the whitened full-rank channel estimate (as discussed below). The ML estimate of the channel and the covariance matrix under the RR constraint is

$$\hat{\mathbf{H}}_{RR} = \hat{\mathbf{W}}^{H/2}\hat{\mathbf{\Pi}}_r\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}, \quad (9)$$

$$\hat{\mathbf{R}}_{ns} = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{H}}_{RR}\hat{\mathbf{R}}_{xy}. \quad (10)$$

$\hat{\mathbf{\Pi}}_r$ is the projector onto the space spanned by the r leading eigenvectors of $\hat{\mathbf{R}}$. The proof is not presented here, however the RR estimate (9) is fully equivalent to the one derived in [10] except that the order of optimization is reversed.

Remark 1: The selection of the r leading eigenvectors that span the signal subspace is performed on the matrix $\hat{\mathbf{H}}_w$, not on $\hat{\mathbf{H}}$, as this implies the decorrelation of the estimation noise $\mathbf{N}_H$ of the FR estimate (see (5)). This whitening is obtained as:

$$\hat{\mathbf{h}}_w = vec\{\hat{\mathbf{H}}_w\} = \hat{\mathbf{R}}_{n_H}^{-H/2}\hat{\mathbf{h}}_{FR} = \quad (11)$$

$$= (\hat{\mathbf{R}}_{xx}^{1/2} \otimes \hat{\mathbf{W}}^{-H/2})\hat{\mathbf{h}}_{FR} = vec\{\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}\hat{\mathbf{R}}_{xx}^{H/2}\},$$

therefore the matrix $\hat{\mathbf{H}}_w$ is referred to as whitened full-rank channel estimate.

Remark 2: The RR ML estimate can be re-written as

$$\hat{\mathbf{H}}_{RR} = \hat{\mathbf{W}}^{H/2}(\hat{\mathbf{\Pi}}_r\hat{\mathbf{H}}_w)\hat{\mathbf{R}}_{xx}^{-H/2}, \quad (12)$$

this highlights that the practical implementation can be carried out as it follows: i) pre-whitening of the full-rank channel estimate (left multiplication by $\hat{\mathbf{W}}^{-H/2}$ and right multiplication by $\hat{\mathbf{R}}_{xx}^{H/2}$ of $\hat{\mathbf{H}}_{FR}$ to obtain $\hat{\mathbf{H}}_w$); ii) truncation of the SVD of $\hat{\mathbf{H}}_w$ to the r largest singular values (projection by $\hat{\mathbf{\Pi}}_r$); iii) cancellation of the pre-whitening (left multiplication by $\hat{\mathbf{W}}^{H/2}$ and right multiplication by $\hat{\mathbf{R}}_{xx}^{-H/2}$).

C. Reduced-rank ML estimate: multi-user

The multi-user estimate under the RR constraint can be obtained similarly to the single-user estimate. The matrices for the ensemble of K users are

$$\hat{\mathbf{R}}^{(k)} = \hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)}\hat{\mathbf{H}}_{FR}^{(k)H}\hat{\mathbf{W}}^{-1/2}, \quad (13)$$

where $\hat{\mathbf{R}}_{xx}^{(k)} = \mathbf{X}^{(k)}\mathbf{X}^{(k)\,H}/N$. The RR estimate is obtained by projecting the full-rank multi-user estimate $\hat{\mathbf{H}}_{FR}^{(k)}$ onto the subspace spanned by the leading (left) eigenvectors of the whitened FR estimate $\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}^{(k)}\hat{\mathbf{R}}_{xx}^{(k)\,H/2}$ as

$$\hat{\mathbf{H}}_{RR} = \hat{\mathbf{W}}^{H/2}\hat{\mathbf{\Pi}}_r^{(k)}\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}; \quad (14)$$

here $\hat{\mathbf{\Pi}}_r^{(k)}$ denotes the corresponding projection matrix onto the subspace spanned by the first $r^{(k)}$ eigenvectors of $\hat{\mathbf{R}}^{(k)}$.

For the MMSE multi-user detector (Section 5) it is needed the estimate of the covariance matrix of noise. The interference due to intercell users is $\hat{\mathbf{N}} = \mathbf{Y} - \sum_{k=1}^{K}\hat{\mathbf{H}}_{RR}^{(k)}\mathbf{X}^{(k)}$ and thus the sample covariance matrix of the noise is $\hat{\mathbf{R}}_{ns} = \hat{\mathbf{N}}\hat{\mathbf{N}}^H/N$.

IV. SELECTION OF RANK-ORDER

In general the rank of the channel matrix is not known and an estimate $\hat{r}$ must be derived. The mean square error of the RR estimate is the sum of the distortion error (that decreases with $\hat{r}$) and the noise error (that increases with $\hat{r}$). The best rank-order is a trade-off between distortion and variance, or equivalently between simplicity and complexity of the channel model. By combining (5) and (9) for $K = 1$ (the user index is omitted), the RR estimate can be written as

$$\hat{\mathbf{H}}_{RR} = \hat{\mathbf{W}}^{H/2}\hat{\mathbf{\Pi}}_{\hat{r}}\hat{\mathbf{W}}^{-H/2}\mathbf{H} + \hat{\mathbf{W}}^{H/2}\hat{\mathbf{\Pi}}_{\hat{r}}\hat{\mathbf{W}}^{-H/2}\mathbf{N}_H. \tag{15}$$

The optimum rank-order r^* for the RR estimate is obtained as

$$r^* = \arg\min_r ||\Delta\mathbf{H}||^2, \tag{16}$$

where $\Delta\mathbf{H} = \mathbf{H} - \hat{\mathbf{H}}_{RR}$. Let consider the MSE $E[||\Delta\mathbf{H}||^2]$ for $N \rightarrow \infty$ and $SNR \rightarrow \infty$ (asymptotic MSE). As $\hat{\mathbf{W}} \rightarrow \mathbf{R}_{ns}$ and $\hat{\mathbf{\Pi}}_{\hat{r}} \rightarrow \mathbf{\Pi}_{\hat{r}}$ ($\mathbf{\Pi}_{\hat{r}}$ is the actual projector onto the space spanned by the $\hat{r}$ largest eigenvalues of $\mathbf{H}_w = \mathbf{R}_{ns}^{-H/2}\mathbf{H}\mathbf{R}_{xx}^{H/2}$) the error becomes:

$$\Delta\mathbf{H} = \mathbf{R}_{ns}^{H/2}(\mathbf{\Pi} - \mathbf{\Pi}_{\hat{r}})\mathbf{R}_{ns}^{-H/2}\mathbf{H} - \mathbf{R}_{ns}^{H/2}\mathbf{\Pi}_{\hat{r}}\mathbf{R}_{ns}^{-H/2}\mathbf{N}_H, \tag{17}$$

where $\mathbf{\Pi} = \mathbf{H}_w(\mathbf{H}_w\mathbf{H}_w^H)^{-1}\mathbf{H}_w^H$. In the trivial case of white noise and training sequence with ideal correlation properties, the asymptotic MSE of the RR estimator reduces to

$$\begin{aligned} E[||\Delta\mathbf{H}||^2] &= tr(\mathbf{H}^H\mathbf{\Pi}_{\hat{r}}^{\perp}\mathbf{H}) + E[tr(\mathbf{N}_H^H\mathbf{\Pi}_{\hat{r}}\mathbf{N}_H)] = \\ &= \sum_{i=\hat{r}+1}^{M} \lambda_i^2 + \frac{\sigma^2\hat{r}W}{N}, \end{aligned} \tag{18}$$

where $\{\lambda_i^2\}_{i=1}^M$ are the square of the singular values of $\mathbf{H}$ arranged in decreasing order. The first term in (18) represents the distortion, the second is the variance of the estimate. It can be noticed that the variance in (18) is reduced by a factor $M/\hat{r}$ with respect to the FR estimate in (6) (with $K = 1$). The optimum value r^* is the one that minimizes the MSE in (18).

According to the statistical properties of the estimate $\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR}\hat{\mathbf{R}}_{xx}^{H/2}$, it can be shown that the rank-order estimated on the base of the MDL criterion [8] on the S-T matrix $\hat{\mathbf{H}}_w$ performs similarly to the optimum rank-order selection. This is illustrated by the example below.

Fig. 1 shows the comparison of the MDL estimate with optimum rank-order r^* (16). In simulations the noise is spatially uncorrelated, the channel matrix $\mathbf{H}$ is randomly generated with $M = 8$, $r = 5$ and $W = 57$. The training sequence is chosen according to UTRA-TDD specifications [9] and its length is $N = 456$. Each simulated value is the result of 500 independent runs of channel and noise. Figure 1(a) compares the MDL estimate with the following rank-order selections: the optimum rank according to (16) (with estimation of the projection matrix) and the rank-order that minimize the expression (18), i.e. for known projection matrix. The figure shows the selected rank-order vs. signal to noise ratio (SNR) defined as the mean signal to noise ratio for single user and single antenna: $SNR = E[(||\mathbf{h}^{(k,m)}||^2)]/\sigma^2$. The results indicate that

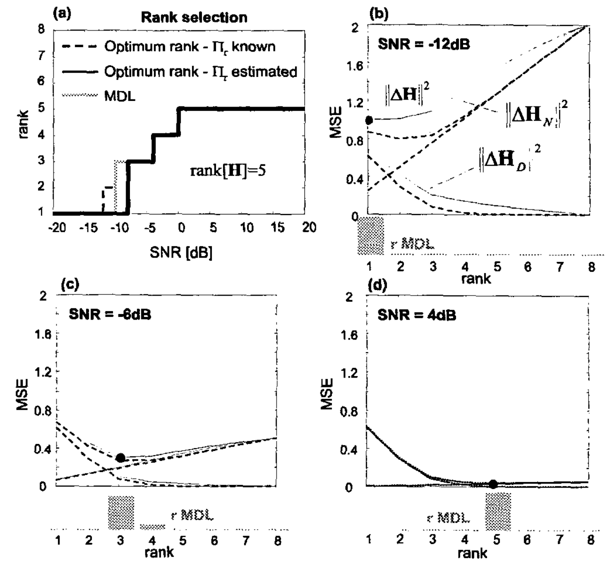

Fig. 1. Selection of the rank order (spatially uncorrelated noise).

for SNR larger than -8dB the MDL estimate performs similarly to the optimum estimate. Figures 1(b), 1(c), 1(d) show the MSE of the RR estimate vs. the rank-order used for the rank reduction, for a few values of SNR $\{-12dB, -6dB, 4dB\}$. Both variance $E[||\Delta\mathbf{H}_N||^2]$ and distortion $E[||\Delta\mathbf{H}_D||^2]$ terms are represented, together with the total MSE $E[||\Delta\mathbf{H}||^2]$ (dashed lines refer to $\hat{\mathbf{\Pi}}_r = \mathbf{\Pi}_r$). In addition, the distribution of the MDL estimate is represented below each graph as an histogram. As expected for low SNR the optimum rank order is $r^* = 1$, for large SNR r^* moves towards the correct rank-order ($r = 5$).

V. SIMULATION RESULTS

The performances of channel estimation techniques are obtained by simulating the uplink of TDD-UTRA standard for IMT-2000 (traffic burst 1), details on the system parameters are not reproposed as can be found in [9]. Numerical results are for $M = 8$ omnidirectional antennas.

For a known rank-order Fig. 2 compares the performances of RR-ML estimate, FR-ML estimate and the rank-r approximation of the FR-ML estimate by SVD truncation to the leading r eigenvalues (ML-FR trunc). Performances are evaluated in terms of normalized MSE $||\Delta\mathbf{H}||^2/||\mathbf{H}||^2$ vs. SNR, for AWGN. The channel matrix $\mathbf{H}$ is randomly generated with $r = 2$ and $W = 57$. The training sequence length is $N = 456$ according to UTRA-TDD specifications [9]. Each simulated MSE value is the result of 500 independent runs of channel and noise, the analytical performances are obtained from (18) for the RR-ML estimate (distortion error is not considered as $\hat{r} = r$) and from (6) for the FR-ML estimate. The SNR degradation due to non ideal correlation properties of the training sequence is negligible as $trace(\mathbf{X}\mathbf{X}^H)^{-1}(N/W) = 0.0837dB$. For low SNR, even thought the rank is known, additional er-

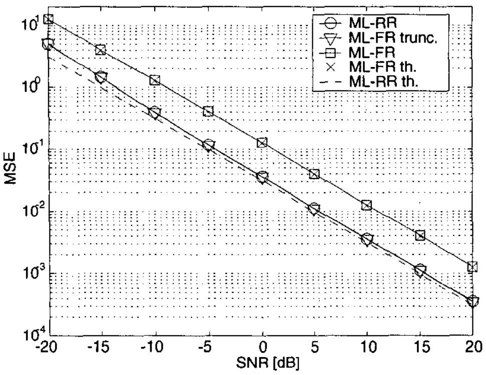

Fig. 2. Channel estimation with $r = 2$ and spatially uncorrelated noise.

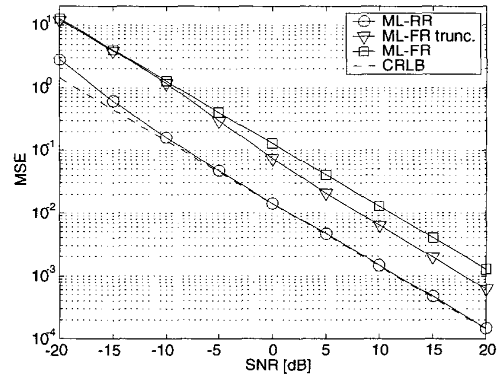

Fig. 3. Channel estimation with $r = 2$ and spatially correlated noise.

rors in the RR estimate are due to $\hat{\mathbf{\Pi}}_r \neq \mathbf{\Pi}_r$. Asymptotically (for SNR>-5dB) distortion becomes negligible and RR estimate outperforms FR estimate by a factor M/r in SNR, i.e. 6dB in this case. The figure indicates that the truncation of the SVD performs similarly to RR estimate in the case of white noise.

With respect to the previous example, in Fig. 3 the Gaussian noise is spatially correlated due to an interferer with DOA 45deg: $[\mathbf{R}_{ns}]_{m,l} = \sigma^2\{0.9\exp(-i\pi\sin(\pi/4))\}^{l-m}$. The truncation of the SVD to the $r = 2$ leading eigenvalues of $\hat{\mathbf{H}}_{FR}$ becomes effective only for high SNR's (for SNR>0dB). However, ML-FR trunc. and ML-FR methods perform notably worse than ML-RR in this scenario. Moreover, for large SNR the RR estimate attains the lower bound (CRLB) derived in [10].

In order to evaluate the performance of the RR-ML estimate with MDL rank selection in realistic propagation environments, the COST-259 Directional Channel Model [11] is considered. This is a stochastic model (an evolution of COST-207) that includes both azimuthal and temporal dispersions. The channel response for k-th user depends on $N_g + 1$ independent groups of scatterers referred to as clusters (user index is understood):

$$\mathbf{H}^{(k)} = \sum_{c=1}^{N_g+1}\sum_{\ell=1}^{N_p(c)} \alpha_{c,\ell}\mathbf{a}(\vartheta_{c,\ell})\mathbf{g}(\tau_{c,\ell})^T$$

the c-th cluster is the superposition of $N_p(c)$ multipaths, each characterized by the direction of arrival ($\vartheta_{c,\ell}$), the delay ($\tau_{c,\ell}$) and the complex amplitude ($\alpha_{c,\ell}$) that accounts for power fluctuations. The vector $\mathbf{g}(\tau_{c,\ell})$ is the temporal channel (of length W) for the ℓ-th path of the c-th cluster and $\mathbf{a}(\vartheta_{c,\ell})$ is the array gain for $\vartheta_{c,\ell}^{(\ell)}$. All the parameters in the model are independent random variables whose probability density functions are assigned according to the specific propagation environment. The line-of-sight cluster is always present, the number of additional clusters is Poisson distributed with mean $\bar{N}_g$, $N_g \sim \mathcal{P}(\bar{N}_g)$. The c-th cluster has azimuth $\bar{\vartheta}_c \sim \mathcal{U}[-\pi/3, \pi/3]$ (for $c = 1, .., N_g + 1$), delay $\bar{\tau}_c = \bar{\tau}_1 + \delta\bar{\tau}_c$ with $\delta\bar{\tau}_c \sim \mathcal{U}[0, \tau_{\max}]$ (for $c = 2, ..., N_g + 1$). The power of each cluster depends on the path loss and has an independent lognormal shadow fading (standard deviation 9dB). According to [12] within the c-th cluster the number of paths is $N_p(c) \sim \mathcal{P}(25)$; the ℓ-th path has delay $\tau_{c,\ell} = \bar{\tau}_c + \delta\tau_{c,\ell}$ with $\delta\tau_{c,\ell}$ exponentially distributed with standard deviation σ_τ, angle $\vartheta_{c,\ell} = \bar{\vartheta}_c + \delta\vartheta_{c,\ell}$ with $\delta\vartheta_{c,\ell} \sim \mathcal{N}(0, \sigma_\vartheta^2)$, complex amplitude $\alpha_{c,\ell}$ subject to fast fading (the power profile function is exponential in both azimuths and delays). Four different generalized radio environments have been identified: GTU (Generalized Typical Urban), GRA (Generalized Rural Area), GBU (Generalized Bad Urban), GHT (Generalized Hill Terrain). The parameters for GTU are $\bar{N}_g = 0.1$, $\sigma_\tau = 1.17\mu s$, $\sigma_\vartheta = 13.4\,\text{deg}$, $\tau_{\max} = 10\mu s$; for GRA, $\bar{N}_g = 0.1$, $\sigma_\tau = 0.23\mu s$, $\sigma_\vartheta = 6.7\,\text{deg}$, $\tau_{\max} = 15\mu s$.

Fig. 4 shows the performance of RR-ML estimate with MDL rank selection for GTU and GRA propagation environments. $K = 6$ users are simultaneously active (same parameters as in previous examples), the noise is either white (top figures) or spatially correlated (below). The spatially correlated noise is

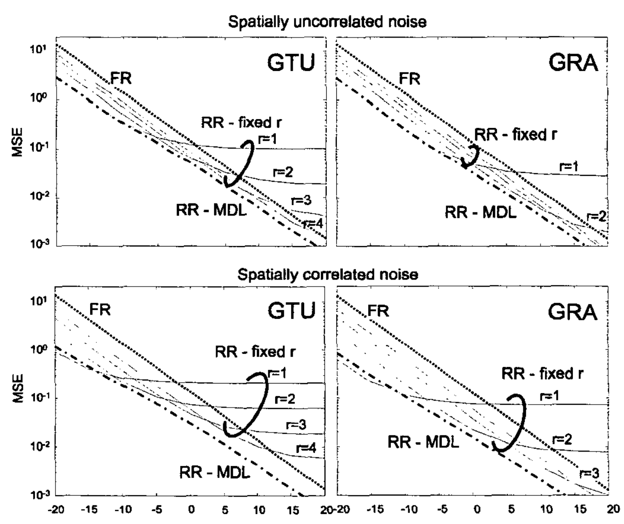

Fig. 4. Performance of channel estimation for COST-259 radio enviroments.

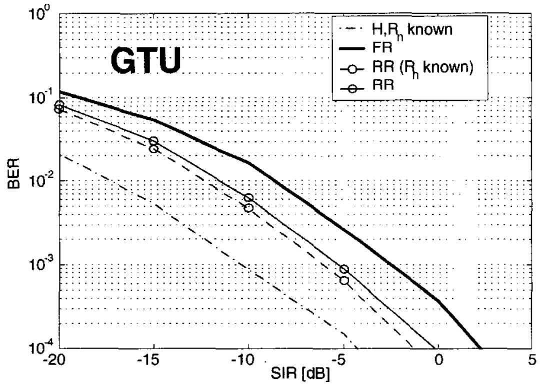

Fig. 5. Performance of MMSE multiuser detector with multi-user ML-RR estimate with MDL selection of the rank and multi-user ML-FR estimate [GTU channel, $K = 6$ intracell users, 6 intercell interferers].

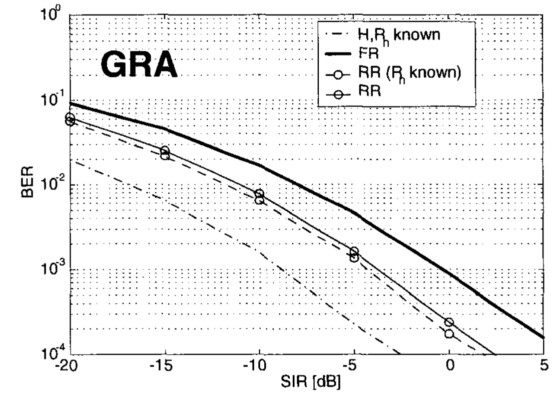

Fig. 6. Performance of MMSE multiuser detector with multi-user ML-RR estimate with MDL selection of the rank and multi-user ML-FR estimate [GRA channel, $K = 6$ intracell users, 6 intercell interferers].

generated by 6 intercell interferers with log-normal shadowing (standard deviation is 13dB as it is increased according to the assumption of perfect power control for all the users). In a severe interfering environment low rank approximation ($r = 1$) is the preferred solution as it has the least number of unknowns to be estimated. For increasing SNR distortion becomes remarkable and higher rank-order is needed. Different environments call for different rank-order depending on the interference level, thus the estimation of optimum rank is mandatory. The MDL selection in the examples shows that the RR-ML estimate achieves the minimum MSE for all SNR values.

The performances of the receiver (complete of channel estimation and MMSE multi-user detection [1]) are evaluated in terms of BER for uncoded bits vs. SIR (a severe interfering environment is simulated, as for Fig. 4). The existence of ISI in conjunction with MAI is assumed, therefore a MMSE detector for multi-user channels subject to intersymbol interference is adopted. The use of the multi-user detection for a sequence of N_b symbols implies the inversion of the $KN_b \times KN_b$ cross-correlation matrix of the KN_b fictitious users [1], the inversion can be efficiently performed by exploiting the block Toeplitz structure of the matrix [13]. According to TDD-UTRA standard [9] OVSF codes have $Q = 16$, raised cosine pulse with roll-off 0.22 and chip rate 3.84 Mchips/s, QPSK modulation is considered. The fading of GTU and GRA propagation models is without Doppler effect. If a fixed low rank order is adopted for the RR-ML estimate (for example rank 1) the distortion becomes remarkable for large SIR and RR-ML estimate performs worse than FR-ML estimate as shown in [7]. In order to overcome this limit here the rank-order is selected adaptively with the interference level by MDL criterion: the RR-ML estimate (solid line with marks) outperforms the FR-ML estimate (solid line) by at least 3dB in SIR either for GTU and GRA and even for large SIR.

VI. Conclusion

A reduced complexity space-time channel estimation for multi-user communication has been presented. The proposed method is based on an under-parameterization of the channel that constraints the ML estimate of each user to have a reduced-rank. The MDL criterion for the rank-order estimation provides the best trade-off between distortion and variance. Simulations proved that reduced-rank estimate outperforms the unconstrained estimate for all SNR values.

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