Adaptive-rank receiver for space-time DS CDMA systems

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Abstract

In this paper we propose a reduced rank space-time receiver for multi-user time slotted CDMA systems. The channel estimation is based on an under-parameterization of the space-time channel model. The channels for all users are estimated jointly by constraining the channel matrix of each user to be low-rank. As the rank depends on the interference level and the characteristics of the propagation environment (delay and angle spreads), the rank-order is estimated adaptively for each user according to the MDL criterion (Minimum Description Length).

1 Introduction

DS CDMA is the preferred multiple access technique for third generation cellular systems. In order to improve the capacity of CDMA systems, integration of space-time processing and multi-user detection (S-T multi-user detection, [1]) can be adopted in the uplink to reduce the interference. This approach requires the estimation of the S-T features of the channel for all the active users. In a time-slotted CDMA system channel estimation is based on the transmission of training sequences known at the receiver. The performance of the estimate is limited by the reduced length of these training sequences and by the large number of channel parameters that have to be estimated. In this paper we propose to reduce the number of unknowns in the channel matrix (parsimonious parametrization) by the reduced rank (RR) approach [2] extended to the multi-user case [3]. The channels of all the users are estimated jointly, with the constraint that the S-T channel matrix for each user is low-rank. The rank-order is estimated according to the MDL (Minimum Description Length) criterion [4]. With respect to [3] here the linear multi-user detection has been modified for S-T receivers where the propagation channel of each user is modelled by the combination of a few rank-one orthogonal S-T matrices (reduced rank channel).

The paper is organized as follows. The discrete-time model for the uplink of a time-slotted CDMA system is considered in Section 2; Section 3 describes the RR channel estimation with rank-order selection and Section 4 presents the S-T multi-user detector based on the RR approach. Simulation results are given in Section 5 for the uplink TDD-UTRA standard [5] and concluding remarks are finally discussed.

2 Problem formulation

The equivalent lowpass model of the uplink of a hybrid TD/CDMA mobile radio system is considered. Within the same cell, in the same frequency band and in the same time slot, K users are simultaneously active. Each mobile station (MS) is equipped with a single transmitter antenna, at the base station (BS) an antenna array of M elements is employed. Here the discrete-time model of the uplink is obtained by sampling at the chip rate $1/T_c$ the signals received by the M antennas after the chip matched filter. The baseband channel between the k-th user's transmitter and the BS receiver can be described by the $M \times W$ S-T matrix $\mathbf{H}_k = [\mathbf{h}_{k,1}, \ldots, \mathbf{h}_{k,M}]^T$ that consists of the M vectors $\mathbf{h}_{k,m}$ $(1 \leq m \leq M)$, representing the discrete-time channel impulse response for the link between the k-th user and the m-th antenna (W is the channel length). The propagation channel can be modelled as the superposition of the L_k multipath each characterized by the direction of arrival $(\vartheta_{k,\ell})$, the delay $(\tau_{k,\ell})$ and the complex gain $(\alpha_{k,\ell})$

$$\mathbf{H}_k = \sum_{\ell=1}^{L_k} \alpha_{k,\ell}\mathbf{a}(\vartheta_{k,\ell})\mathbf{g}(\tau_{k,\ell})^T, \tag{1}$$

here the vector $\mathbf{g}(\tau_{k,\ell})$ of length W represents the chip pulse shape delayed by $\tau_{k,\ell}$, $\mathbf{a}(\vartheta_{k,\ell})$ is the array gain for $\vartheta_{k,\ell}$.

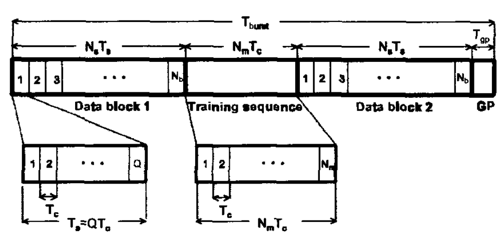

Figure 1. Burst structure for the uplink of a hybrid TD/CDMA system.

In the hybrid TD/CDMA system the users transmit in bursts, each consisting of two data blocks and a user specific training sequence (midamble) for channel estimation (see Fig. 1). At the first order the propagation channel can be assumed to be time-invariant within the burst interval so that the channel can be estimated for the whole burst from the training sequence. Each data block contains N_s QPSK symbols of duration $T_s = QT_c$ spread by a user-specific signature of Q chips, $\mathbf{c}_k = [c_k(1), \ldots, c_k(Q)]^T$ for the k-th user. The midamble consists of $N_m = N + W - 1$ chips. At the BS the received signals are separated into two subsets of samples, the subset depending on the midamble training sequences and the subset depending on the two data blocks of symbols. The estimation of the S-T channels $\mathbf{H}_k$, for $k = 1, ..., K$, from the first subset of signals is developed by the RR method as described in Section 2. Then S-T multiuser detection is carried out on the two blocks of signals received before and after the midamble in order to estimate the $2N_sK$ data symbols in the burst; this is discussed in Section 3 using the knowledge of the RR channel estimates $\hat{\mathbf{H}}_k$ and the signature sequences $\mathbf{c}_k$, for $k = 1, ..., K$.

3 Channel estimation

3.1 RR channel model

Let consider the received signals that depend exclusively on the transmitted training sequences, each training sequence is arranged in a $W \times N$ Toeplitz matrix $\mathbf{X}_k$ that represents the convolution operation. Let $\mathbf{Y} = [\mathbf{y}(1), \ldots, \mathbf{y}(N)]$ be the $M \times N$ matrix containing N samples of the signals received by the M antennas, $\mathbf{H} = [\mathbf{H}_1, \ldots, \mathbf{H}_K]$ and $\mathbf{X} = [\mathbf{X}_1^T, \ldots, \mathbf{X}_K^T]^T$ are multiuser matrices suitably defined to include, respectively, the K channel matrices and the K training sequence matrices. The received signal can be written as

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{X}_k + \mathbf{N} = \mathbf{H}\mathbf{X} + \mathbf{N}, \tag{2}$$

where $\mathbf{N} = [\mathbf{n}(1), \ldots, \mathbf{n}(N)]$ models both the additive ambient noise and the inter-cell interference. The noise is temporally uncorrelated and spatially correlated $\mathbf{n}(n) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{R}_{ns})$, σ^2 is the variance at each antenna element, the space covariance matrix $\mathbf{R}_{ns}$ is normalized such that $[\mathbf{R}_{ns}]_{i,i} = 1$, for $i = 1, ..., M$.

The unconstrained (full-rank, FR) maximum likelihood estimate of the multi-user channel is

$$\hat{\mathbf{H}}_{FR} = [\hat{\mathbf{H}}_{FR,1}, ..., \hat{\mathbf{H}}_{FR,K}] = \mathbf{Y}\mathbf{X}^H(\mathbf{X}\mathbf{X}^H)^{-1}, \tag{3}$$

where $(.)^H$ denotes the Hermitian transposition. The mean square error (MSE) of the FR estimate depends on the ratio KMW/N between the total number of unknown coefficients and the training sequence length. As the latter is limited and fixed, estimation performance can be improved only by reducing the number of the parameters describing the channel matrix, or equivalently by reducing the complexity of the channel model [3].

This is carried out classically by exploiting the multipath structure of the channel (structured approach), i.e. by estimating the $3L_k$ multipath unknowns in (1) by any angle/delay estimation algorithm, such as [6]. Here we consider a different unstructured method that exploits the low rank nature of the channel matrix. Let the S-T channel be described by a matrix $\mathbf{H}_k$ of rank $r_k \ll \min(W, M)$; the rank-r_k matrix can be rewritten as the combination of two full-rank matrices

$$\mathbf{H}_k = \sum_{r=1}^{r_k} \mathbf{a}_{k,r} \mathbf{b}_{k,r}^H = \mathbf{A}_k \mathbf{B}_k^H \tag{4}$$

where the space and time (unstructured) matrices $\mathbf{A}_k = [\mathbf{a}_{k,1}, ..., \mathbf{a}_{k,r_k}]$ and $\mathbf{B}_k = [\mathbf{b}_{k,1}, ..., \mathbf{b}_{k,r_k}]$ have dimensions $M \times r_k$ and $W \times r_k$, respectively. As an example, a rank-1 channel ($r_k = 1$) is obtained from (1) by setting $\vartheta_{k,\ell} = \vartheta_k$, $\forall \ell$ (no angle spread) or equivalently by assuming $\tau_{k,\ell} = \tau_k$, $\forall \ell$ (no delay spread). This type of rank-1 channel seems to never occur in realistic propagation environments as the channel impulse response is always characterized by both angular and temporal dispersions. However, paths can be grouped into clusters of scatterers that can no longer be resolved in τ or ϑ (i.e., clusters have reduced angle/delay spread). In this case the rank-order is approximately given by the number of clusters, that is usually lower than $\min(W, M)$. Clearly rank-order depends on the resolution of the array and on the pulse bandwidth. In [3] it was proved that the rank order of realistic channels is in the order of $1 \div 4$ for an array of $M = 8$ antennas and bandwidth as for the TDD-UTRA standard [5]. Hence, the estimation of the $(M + W)r_k$ parameters, that describe the channel matrix in (4), can be carried out by the RR multi-user algorithm reproposed below from [3].

3.2 RR channel estimation and rank-order selection

Let us define the following matrices :

$$\begin{aligned} \hat{\mathbf{W}} &= \mathbf{Y}\mathbf{\Pi}^{\perp}(\mathbf{X}^H)\mathbf{Y}^H, & (5) \\ \hat{\mathbf{R}}_k &= \hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_{FR,k}\mathbf{X}_k\mathbf{X}_k^H\hat{\mathbf{H}}_{FR,k}^H\hat{\mathbf{W}}^{-1/2}, & (6) \\ \hat{\mathbf{U}}_k &= eigv_{r_k}\{\hat{\mathbf{R}}_k\}, & (7) \end{aligned}$$

$\hat{\mathbf{W}}^{1/2}$ is the Cholesky factor of $\hat{\mathbf{W}}$, $eigv_r\{\hat{\mathbf{R}}_k\}$ denotes the $M \times r_k$ matrix of the r_k leading eigenvectors of the matrix $\hat{\mathbf{R}}_k$, $\hat{\mathbf{H}}_{FR,k}$ is the FR channel estimate for the k-th user, $\mathbf{\Pi}^{\perp}(\mathbf{X}^H)$ is the orthogonal projector onto the null space of $\mathbf{X}$. The joint estimate of the K channels under the

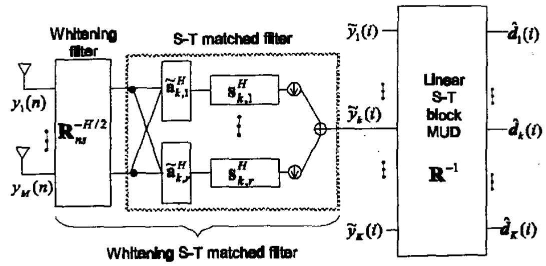

Figure 2. S-T multi-user receiver with RR channel model.

constraint that the S-T matrix of the k-th user has rank r_k, for $k = 1, ..., K$, is [7]:

$$\hat{\mathbf{A}}_k = \hat{\mathbf{W}}^{H/2}\hat{\mathbf{U}}_k \quad (8)$$

$$\hat{\mathbf{B}}_k = \hat{\mathbf{H}}_k^H \hat{\mathbf{W}}^{-1/2}\hat{\mathbf{U}}_k \quad (9)$$

$$\hat{\mathbf{H}}_k = \hat{\mathbf{A}}_k\hat{\mathbf{B}}_k^H = \hat{\mathbf{W}}^{H/2}\mathbf{\Pi}_{r_k}(\hat{\mathbf{R}}_k)\hat{\mathbf{W}}^{-H/2}\hat{\mathbf{H}}_k \quad (10)$$

where $\mathbf{\Pi}_{r_k}(\hat{\mathbf{R}}_k) = \hat{\mathbf{U}}_k\hat{\mathbf{U}}_k^H$ is the projector onto the space spanned by the r_k leading eigenvectors of $\hat{\mathbf{R}}_k$. The noise covariance matrix is estimated from the residuals of the channel estimation $\hat{\mathbf{N}} = \mathbf{Y} - \sum_{k=1}^{K}\hat{\mathbf{H}}_k\mathbf{X}_k$, as: $\hat{\mathbf{R}}_n = \hat{\mathbf{N}}\hat{\mathbf{N}}^H/N$.

The rank order r_k is not known and an estimate $\hat{r}_k$ has to be derived. The optimum $\hat{r}_k$ is the one that minimizes the mean squared error of the RR estimate. The latter is the sum of two terms [3]: a distortion error, that is large for low $\hat{r}_k$ (i.e. $\hat{r}_k < r_k$) and depends on the characteristics of the propagation channel (delay/angle spreads, number of clusters), and a noise error that increases with r_k and depends on the interference level. For this reason the rank order has to be estimated adaptively with the interference level and the propagation environment, as a trade-off between distortion and noise variance. Here the estimation of the rank-order is carried out according to the detection criterion derived in [4] for the estimation of the number of sources impinging on an array. This criterion is based of the minimum description length principle (MDL). The estimation is performed on the sample covariance matrix $\hat{\mathbf{R}}_k$ and depends on the eigenvalues of $\hat{\mathbf{R}}_k$. In [3] it was proved that the rank-order selection by the MDL criterion is uniformly optimum with respect to fixed-rank or user-independent rank-order selection.

4 Data estimation

In this section S-T multi-user detection with the RR channel model is performed as in [1], with the difference that here the channel matrix is not structured, i.e., each spatial component $\mathbf{a}_{k,\ell}$ of the channel is different from the steering vector $\mathbf{a}(\vartheta_{k,\ell})$ and each temporal component $\mathbf{b}_{k,\ell}$ is not the single delayed signature $\mathbf{g}(\tau_{k,\ell})$.

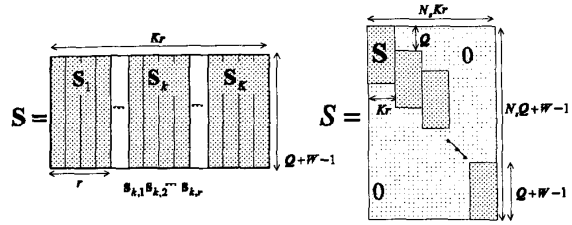

Figure 3. Structure of matrices S **and** $\mathcal{S}$.

4.1 System model

The transmission of one single data block (before or after the midamble) is considered and the influence of the midamble on the data block due to the temporal dispersion of the channel is assumed to be perfectly cancelled for all the users. Let $d_k(i)$ denote the i-th symbol transmitted by the k-th user, $\mathbf{d}(i) = [d_1(i) \cdots d_K(i)]^T$ is the vector containing the i-th symbol for all the K users, $\mathbf{d} = [\mathbf{d}(1)^T \cdots \mathbf{d}(N_s)^T]^T$ is the overall data vector of length N_sK. User k is assigned a specific spreading sequence $\mathbf{c}_k$, the convolution of the latter with the ℓ-th temporal channel $\mathbf{b}_{k,\ell}$ is denoted by the composite signature $\mathbf{s}_{k,\ell} = [s_{k,\ell}(1), \ldots, s_{k,\ell}(Q+W-1)] = \mathbf{C}_k\mathbf{b}_{k,\ell}$ ($\mathbf{C}_k$ is the convolution matrix for the k-th code), with $\ell = 1, ..., r$. For simplicity of notation all the users are assumed to have the same rank order $r_k = r, \forall k$.

The complex baseband representation for the n-th sample $\mathbf{y}(n)$ of the signal received by the antenna array (after chip-matched filtering and chip-rate sampling) can be written as

$$\mathbf{y}(n) = \sum_{i=1}^{N_s}\sum_{k=1}^{K} d_k(i)\sum_{\ell=1}^{r}\mathbf{a}_{k,\ell}s_{k,\ell}(n-(i-1)Q)+\mathbf{n}(n), \quad (11)$$

the noise is temporally uncorrelated and spatially correlated, $\mathbf{n}(n) \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{R}_{ns})$. A sufficient statistic for the detection of the user data $\mathbf{d}$ in (11) for is the output of the S-T whitening matched filter for the K users, $\tilde{\mathbf{y}}(i) = [\tilde{y}_1(i) \cdots \tilde{y}_K(i)]^T$ for $i = 1, ..., N_s$ [1]. As shown in Fig. 2, $\tilde{\mathbf{y}}(i)$ is obtained by whitening the received signal $\mathbf{y}(n)$ by the spatial filter $\mathbf{R}_{ns}^{-H/2}$, then the whitened signal is passed through the Kr beamformers $\tilde{\mathbf{a}}_{k,\ell}^H = (\mathbf{R}_{ns}^{-H/2}\mathbf{a}_{k,\ell})^H$, for $k = 1, ..., K$ and $\ell = 1, .., r$ (whitening space matched filters). Each beamformer is followed by the corresponding temporal filter $\mathbf{s}_{k,\ell}^H$. Finally $\tilde{y}_k(i)$ is derived by combining the outputs of these r filters according to the maximal ratio combining criterion [1]. The discrete time model for the signals after S-T matched filtering is derived below.

For the k-th user the r temporal signatures and the r whitening spatial filters are $\mathbf{S}_k = \mathbf{C}_k\mathbf{B}_k$ and $\tilde{\mathbf{A}}_k = \mathbf{R}_{ns}^{-H/2}\mathbf{A}_k$, respectively; the corresponding spatial and temporal multi-user matrices are $\mathbf{S} = [\mathbf{S}_1 \cdots \mathbf{S}_K]$ and $\tilde{\mathbf{A}} = [\tilde{\mathbf{A}}_1 \cdots \tilde{\mathbf{A}}_K]$. The spatial correlation matrix is given by $\mathbf{R}_s = \tilde{\mathbf{A}}^H \tilde{\mathbf{A}}$; the temporal correlation matrix for the overall data block is $\mathbf{R}_t = \mathcal{S}^H \mathcal{S}$, where $\mathcal{S}$ is the matrix of dimensions $(N_s Q + W - 1) \times N_s K r$ defined as (see Fig. 3)

$$\mathcal{S}_{Q(i-1)+n, Kr(i-1)+\ell} = \begin{cases} \mathbf{S}_{n,\ell} & \begin{array}{l} i = 1, \ldots, N \\ n = 1, \ldots, Q + W - 1 \\ \ell = 1, \ldots, Kr \end{array} \\ 0 & \text{else} \end{cases} \tag{12}$$

$\mathbf{R}_t$ has the block Toeplitz structure denoted by $\mathbf{R}_t = \mathcal{T}_{N_s}([\mathbf{R}_t^{(0)}, ..., \mathbf{R}_t^{(\Delta)}])$ as shown below

$$\mathbf{R}_t = \begin{bmatrix} \mathbf{R}_t^{(0)} & \mathbf{R}_t^{(1)} & \ldots & \mathbf{R}_t^{(\Delta)} & & \\ \mathbf{R}_t^{(-1)} & \mathbf{R}_t^{(0)} & \mathbf{R}_t^{(1)} & \ldots & \mathbf{R}_t^{(\Delta)} & \\ \ddots & & & & \ddots & \\ & \mathbf{R}_t^{(-\Delta)} & \ldots & \mathbf{R}_t^{(-1)} & \mathbf{R}_t^{(0)} & \mathbf{R}_t^{(1)} \\ & & \mathbf{R}_t^{(-\Delta)} & \ldots & \mathbf{R}_t^{(-1)} & \mathbf{R}_t^{(0)} \end{bmatrix}, \tag{13}$$

where $\Delta = \left\lfloor \frac{Q+W-1}{Q} \right\rfloor$ is the temporal spread in symbol intervals, each block $\mathbf{R}_t^{(i)} = \mathbf{R}_t^{(-i)H}$ has dimensions $Kr \times Kr$ $(i = 0, ..., \Delta)$. The overall S-T correlation matrix is $\mathbf{R} = \mathcal{T}_{N_s}([\mathbf{R}^{(0)}, ..., \mathbf{R}^{(\Delta)}])$, each $K \times K$ block $\mathbf{R}^{(i)}$ depends on the Hadamard (element-wise) product between the space and the time correlation matrices previously defined:

$$\mathbf{R}^{(i)} = \mathbf{G}^H \mathbf{R}_s \circ \mathbf{R}_t^{(i)} \mathbf{G}$$

with $\mathbf{G} = \mathbf{I}_{KN} \otimes \mathbf{1}_r$, $\mathbf{1}_r$ is a r-dimensional vector of ones. By arranging the matched filter outputs for all symbols into the vector $\tilde{\mathbf{y}} = [\tilde{\mathbf{y}}^T(1) \ldots \tilde{\mathbf{y}}^T(N_s)]^T$ the system model for the overall data block reduces to

$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{d} + \tilde{\mathbf{n}} \tag{14}$$

where the noise vector is $\tilde{\mathbf{n}} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$.

4.2 S-T multi-user detection

From Fig. 2 the outputs of the S-T matched filter (14) are fed into a linear S-T multi-user detector that estimates the user data $\mathbf{d}$. The ideal detector removes both inter-symbol interference and multiple access interference by processing the whole received data block $\tilde{\mathbf{y}}$: $\hat{\mathbf{d}} = \mathbf{M}\tilde{\mathbf{y}}$. In particular, the decorrelating detector is $\mathbf{M} = \mathbf{R}^{-1}$, the MMSE detector is $\mathbf{M} = (\mathbf{R} + \sigma^2 \mathbf{I}_{NK})^{-1}$ if the data symbols are independent and uniformly distributed.

In both cases, most of the computational complexity of the linear system resolution comes from the Cholesky factorization of the sparse and large correlation matrix. As $\mathbf{R}$ has a block Toeplitz structure, this factorization requires in the order of $O[K^3 N_s \Delta]$ operations [8]. If the size of the data block N_s is very large ideal detection may not be feasible. A more practical solution can be derived by using a sliding observation window that spans $N_a \ll N_s$ symbol lengths [7]. An alternative solution is an approximation of the Cholesky factor that exploits the block Toeplitz structure of $\mathbf{R}$ and the property that $\mathbf{R}^{(i)} = \mathbf{0}$ for $|i| > \Delta$, with $\Delta \ll N_s$. It turns out that the Cholesky factor $\mathbf{R}^{1/2}$ is nearly block Toeplitz and can be calculated from the factorization of a $N_a K \times N_a K$ matrix [8]. For both methods the computational complexity is reduced to $O[K^3 N_a \Delta]$, with a negligible degradation in performance.

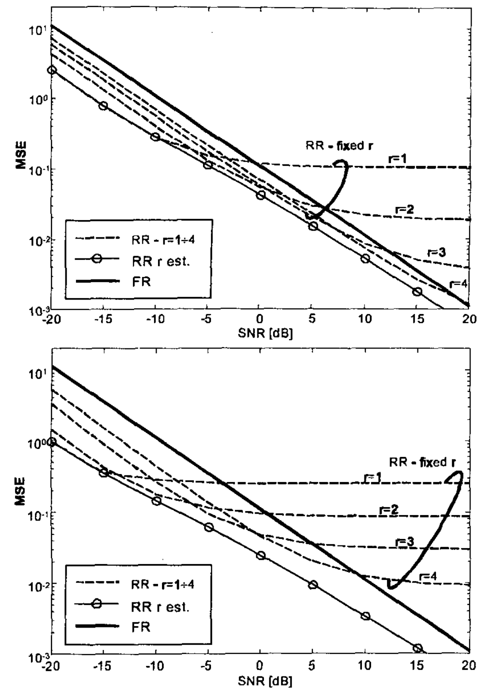

Figure 4. Performance of the RR channel estimation for GTU channel, spatially uncorrelated noise (top) and spatially correlated noise (bottom).

5 Simulation results

In the examples below the performance of RR channel estimation and S-T MUD are evaluated by simulating the uplink of the UTRA-TDD standard [5]. Numerical results are for $M = 8$ omnidirectional antennas, $K = 8$ users are active in the cell, the channel-length is $W = 57$. The traffic burst 1 of TDD-UTRA is considered, spreading codes have $Q = 16$, raised cosine pulse with roll-off 0.22 and chip rate

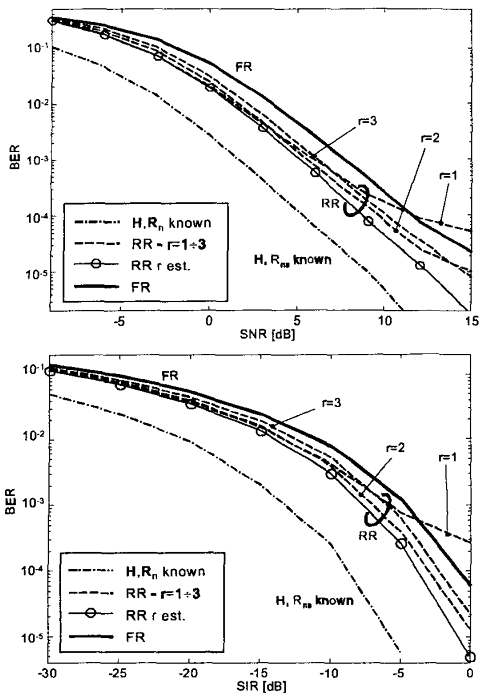

Figure 5. Performance of the MMSE S-T multi-user detector with RR and FR estimates: GTU channel, K=8, spatially uncorrelated noise (top) and spatially correlated noise (bottom).

3.84 Mchips/s. Data blocks of $N_s = 61$ QPSK modulated symbols are considered, training sequences are chosen according to standard specifications with length $N_m = 512$. The propagation channel is simulated as defined by the Generalized Typical Urban environment (GTU) of COST-259 Directional Channel Model [3]. In the simulations both spatially uncorrelated and correlated noise (generated by 3 intercell interferers) are considered.

In Fig. 4 the performance of the RR channel estimate are compared with the FR estimate in terms of the normalized MSE $||\Delta\mathbf{H}||^2/\,||\mathbf{H}||^2$ vs. the signal to noise ratio $SNR = E[(||\mathbf{h}^{(k,m)}||^2)]/\sigma^2$. As expected for low SNR the rank-1 approximation is the preferred solution as it has the least number of unknowns to be estimated. For large SNR the distortion becomes remarkable and a higher rank-order is needed. The RR channel estimate with MDL selection of rank order (circle line) outperforms the fixed-rank estimates (with $r = 1, 2, 3, 4$, dashed lines) and the FR estimate (thick line) for all the SNR values.

The receiver, complete with channel estimation and S-T MMSE multi-user detection, is considered in Fig. 5. At the top performance is evaluated in terms of BER for uncoded bits vs. $SNR = E_{bit}/N_0$, for uncorrelated noise, at the bottom 3 intercell interferers are considered and performance is given in terms of BER vs. signal interference ratio (SIR). If a fixed rank order is adopted for the RR estimate (dashed lines) the distortion becomes very marked for large SIR/SNR, and the RR estimate performs worse than the FR (thick line); the adaptive selection of rank order by MDL criterion (circle line) guarantees a gain with respect to a FR of approx. 3dB for all SNR/SIR values.

6 Conclusion

A S-T multi-user receiver based on RR channel estimation has been presented. Rank-order is selected adaptively according to the interference level and the characteristics of the propagation environment. Simulations have proved that the RR channel estimation adopted in a S-T MMSE multi-user detector provides improvements in system performance (evaluated in terms of BER) by at least 3dB in signal to noise ratio (SNR).

Acknowledgment

The authors would like to thank the SIEMENS-ICN Italy for supporting the research.

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