Multislot Estimation of Fast-Varying Space-Time Channels in TD-CDMA Systems

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***Abstract*—In this letter we propose a subspace method to estimate the time-varying space-time channel by taking into account the different rates of variation between delays/directions of arrival (slowly-varying) and faded amplitudes (fast-varying) of the multipath propagation. The stationarity of angles and delays across several time-slots is exploited to perform an unstructured estimation that avoids the explicit computation of the path parameters. Numerical analysis for a TD-CDMA system shows that the asymptotic performance is reached with a reasonable number of slots.**

***Index Terms*—Antenna array, channel estimation, multi-slot, subspace method, TD-CDMA, TD-SCDMA, ULTRA-TDD.**

I. Introduction

In MOBILE communication systems the channels are time-varying due to the movement of the terminals. Each path of the multipath propagation channel can be characterized by an angle (or direction of arrival), a delay and an amplitude. The angle/delay pattern is stationary compared to the amplitudes (even for fast moving mobile terminals) and therefore a receiver with a linear antenna array could exploit these different varying rates to improve the accuracy in the estimate of the space-time channel matrix. This is even more important when the training sequences used for the estimation have to be kept short to preserve the transmission efficiency. In this letter we propose a method that exploits these slow/fast variations to improve the estimate performance. We focus on time-slotted CDMA systems (such as the time-division synchronous CDMA standard, TD-SCDMA [1]), nonetheless the method appears to be a valid solution for a wider class of systems. The space-time bases that describe the angle/delay pattern are estimated by assuming the stationary over a set of L slots, while the amplitudes can be fast-varying and have to be calculated on a slot-by-slot basis.

In multislot (MS) processing, the accuracy of the channel estimate can be increased by averaging the information from successive slots so as to extend the effective training data length. MS averaging has been proposed for slow-fading channels [2]. For fast fading channels the slowly varying features of the propagation channel can still be exploited by explicitly estimating angles and delays with high resolution (and computationally expensive) methods [3]. Here we propose to estimate not the angle/delay pattern but the subspace spanned by the corresponding spatial/temporal channel signatures from a set of L slots where the subspace remains stationary.

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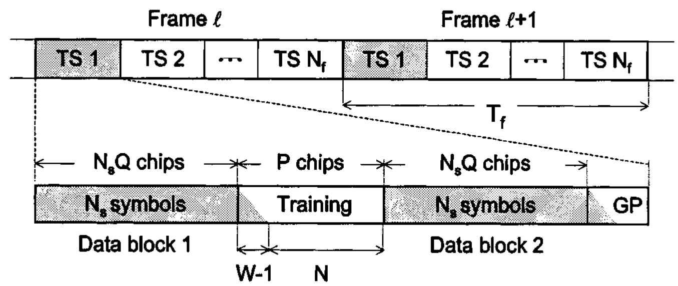

Fig. 1. Frame structure of a TD-CDMA system.

The angle (or delay) stationarity should be compared to the spatial (or temporal) resolution of the array (or the waveform). To illustrate the meaning, let us consider the uplink of a TD-SCDMA [1] system with an array of $M = 8$ half wavelength spaced elements and a mobile terminal at a distance of 500 m from the base station. The channel can be considered as stationary within $L = 10$ frames (100 ms) as far as the velocity of the mobile terminal is below 1000 km/h (in this case, the temporal resolution is more restrictive compared to spatial resolution).

II. TD-CDMA Signal Model

The frame of a hybrid TD-CDMA system (see Fig. 1) contains N_f time-slots, each can be allocated to either the uplink or the downlink. We will concentrate on the uplink and assume that the base station is equipped with a linear antenna array of M half wavelength spaced elements. Within each frame one time-slot is considered in which K users are transmitting data at the same carrier frequency but using different spreading codes. During the time-slot the users transmit a burst that consists of two data fields of N_s information symbols (spread by a code of length Q), a midamble of P chips and a guard period. To simplify, only one time-slot is allocated in each frame to the K users, so that the temporal interval between two successive bursts equals the frame duration T_f. In this case the multi-slot approach is carried out on a frame-by-frame basis as multiframe processing.

The midamble of the kth user ($k = 1, \ldots, K$) contains a training sequence of N chips $\{x_k(i)\}_{i=0}^{N-1}$ that is known at the receiver and is used for channel estimation. The last $W-1$ chips of the sequence are repeated at the beginning of the midamble as a cyclic prefix ($P = N + W - 1$). The multipath channel from the kth user to the receiving antennas is assumed to be quasi-static within the burst interval (or at least within the training period) but varying from frame to frame. It is modeled by the

$M \times 1$ impulse response vector $\mathbf{h}_k(t;\ell)$ (ℓ denotes the frame index) whose temporal support is $t \in [0, WT)$ (T is the chip interval). This implies that the training sequences are received free from the interference of data symbols.

After chip matched filtering and sampling at chip rate, the N time samples received by the M antennas during the ℓth training period can be organized into a $M \times N$ matrix $\mathbf{Y}(\ell)$ (each row corresponds to the signal received by an antenna). Similarly, the discrete-time channel relative to the kth user can be organized into the $M \times W$ matrix $\mathbf{H}_k(\ell) = [\mathbf{h}_k(0;\ell)\mathbf{h}_k(T;\ell)\ldots\mathbf{h}_k((W-1)T;\ell)]$, referred to as the space-time channel matrix. It follows the signal model

$$\mathbf{Y}(\ell) = \sum_{k=1}^{K} \mathbf{H}_k(\ell)\mathbf{X}_k + \mathbf{N}(\ell) = \mathbf{H}(\ell)\mathbf{X} + \mathbf{N}(\ell) \quad (1)$$

where $\mathbf{H}(\ell) = [\mathbf{H}_1(\ell)\ldots\mathbf{H}_K(\ell)]$, $\mathbf{X} = [\mathbf{X}_1^{\mathrm{T}}\ldots\mathbf{X}_K^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{X}_k$ is the $W \times N$ convolution matrix with $[\mathbf{X}_k]_{m,n} = x_k(n-m)$ (recall that $x_k(n+N) = x_k(n)$). The additive noise $\mathbf{N}(\ell)$ is temporally uncorrelated but spatially correlated with covariance $\mathbf{R}: \mathrm{E}[\mathbf{N}(\ell)\mathbf{N}^{\mathrm{H}}(\ell+m)]/N = \mathbf{R}\delta(m)$.

III. Multipath Model of the Space-Time Channel

According to the multipath propagation model, the channel matrix $\mathbf{H}_k(\ell)$ can be written as the sum of the contributions relative to d_k paths, the ith path ($i = 1, \ldots, d_k$) is characterized by a direction of arrival $\alpha_{k,i}$, a delay $\tau_{k,i}$ and a complex amplitude $\beta_{k,i}(\ell)$. If the number L of slots is properly selected according to the mobility of the user, the pair $(\alpha_{k,i}, \tau_{k,i})$ can be considered as slot-independent and $\mathbf{H}_k(\ell)$ is expressed as

$$\mathbf{H}_k(\ell) = \mathbf{A}_k\mathbf{D}_k(\ell)\mathbf{G}_k^{\mathrm{T}}, \qquad \ell = 1, \ldots, L. \quad (2)$$

The $W \times d_k$ temporal matrix $\mathbf{G}_k = [\mathbf{g}(\tau_{k,1}), \ldots, \mathbf{g}(\tau_{k,d_k})]$ depends on the set of delays $\tau_k = [\tau_{k,1}, \ldots, \tau_{k,d}]^{\mathrm{T}}$, each column contains the delayed pulse waveform $[\mathbf{g}(\tau_{k,i})]_{n,1} = g((n-1)T - \tau_{k,i})$. Similarly, the $M \times d_k$ spatial response matrix $\mathbf{A}_k = [\mathbf{a}(\alpha_{k,1}), \ldots, \mathbf{a}(\alpha_{k,d_k})]$ depends on the set of angles $\alpha_k = [\alpha_{k,1}, \ldots, \alpha_{k,d_k}]^{\mathrm{T}}$, where $[\mathbf{a}(\alpha_{k,i})]_{m,1} = \exp(j\pi(m-1)\sin\alpha_{k,i})$. The diagonal matrix $\mathbf{D}_k(\ell) = \mathrm{diag}(\beta_{k,1}(\ell), \ldots, \beta_{k,d_k}(\ell))$ contains the amplitudes that are assumed to follow the WSSUS channel model, with $\mathrm{E}[\mathbf{D}_k(\ell+m)\mathbf{D}_k^{\mathrm{H}}(\ell)] = \rho_k(m) \cdot \mathrm{diag}(\sigma_{k,1}^2, \ldots, \sigma_{k,d}^2)$. According to the assumption of quasi-static channel within the time-slot, the normalized correlation function $\rho_k(m)$ depends only on the time interval mT_f and on the terminal mobility (velocity v_k). Indeed, according to the Clarke's isotropic scattering model, it is $\rho_k(m) = J_0(2\pi f_k mT_f)$, where $f_k = v_k/\lambda$ is the Doppler shift. The order of spatial diversity $r_{\mathrm{S},k} = \mathrm{rank}(\mathbf{A}_k) \le M$ accounts for the number of resolvable angles in α_k (given the array aperture), while the order of temporal diversity $r_{\mathrm{T},k} = \mathrm{rank}(\mathbf{G}_k) \le W$ equals the number of resolvable delays in τ_k (given the bandwidth of the transmitted signal). For d_k paths it is $r_{\mathrm{S},k} \le d_k$ and $r_{\mathrm{T},k} \le d_k$.

IV. Multislot (MS) Channel Estimate

The unconstrained maximum-likelihood estimates (MLE) of $\mathbf{H}(\ell)$ and $\mathbf{R}$ obtained without imposing any additional structure on the unknowns are known to be

$$\mathbf{H}_{\mathrm{LS}}(\ell) = [\mathbf{H}_{\mathrm{LS},1}(\ell)\ldots\mathbf{H}_{\mathrm{LS},K}(\ell)] = \mathbf{R}_{yx}(\ell)\mathbf{R}_{xx}^{-1} \quad \text{(3a)}$$

$$\mathbf{R}_{\mathrm{LS}} = \frac{1}{L}\sum_{\ell=1}^{L}(\mathbf{R}_{yy}(\ell) - \mathbf{R}_{yx}(\ell)\mathbf{R}_{xx}^{-1}\mathbf{R}_{yx}^{\mathrm{H}}(\ell)) \quad \text{(3b)}$$

where $\mathbf{R}_{xx} = \mathbf{X}\mathbf{X}^{\mathrm{H}}/N$, $\mathbf{R}_{yx}(\ell) = \mathbf{Y}(\ell)\mathbf{X}^{\mathrm{H}}/N$ and $\mathbf{R}_{yy}(\ell) = \mathbf{Y}(\ell)\mathbf{Y}^{\mathrm{H}}(\ell)/N$. Notice that $\mathbf{H}_{\mathrm{LS}}(\ell)$ is the conventional least squares (LS) estimate.

Based on the model (2), we introduce the MS parametrization of the channel matrix:

$$\mathbf{H}_k(\ell) = \mathbf{U}_{\mathrm{S},k}\mathbf{\Gamma}_k(\ell)\mathbf{U}_{\mathrm{T},k}^{\mathrm{H}} \quad (4)$$

$\mathbf{U}_{\mathrm{S},k}$ ($M \times r_{\mathrm{S},k}$) and $\mathbf{U}_{\mathrm{T},k}$ ($W \times r_{\mathrm{T},k}$) are the burst-independent orthonormal basis for the column spaces $\mathcal{R}(\mathbf{A}_k)$ and $\mathcal{R}(\mathbf{G}_k)$, $\mathbf{\Gamma}_k(\ell)$ is a burst-dependent $r_{\mathrm{S},k} \times r_{\mathrm{T},k}$ matrix (nondiagonal). The parameterization (4) can be easily obtained from the model (2) by considering the singular value decompositions $\mathbf{A}_k = \mathbf{U}_{\mathrm{S},k}\mathbf{\Sigma}_{\mathrm{S},k}\mathbf{V}_{\mathrm{S},k}^{\mathrm{H}}$, $\mathbf{G}_k = \mathbf{U}_{\mathrm{T},k}\mathbf{\Sigma}_{\mathrm{T},k}\mathbf{V}_{\mathrm{T},k}^{\mathrm{H}}$, and by further defining $\mathbf{\Gamma}_k(\ell) = \mathbf{\Sigma}_{\mathrm{S},k}\mathbf{V}_{\mathrm{S},k}^{\mathrm{H}}\mathbf{D}_k(\ell)\mathbf{V}_{\mathrm{T},k}\mathbf{\Sigma}_{\mathrm{T},k}^{\mathrm{H}}$. In the following, $\mathcal{R}(\mathbf{U}_{\mathrm{S},k})$ and $\mathcal{R}(\mathbf{U}_{\mathrm{T},k})$ will be referred to as the spatial and temporal subspaces.

Let the lower triangular Cholesky factor $\mathbf{R}_{xx}^{\mathrm{H}/2}$ be partitioned into blocks of dimension $K \times K$, $\mathbf{Q}_{k,h}$ denotes the block (k,h) of $\mathbf{R}_{xx}^{\mathrm{H}/2}$ with $\mathbf{Q}_{k,h} = \mathbf{0}$ for $h > k$ and $k, h = 1, \ldots, K$. For $N \to \infty$ the optimization of the log-likelihood function can be shown to be equivalent to the minimization of

$$\Psi = \sum_{k=1}^{K}\sum_{\ell=1}^{L}\left\|\widetilde{\mathbf{H}}_{\mathrm{LS},k}(\ell) - \widetilde{\mathbf{U}}_{S,k}\mathbf{\Gamma}_k(\ell)\widetilde{\mathbf{U}}_{T,k}^{\mathrm{H}}\right\|^2 \quad (5)$$

where $\widetilde{\mathbf{U}}_{\mathrm{S},k} = \mathbf{R}_{\mathrm{LS}}^{-\mathrm{H}/2}\mathbf{U}_{\mathrm{S},k}$, $\widetilde{\mathbf{U}}_{\mathrm{T},k} = \mathbf{Q}_{k,k}^{\mathrm{H}}\mathbf{U}_{\mathrm{T},k}$, $\widetilde{\mathbf{H}}_{\mathrm{LS},k}(\ell) = \mathbf{R}_{\mathrm{LS}}^{-\mathrm{H}/2}\mathbf{H}_{\mathrm{LS},k}(\ell)\mathbf{Q}_{k,k} + \widetilde{\mathbf{H}}_{\mathrm{MAI},k}(\ell)$, and

$$\widetilde{\mathbf{H}}_{\mathrm{MAI},k}(\ell) = \sum_{h=k+1}^{K}\mathbf{R}_{\mathrm{LS}}^{-\mathrm{H}/2}(\mathbf{H}_{\mathrm{LS},h}(\ell) - \mathbf{U}_{\mathrm{S},h}\mathbf{\Gamma}_h(\ell)\mathbf{U}_{\mathrm{T},h}^{\mathrm{H}})\mathbf{Q}_{h,k}$$

accounts for the multiple access interference (MAI). The optimization of Ψ in closed form is not an easy task but the analysis of (5) suggests an approximated solution based on a successive cancellation of the MAI (similar to interference cancellation multiuser detection). This solution is obtained by minimizing separately the terms corresponding to each user, starting from the Kth user down to the first. The channel estimate is calculated iteratively for $k = K, K-1, \ldots, 1$ as

$$\hat{\mathbf{H}}_k(\ell) = \mathbf{R}_{\mathrm{LS}}^{\mathrm{H}/2}\hat{\mathbf{\Pi}}_{\mathrm{S},k}\widetilde{\mathbf{H}}_{\mathrm{LS},k}(\ell)\hat{\mathbf{\Pi}}_{\mathrm{T},k}\mathbf{Q}_{k,k}^{-1} \quad (6)$$

where $\hat{\mathbf{\Pi}}_{\mathrm{S},k}$ (and $\hat{\mathbf{\Pi}}_{\mathrm{T},k}$) is the projector onto the subspace spanned by the $r_{\mathrm{S},k}$ (and $r_{\mathrm{T},k}$) principal eigenvectors of the spatial correlation matrix $\mathbf{R}_{\mathrm{S},k} = \sum_{\ell=1}^{L}\widetilde{\mathbf{H}}_{\mathrm{LS},k}(\ell)\widetilde{\mathbf{H}}_{\mathrm{LS},k}^{\mathrm{H}}(\ell)/L$ (and of the temporal correlation matrix $\mathbf{R}_{\mathrm{T},k} = \sum_{\ell=1}^{L}\widetilde{\mathbf{H}}_{\mathrm{LS},k}^{\mathrm{H}}(\ell)\widetilde{\mathbf{H}}_{\mathrm{LS},k}(\ell)/L$). As the eigenvalue decomposition is expensive in terms of computational complexity, subspace tracking methods can be adopted for the evaluation of the projectors.

V. Numerical Results

In order to validate the performance of the MS algorithm we simulate the uplink of a TD-SCDMA system with a ZF-BLE receiver [4] for $M = 8$ antennas. The parameters of interest are [1]: $T_f = 10$ ms, $W = 16$, $N = 129$, $P = N + W - 1 = 144$, roll-off equal to 0.22, carrier frequency 1950 MHz. Moreover, each one of the $K = 8$ users transmits at the same rate using a spreading factor $Q = 16$: the data field thus contains $N_S = 22$ QPSK information symbols (352 chips). The K training sequences are chosen according to the standard specifications. We consider a simple radio environment characterized by two clusters of scatterers with angles randomly chosen within the angular support $[-60, 60]$ deg, four paths per cluster ($d_k = 8$) with angular dispersion of 5 deg and exponential power profile $\sigma_{k,i}^2 = (0.5)^{i-1}$ (paths $i = 1, \ldots, 4$ correspond to the first cluster and paths $i = 5, \ldots, 8$ to the second). In each cluster the delays are chip interval spaced and the first delay is randomly selected in the interval $[2T, 10T]$. The fading variation is simplified by assuming the same velocity v for all the users. The covariance matrix of noise is $\mathbf{R} = \mathbf{R}_\mathrm{i} + \mathbf{R}_\mathrm{n}$, where $\mathbf{R}_\mathrm{i} = \sigma_\mathrm{i}^2 \sum_{h=1}^{6} \mathbf{a}(\vartheta_h)\mathbf{a}^\mathrm{H}(\vartheta_h)$ models the interference from 6 out-of-cell terminals with angles ϑ_h uniformly spaced in $[-60, 60]$ deg, while $\mathbf{R}_\mathrm{n} = \sigma_\mathrm{n}^2 \mathbf{I}_M$ accounts for the background AWGN noise. The signal-to-interference ratio is defined as $\mathrm{SIR} = \sigma_x^2/\sigma_\mathrm{i}^2$ and the signal-to-(background) noise as $\mathrm{SNR} = \sigma_x^2/\sigma_\mathrm{n}^2 = 30$ dB, where $\mathrm{tr}(\mathbf{R}_{xx}) = \sigma_x^2 KWN$. The channel of each user is normalized so that $\mathrm{E}[||\mathbf{H}_k(\ell)||^2] = 1$. The diversity orders ($r_{\mathrm{S},k}$ and $r_{\mathrm{T},k}$) are estimated by using the minimum description length (MDL) criterion from the covariance matrices $\mathbf{R}_{\mathrm{S},k}$ and $\mathbf{R}_{\mathrm{T},k}$.

In Fig. 2 the performance of the MS algorithm is evaluated in terms of average bit error rate (BER) and mean square error (MSE) of the channel estimate. The MS estimate for $L = 10$ is compared to the single-slot techniques LS and RR (Reduced Rank [5]). For any mobility of the terminal, from the pedestrian environment ($v = 3$ km/h), up to the vehicular ($v = 120$ km/h), the performance of the MS method are the same. On the other hand, for a static channel ($v = 0$ km/h) the MS technique performs better since the subspaces dimensions $r_{\mathrm{S},k}$ and $r_{\mathrm{T},k}$ can be reduced [6]. Fig. 2 (upper figure) shows that for $L = 10$ and time-varying channel ($v \geq 3$ km/h) the MSE is very close to the theoretical limit computed in [6] for $L \to \infty$. Compared to the ZF-BLE receiver based on the LS channel estimate (3a), the MS method shows a meaningful advantage in term of SIR (3–4 dB) that is practically independent on the variation of the faded amplitudes (lower figure). In addition, the loss with respect to the ideal case of known channel (dashed line) is 1.5 dB.

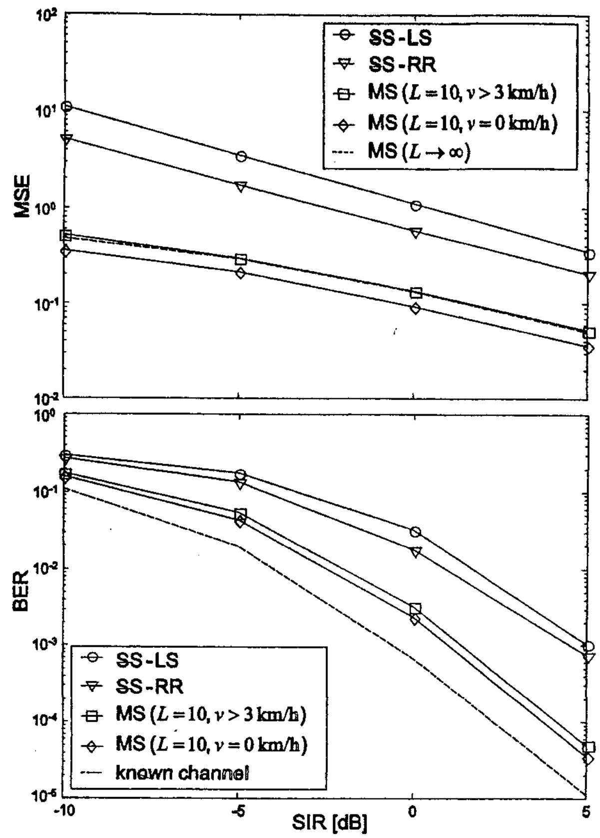

Fig. 2. MSE (upper figure) and BER (lower figure) versus SIR for single-slot (SS) techniques (LS and RR) and for the MS algorithm with $L = 10$ slot. The BER for known channel is shown as reference.

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