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Abstract

In mobile communications, coherent detection requires the estimate of an increasing number of channel parameters due to the promising performance of systems that deploy multiple antennas. However, the estimate of the space-time channels requires a number of training symbols that grows with the number of unknowns. To overcome this problem, we propose a subspace-based estimation method that exploits the different varying rates in the structure of the space-time channel for moving terminals. Since the channel has some fast-varying (faded amplitudes of the paths) and slowly-varying (delays and directions of arrival) features, the multi-slot estimate is composed of two terms: the slowly-varying spatial and temporal bases estimated from L consecutive slots and the fast-varying amplitudes estimated on a slot-by-slot basis. Performance analysis and simulations confirm the expected benefits of the multi-slot approach and demonstrate that for large L the mean square error (MSE) on the channel estimate depends only on the number of fast-varying parameters.

I. INTRODUCTION

In most of existing and next-generation mobile communication systems the multipath fading channel is estimated from known training symbols in order to allow a coherent detection. The accuracy of the estimate depends on the ratio between the number of channel parameters that have to be estimated and the length of the training sequence. Any increase of the latter comes at the price of a reduced transmission efficiency. In systems that deploy multiple antennas at the receiver and/or the transmitter the augmented number of antennas leads to a proportionally larger number of channel parameters. Due to the prominent role of such systems in the actual and future generation of wireless communication networks, it is mandatory to design a parametric channel estimation technique that is simple enough to overcome this problem.

In this paper we propose novel techniques for the estimation of the propagation channel. These methods are here designed for single-user channels in the uplink of time slotted systems and are based on the stationarity of the spatial-temporal channel subspaces across several slots (see also [2]). The approach can be easily extended to multi-user systems (e.g., hybrid TDMA-CDMA systems), MIMO systems, or other transmission schemes such as multicarrier systems. All these applications are straightforward and appear very promising. The multi-slot (MS) method is based on the recognition that in a multipath channel the angles of arrival (or, in short, angles) and the delays of the paths are slowly varying, while the faded amplitudes of the paths are fast-varying. Even if the degree of fading decorrelation depends on the terminals' movement according to the Doppler effect, the amplitudes can be considered quasi-static within each slot (fading variations within the slot are not considered here), while the variations of angles and delays can be modelled as quasi-static in a larger time scale, say L slots. The multi-slot method estimates the slowly-varying features (angles and delays) from the ensemble of L slots and the fast-varying amplitudes in a slot-by-slot fashion. The slowly varying terms are obtained without the explicit computation of the angles and delays but by estimating the corresponding spatial and temporal subspaces.

In a time-slotted system the number of slots L can be chosen to have negligible variations of angles and delays compared to the angular and temporal resolution of the receiver. Notice that the angular resolution depends on the aperture of the antenna array while the temporal resolution is approximately given by the inverse of the signal bandwidth. As an example, let us consider the uplink of a system with inter-slot interval of 5ms, signal bandwidth of 1.28 MHz (e.g., TD-SCDMA system [3]) and an antenna array of $M = 8$ half wavelength spaced elements. If the mobile terminal is 250 m (or 500 m) from the base station (as for an outdoor environment macro cell), the angles and the delays can be considered stationary within $L = 40$ slots (or $L = 80$ slots), provided that the velocity of the mobile terminal is less than 500 km/h.

The problem of estimating the mobile-to-base station channel in time-slotted systems has been classically solved by using a slot-by-slot approach, such as the least squares estimate (LSE). The accuracy of the LSE can be improved by reducing the number of unknowns through the exploitation of the low rank property of the space-time channel (parametric estimate) [4], [5], [6]. In principle, the estimate accuracy can be increased also by considering training sequences that are apparently longer, as obtained by appropriately merging information from multiple slots. The simplest way to implement such an approach is averaging over the channel estimates relative to consecutive slots [7]. This choice leads to good performance only under rather restrictive assumptions about the terminal mobility and it cannot cope with time varying channels, neither with moderately slow fading variations.

In order to make the multislot approach effective, the estimation should be based on the multipath structure of the channel so as to account for the fading amplitude variations over the slots. The joint estimation of angles and delays based on their invariance across multiple slots has been recently proposed in [8] as an extension of the 2D-ESPRIT method [9]. When the receiver employs the multi-slot angle-delay estimation the system performance slightly degrades with respect to the case of perfectly known channel [10]. However, the multi-slot angle-delay estimation is computationally prohibitive for communication systems and does not take into account the correlation properties of the noise. In this paper we translate the slot-invariance property of angles and delays into the stationarity of the spatial and temporal channel subspaces, since this permits to avoid the expensive angle-delay estimation [8]. The exploitation of the stationarity of the spatial subspace in a multi-slot approach was introduced in [11] (without proof of optimality). Here we show that the maximum likelihood estimate of the space-time channel can be reduced to the projection of the LSE onto the spatial and temporal subspaces obtained from multiple slots. If the number of slots L is large enough (in practice $L = 10 \div 20$) these projectors can be estimated with enough accuracy, so that the mean square error (MSE) for the channel estimate after the projection depends only on the (small) residual noise. Furthermore, differently from [10] here both the (spatial) correlation properties of the noise and the (temporal) correlation of the training sequence can be taken into account in the estimation of the subspaces. Compared to the standard LSE, the benefits of the multi-slot approach are relevant when the radio channel allows a significant reduction of complexity. It is proved that this occurs when the degree of spatial diversity is smaller than the number of antennas M and the degree of temporal diversity is smaller than the temporal support of the channel W.

The outline of the paper is as follows. Basic model and signals definitions, notations, and unconstrained MLE are in Section II. Section III highlights the different varying rates in the multipath channel and introduces the corresponding spatial and temporal subspaces for a finite number of slots L. The multislot method for channel estimation is proposed in Section IV for joint spatial and temporal subspaces (MS-ST method), and for spatial (MS-S method) or temporal (MS-T method) subspaces. Section V investigates the properties of the asymptotic (i.e., $L \to \infty$) MSE bound for all the proposed estimators and compares analytically the performance with other methods. Since the multi-slot methods are based on the computations of the spatial and/or temporal subspaces from the ensemble of single-slot estimates, the fast subspace tracking method appears to be the mandatory for real time implementations, this is discussed in Section VI. Section VII contains the numerical results to validate the advantage of the method and the rate of convergence to the analytic MSE bounds.

II. Notations and preliminaries

Let us introduce some notational conventions that will be used in this paper. Lowercase (uppercase) bold denotes column vector (matrix), $(\cdot)^{\mathrm{T}}$ is the matrix transpose, $(\cdot)^*$ is the complex conjugate, $(\cdot)^{\mathrm{H}}$ is the Hermitian transposition, $[\cdot]_{m,n}$ is the element (m, n) of the matrix argument. $||\mathbf{X}||^2$ is the matrix Frobenius norm, $\mathbf{R}^{1/2}$ is the Cholesky factorization of a positive definite matrix $\mathbf{R}$: $\mathbf{R} = \mathbf{R}^{\mathrm{H}/2}\mathbf{R}^{1/2}$. $\otimes$ is the Kronecker matrix product, $\mathbf{v} = \mathrm{vec}\{\mathbf{V}\}$ is the stacking operator, the following property will be used: $\mathrm{vec}\{\mathbf{ABC}\} = (\mathbf{C}^{\mathrm{T}} \otimes \mathbf{A})\,\mathrm{vec}\{\mathbf{B}\}$ (see [12] for additional properties). $\mathbf{I}_P$ is the $P \times P$ unit matrix, $\mathbf{A}^\dagger$ is the pseudoinverse of $\mathbf{A}$, $\mathcal{R}\{\mathbf{A}\}$ is the subspace spanned by the columns of $\mathbf{A}$, $\Pi_{\mathbf{A}} = \mathbf{A}\mathbf{A}^\dagger$ is the projection matrix onto $\mathcal{R}\{\mathbf{A}\}$. For a matrix $\mathbf{A}(L)$ such that $\mathbf{A}(L) - \mathbf{B} = O(1/L^\alpha)$ for $\alpha > 0$ the following asymptotic (for $L \to \infty$) notation will be used: $\mathbf{A}(L) \to \mathbf{B}$ and $\mathcal{R}\{\mathbf{A}(L)\} \to \mathcal{R}\{\mathbf{B}\}$ (or equivalently $\mathbf{A}(L)\mathbf{A}^\dagger(L) \to \mathbf{B}\mathbf{B}^\dagger$); the same properties hold with probability 1 for random matrices.

A. Signal model

Let us consider a time-slotted wireless communication system in which a mobile user transmits bursts to a base station equipped with an antenna array of M elements. The frame structure is illustrated in Fig. 1, within each frame a single slot is allocated to each user so that the time interval between two consecutive slots for the same user equals the frame duration T_f. During the time slot the mobile terminal transmits a burst consisting of two data blocks, a training sequence (midamble) and a guard period. At each antenna the baseband received signal is passed through a filter matched to the pulse waveform, the signal at the output of the matched filter is modelled as

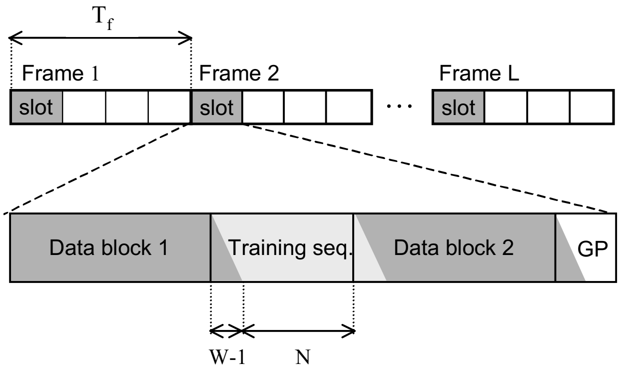

Fig. 1. Frame structure of time slotted systems.

the $M \times 1$ vector:

$$\mathbf{y}(t;\ell) = \sum_i x(i;\ell)\mathbf{h}(t - iT;\ell) + \mathbf{n}(t;\ell), \tag{1}$$

here t denotes the time variable within the time-slot of the ℓth frame, while the complex value $x(i;\ell)$ denotes the ith symbol of the transmitted sequence (either information or training data) at rate $1/T$. The space-time channel from the mobile transmitter to the antenna array is described by the $M \times 1$ impulse vector $\mathbf{h}(t;\ell)$ which accounts for the array response, the fading channel, the symbol waveform and the matched filter at the receiver. In general the channel is time-varying, however in this paper the time slot is assumed to be short enough so that the channel can be simplified as invariant within the burst interval, while it can vary from frame to frame. The index ℓ denotes the dependence to the frame. Furthermore, the $M \times 1$ zero mean circularly symmetric Gaussian vector $\mathbf{n}(t;\ell)$ models both the co-channel interference and the background noise, it is temporally uncorrelated but spatially correlated.

During the training period the transmitted sequence $\{x(i;\ell)\}_{i=-W+1}^{N-1}$ is known at the receiver in order to allow the estimation of the propagation channel. The discrete-time model for the signal received within the midamble is obtained by sampling at the symbol rate $1/T$ the output of the matched filter. The temporal support of the channel is assumed to be $[0, WT)$. Notice that the first $W-1$ samples of the received signal cannot be used for channel estimation as they are affected by the interference from the preceding data symbols. Hence, by discarding the first $W-1$ samples of the signal $\mathbf{y}(t;\ell)$ and arranging the remaining samples into the $M \times N$ matrix $\mathbf{Y}(\ell) = [\mathbf{y}(0;\ell) \cdots \mathbf{y}((N-1)T;\ell)]$, the data model (1) can be rewritten by using the standard notation

$$\mathbf{Y}(\ell) = \mathbf{H}(\ell)\mathbf{X}(\ell) + \mathbf{N}(\ell). \tag{2}$$

Here $\mathbf{H}(\ell) = [\mathbf{h}(0;\ell), \mathbf{h}(T;\ell), \ldots, \mathbf{h}((W-1)T;\ell)]^{\mathrm{T}}$ is the $M \times W$ space-time channel matrix, and $\mathbf{X}(\ell)$ is the $W \times N$ Toeplitz matrix that represents the convolution of the channel with the training sequence, i.e. $[\mathbf{X}(\ell)]_{m,n} = x(n-m;\ell)$. Moreover, the discrete-time noise $\mathbf{N}(\ell) = [\mathbf{n}(0;\ell) \cdots \mathbf{n}((N-1)T;\ell)]$ is Gaussian, still temporally uncorrelated (the transmitted symbol waveforms are orthogonal), but spatially correlated with covariance $\mathrm{E}[\mathbf{n}(iT;\ell)\mathbf{n}^{\mathrm{H}}((i+m)T;\ell)] = \mathbf{Q}\delta(m)$. The noise covariance matrix $\mathbf{Q}$ is full-rank, it accounts for the thermal noise, the mean power and the spatial arrangement of the interferers (see Sec. VII). Since the spatial features of the interference are slowly varying, $\mathbf{Q}$ is assumed to be independent of the slot. All the antennas have the same noise power $[\mathbf{Q}]_{m,m} = \sigma_n^2$.

B. Unconstrained channel estimate

From the model (2) the unconstrained maximum likelihood estimate (MLE) of the channel and the noise covariance matrix can be shown to be

$$\mathbf{H}_{\mathrm{u}}(\ell) = \mathbf{R}_{yx}(\ell)\mathbf{R}_{xx}^{-1}, \tag{3a}$$

$$\mathbf{Q}_{\mathrm{u}} = \frac{1}{L}\sum_{\ell=1}^{L}\left(\mathbf{R}_{yy}(\ell) - \mathbf{R}_{yx}(\ell)\mathbf{R}_{xx}^{-1}\mathbf{R}_{yx}^{\mathrm{H}}(\ell)\right), \tag{3b}$$

where $\mathbf{R}_{yx}(\ell) = \mathbf{Y}(\ell)\mathbf{X}^{\mathrm{H}}(\ell)/N$, $\mathbf{R}_{yy}(\ell) = \mathbf{Y}(\ell)\mathbf{Y}^{\mathrm{H}}(\ell)/N$, and the training sequence $\mathbf{X}(\ell)$ is assumed to have correlation matrix $\mathbf{R}_{xx} = \mathbf{X}(\ell)\mathbf{X}^{\mathrm{H}}(\ell)/N$ independent on the slot. Notice that the estimate (3a) coincides with the LSE of the space-time channel matrix when this is carried out on a slot-by-slot basis.

The estimate (3a) is known to be unbiased and $\mathbf{Q}_{\mathrm{u}}-\mathbf{Q}=O(1/NL)$, therefore it is $\mathbf{Q}_{\mathrm{u}}\rightarrow\mathbf{Q}$ for $NL\rightarrow\infty$ [4]. The mean square error (MSE) of the LSE can be obtained from the vectorized error $\Delta\mathbf{h}_{\mathrm{u}}=\mathrm{vec}\{\mathbf{H}_{\mathrm{u}}(\ell)-\mathbf{H}(\ell)\}=\frac{1}{N}[(\mathbf{R}_{xx}^{-1}\mathbf{X}(\ell))^{*}\otimes\mathbf{I}_{M}]\mathbf{n}(\ell)$, where $\mathbf{n}(\ell)=\mathrm{vec}\{\mathbf{N}(\ell)\}$. Indeed the covariance of the channel estimator is $\mathrm{Cov}\{\mathrm{vec}\{\mathbf{H}_{\mathrm{u}}(\ell)\}\}=\mathrm{E}\left[\Delta\mathbf{h}_{\mathrm{u}}\Delta\mathbf{h}_{\mathrm{u}}^{\mathrm{H}}\right]=\frac{1}{N}\mathbf{R}_{xx}^{-*}\otimes\mathbf{Q}$ and therefore the MSE is given by

$$\mathrm{MSE}_{\mathrm{u}}=\mathrm{E}\left[\left\|\mathbf{H}_{\mathrm{u}}(\ell)-\mathbf{H}(\ell)\right\|^{2}\right]=\mathrm{tr}\left\{\mathrm{Cov}\left\{\mathbf{h}_{\mathrm{u}}\right\}\right\}=\frac{1}{N}\mathrm{tr}\left\{\mathbf{Q}\right\}\mathrm{tr}\left\{\mathbf{R}_{xx}^{-1}\right\}=\frac{M\sigma_{n}^{2}}{N}\mathrm{tr}\left\{\mathbf{R}_{xx}^{-1}\right\}. \quad (4)$$

For training sequences with ideal correlation properties ($\mathbf{R}_{xx}=\sigma_{x}^{2}\,\mathbf{I}_{W}$) the MSE (4) reduces to

$$\mathrm{MSE}_{\mathrm{u}}=\frac{\sigma_{n}^{2}}{\sigma_{x}^{2}}\cdot\frac{MW}{N}=\rho MW, \quad (5)$$

with $\rho=\sigma_{n}^{2}/N\sigma_{x}^{2}$. As a result the accuracy of the LSE depends on the ratio between the channel parameters (MW) and the training sequence length (N). Purpose of the paper is to reduce the MSE by a parsimonious parameterization of the space-time channel that takes advantage of the multi-slot redundancy.

III. Subspace channel model

Within a set of L consecutive bursts the channel can be modelled as a combination of P paths, each of them is characterized by a delay τ_{p}, an angle α_{p} and an amplitude $\beta_{p}(\ell)$ that accounts for the fading variations:

$$\mathbf{h}(t;\ell)=\sum_{p=1}^{P}\beta_{p}(\ell)\mathbf{a}(\alpha_{p})g(t-\tau_{p}). \quad (6)$$

The waveform $g(t)$ is the convolution of the transmitted pulse and the matched filter at the receiver, the $M\times 1$ vector $\mathbf{a}(\alpha_{p})$ is the array response to a plane wave with direction of arrival α_{p}. For instance, for a uniform linear array of half-wavelength spaced antennas it is $\mathbf{a}(\alpha_{p})=[1,\exp(-j\pi\sin\alpha_{p}),\cdots,\exp(-j\pi\sin\alpha_{p}(M-1))]^{\mathrm{T}}$.

The variations of the angles and delays over the L slots are assumed to be smaller than the angular-temporal resolution so that the parameters $\{\alpha_{p},\tau_{p}\}_{p=1}^{P}$ in model (6) can be considered slot-independent. On the other hand, the slot-dependent amplitudes $\boldsymbol{\beta}(\ell)=[\beta_{1}(\ell),\ldots,\beta_{P}(\ell)]^{\mathrm{T}}$ are assumed to be uncorrelated with $\mathbf{R}_{\boldsymbol{\beta}}=\mathrm{E}[\boldsymbol{\beta}(\ell)\boldsymbol{\beta}^{\mathrm{H}}(\ell)]=\mathrm{diag}\{\sigma_{1}^{2},\ldots,\sigma_{P}^{2}\}$ according to the WSSUS channel model [13]. Each amplitude $\beta_{p}(\ell)$ is a stationary and ergodic (up to the second order) process with $\mathrm{E}[\beta_{p}^{*}(\ell)\boldsymbol{\beta}_{p}(\ell+m)]=\sigma_{p}^{2}\cdot\varphi(m)$. According, for instance, to Clarke's isotropic scattering model [13], the normalized correlation function $\varphi(m)$ depends only on the time interval mT_{f} and on the terminal mobility. Except for the static channel ($\varphi(m)=1$), for any moving terminal the fading is uncorrelated over a large number of slots as $\varphi(m)\rightarrow 0$ for $m\rightarrow\infty$. It is worth anticipating that most of the reasonings proposed throughout the paper largely simplify when the fading is uncorrelated from slot-to-slot, i.e., $\varphi(m)=\delta(m)$. This occurs when the frame duration T_{f} is large compared to the channel coherence time or, as an extension of the model discussed here, when each slot is transmitted on a different frequency bandwidth (e.g., in frequency hopping or multicarrier system).

The space-time channel matrix is obtained by sampling at the symbol rate the response (6) and rearranging the terms as

$$\mathbf{H}(\ell)=\sum_{p=1}^{P}\beta_{p}(\ell)\mathbf{a}(\alpha_{p})\mathbf{g}^{\mathrm{T}}(\tau_{p})=\mathbf{A}\mathbf{D}(\ell)\mathbf{G}^{\mathrm{T}}, \quad (7)$$

the $W\times 1$ vector $\mathbf{g}(\tau_{p})=[g(-\tau_{p}),g(T-\tau_{p}),\ldots,g((W-1)T-\tau_{p})]^{\mathrm{T}}$ is the sampled delayed waveform, the set of P vectors for the delays $\boldsymbol{\tau}=[\tau_{1},\ldots,\tau_{P}]^{\mathrm{T}}$ are collected into the real-valued temporal matrix $\mathbf{G}=[\mathbf{g}(\tau_{1}),\ldots,\mathbf{g}(\tau_{P})]$. Similarly, the spatial (or steering) matrix $\mathbf{A}=[\mathbf{a}(\alpha_{1}),\ldots,\mathbf{a}(\alpha_{P})]$ depends on the P angles $\boldsymbol{\alpha}=[\alpha_{1},\ldots,\alpha_{P}]^{\mathrm{T}}$; the diagonal matrix $\mathbf{D}(\ell)=\mathrm{diag}\{\boldsymbol{\beta}(\ell)\}$ contains the amplitudes.

The property of model (7) to separate the slot-dependent term $\mathbf{D}(\ell)$ from the slot-independent matrices $\mathbf{A}$ and $\mathbf{G}$ was exploited in [9] to jointly estimate $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$. Here, in order to avoid the computationally expensive estimation of the angle-delay pairs, we propose to re-parameterize the channel (7) in terms of unstructured slot-varying/unvarying matrices. This approach is derived from the multipath model (7) by exploiting the diversity of the propagation channel. The order of the spatial (r_{S}) and the temporal (r_{T}) diversity are

$$r_{\mathrm{S}}=\dim\mathcal{R}\{\mathbf{A}\}=\mathrm{rank}\{\mathbf{A}\}\leq M, \quad (8a)$$

$$r_{\mathrm{T}}=\dim\mathcal{R}\{\mathbf{G}\}=\mathrm{rank}\{\mathbf{G}\}\leq W, \quad (8b)$$

respectively. The first accounts for the number of angles that can be resolved in $\boldsymbol{\alpha}$ (given the array aperture), while the second equals the number of the resolvable delays in $\boldsymbol{\tau}$ (given the bandwidth of the transmitted signal). For P paths it is $r_{\mathrm{S}} \leq P$ and $r_{\mathrm{T}} \leq P$. In many practical situations P can be very large, but the order of diversity depends only on few clusters of scatterers with moderate angle-delay spread. This makes the diversity orders r_{S} and r_{T} smaller than P and it allows the use of the parsimonious parameterization described below.

Let us define the spatial and temporal correlation of the multi-slot channel for a set of L slots as

$$\mathbf{R}_{\mathrm{S}}(L) = \frac{1}{L}\sum_{\ell=1}^{L}\mathbf{H}(\ell)\mathbf{H}^{\mathrm{H}}(\ell), \tag{9a}$$

$$\mathbf{R}_{\mathrm{T}}(L) = \frac{1}{L}\sum_{\ell=1}^{L}\mathbf{H}^{\mathrm{H}}(\ell)\mathbf{H}(\ell), \tag{9b}$$

so that $\mathrm{rank}\{\mathbf{R}_{\mathrm{S}}(L)\} = r_{\mathrm{S}}(L)$ and $\mathrm{rank}\{\mathbf{R}_{\mathrm{T}}(L)\} = r_{\mathrm{T}}(L)$. Given the eigenvector sets $\mathbf{U}_{\mathrm{S}}$ and $\mathbf{U}_{\mathrm{T}}$ for the correlation matrices (9a)-(9b), respectively, the channel matrix can be parametrized as

$$\mathbf{H}(\ell) = \mathbf{U}_{\mathrm{S}}\Gamma(\ell)\mathbf{U}_{\mathrm{T}}^{\mathrm{H}}, \tag{10}$$

where the $r_{\mathrm{S}}(L) \times r_{\mathrm{T}}(L)$ full-rank matrix $\Gamma(\ell)$ is slot-dependent, $\mathbf{U}_{\mathrm{S}}$ and $\mathbf{U}_{\mathrm{T}}$ are slot-independent matrices of dimension $M \times r_{\mathrm{S}}(L)$ and $W \times r_{\mathrm{T}}(L)$, respectively. Notice that, differently from (7), here $\mathbf{U}_{\mathrm{S}}$ and $\mathbf{U}_{\mathrm{T}}$ are non-structured matrices. In the following $\mathcal{R}\{\mathbf{R}_{\mathrm{S}}(L)\} = \mathcal{R}\{\mathbf{U}_{\mathrm{S}}\}$ will be referred to as the spatial subspace, and $\mathcal{R}\{\mathbf{R}_{\mathrm{T}}(L)\} = \mathcal{R}\{\mathbf{U}_{\mathrm{T}}\}$ as the temporal subspace.

For finite L the structure of the sample correlation matrices (and therefore $\mathbf{U}_{\mathrm{S}}$ and $\mathbf{U}_{\mathrm{T}}$) cannot be ascribed only to the angle-delay pattern, as the interaction between the faded amplitudes can make the spatial and temporal subspaces interfere with each other. It can be shown that the spatial-temporal subspaces are $\mathcal{R}\{\mathbf{U}_{\mathrm{S}}\} \subseteq \mathcal{R}\{\mathbf{A}\}$ and $\mathcal{R}\{\mathbf{U}_{\mathrm{T}}\} \subseteq \mathcal{R}\{\mathbf{G}\}$ and the rank orders fulfil the following relations [1]

$$\min\{r_{\mathrm{S}}, r_{\mathrm{T}}\} \leq r_{\mathrm{S}}(L) \leq r_{\mathrm{S}}, \tag{11a}$$

$$\min\{r_{\mathrm{S}}, r_{\mathrm{T}}\} \leq r_{\mathrm{T}}(L) \leq r_{\mathrm{T}}. \tag{11b}$$

The left and right equalities in (11a)-(11b) correspond to the following extreme cases. For a static channel ($v = 0$) or $L = 1$ the subspace dimension is $r_{\mathrm{S}}(L) = r_{\mathrm{T}}(L) = \min\{r_{\mathrm{S}}, r_{\mathrm{T}}\}$, that equals the rank-order of the single-slot channel matrix [6]. For moving terminals and $L \to \infty$ the spatial-temporal subspaces depend only on the slowly varying features of the propagation channel (i.e., the angle-delay pattern), while the ranks of (9a)-(9b) approach the order of spatial-temporal diversity: $\mathcal{R}\{\mathbf{U}_{\mathrm{S}}\} \to \mathcal{R}\{\mathbf{A}\}$, $\mathcal{R}\{\mathbf{U}_{\mathrm{T}}\} \to \mathcal{R}\{\mathbf{G}\}$, $r_{\mathrm{S}}(L) \to r_{\mathrm{S}}$, $r_{\mathrm{T}}(L) \to r_{\mathrm{T}}$.

IV. MULTI-SLOT ESTIMATE OF SPATIAL AND TEMPORAL SUBSPACES

In this Section we derive a multi-slot estimation for the channel matrices $\mathbf{H}(\ell)$ ($\ell = 1, .., L$) that accounts for both the slow and the fast-varying features of the multipath structure (10). By using the parameterization (10), the signal model (2) can be re-written as

$$\mathbf{Y}(\ell) = \mathbf{U}_{\mathrm{S}}\Gamma(\ell)\mathbf{U}_{\mathrm{T}}^{\mathrm{H}}\mathbf{X}(\ell) + \mathbf{N}(\ell), \quad \ell = 1, \ldots, L. \tag{12}$$

The spatial covariance matrix $\mathbf{Q}$ (not known) and the correlation of the training sequence $\mathbf{R}_{xx}$ are both assumed to be constant over the L slots. The correlation of the fading amplitudes over the slots is not explicitly modelled.

In the following we first discuss the multi-slot maximum likelihood method that exploits the stationarity of the spatial *and* the temporal subspace (MS-ST algorithm). Then we simplify the method by imposing only one constraint of stationarity, for either the spatial (MS-S) *or* the temporal (MS-T) subspace. An example will be used to illustrate the advantages in terms of noise reduction.

A. MS-ST

According to the model (12), the maximum likelihood estimate (MLE) under the constraint (10) reduces asymptotically (for $NL \to \infty$) to the minimizer of the loss function [14]

$$\Psi(\boldsymbol{\theta}) = \sum_{\ell=1}^{L}\left\|\tilde{\mathbf{H}}_{\mathrm{u}}(\ell) - \tilde{\mathbf{U}}_{\mathrm{S}}\Gamma(\ell)\tilde{\mathbf{U}}_{\mathrm{T}}^{\mathrm{H}}\right\|^{2}, \tag{13}$$

where $\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)=\mathbf{Q}_{\mathrm{u}}^{-\mathrm{H}/2}\mathbf{H}_{\mathrm{u}}(\ell)\mathbf{R}_{xx}^{\mathrm{H}/2}$, $\tilde{\mathbf{U}}_{\mathrm{S}}=\mathbf{Q}_{\mathrm{u}}^{-\mathrm{H}/2}\mathbf{U}_{\mathrm{S}}$ and $\tilde{\mathbf{U}}_{\mathrm{T}}=\mathbf{R}_{xx}^{1/2}\mathbf{U}_{\mathrm{T}}$. The parameter vector is $\boldsymbol{\theta}=[\boldsymbol{\theta}_{\mathbf{U}}^{\mathrm{T}},\boldsymbol{\theta}_{\Gamma}^{\mathrm{T}}]^{\mathrm{T}}$, where $\boldsymbol{\theta}_{\mathbf{U}}=[\mathrm{vec}\{\mathbf{U}_{\mathrm{S}}\}^{\mathrm{T}},\mathrm{vec}\{\mathbf{U}_{\mathrm{T}}\}^{\mathrm{T}}]^{\mathrm{T}}$ of size $(Mr_{\mathrm{S}}(L)+Wr_{\mathrm{T}}(L))\times 1$ contains the slot-independent parameters and $\boldsymbol{\theta}_{\Gamma}=\mathrm{vec}\{[\Gamma(1)\cdots\Gamma(L)]\}$ of size $(Lr_{\mathrm{S}}(L)r_{\mathrm{T}}(L))\times 1$ the slot-dependent terms. The model orders $r_{\mathrm{S}}(L)$ and $r_{\mathrm{T}}(L)$ are assumed known.

We observe that, according to (13), the constrained MLE is equivalent to a parametric re-estimation computed from the whitened LSE's $\{\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\}_{\ell=1}^{L}$. The whitening operation is performed on the LSE $\mathbf{H}_{\mathrm{u}}(\ell)$, for $\ell=1,\ldots,L$, to take into account the noise correlation and the correlation properties of the training sequence. This is obtained by the spatial ($\mathbf{Q}_{\mathrm{u}}^{-\mathrm{H}/2}$) and the temporal ($\mathbf{R}_{xx}^{\mathrm{H}/2}$) whitening factors, as the covariance of $\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)$ can be derived from the covariance of the LSE: $\mathrm{Cov}\{\mathrm{vec}\{\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\}\}=\mathrm{Cov}\{(\mathbf{R}_{xx}^{*/2}\otimes\mathbf{Q}_{\mathrm{u}}^{-\mathrm{H}/2})\mathbf{H}_{\mathrm{u}}(\ell)\}=\mathbf{I}_{W}\otimes(\mathbf{Q}_{\mathrm{u}}^{-\mathrm{H}/2}\mathbf{Q}\mathbf{Q}_{\mathrm{u}}^{-1/2})$ which tends to $\mathbf{I}_{MW}$ for $NL\rightarrow\infty$.

The optimization of $\Psi(\boldsymbol{\theta})$ in (13) yields the *MS-ST (Multi-Slot Space-Time) estimator* [1]

$$\hat{\mathbf{H}}(\ell)=\mathbf{Q}_{\mathrm{u}}^{\mathrm{H}/2}\hat{\Pi}_{\mathrm{S}}\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\hat{\Pi}_{\mathrm{T}}\mathbf{R}_{xx}^{-\mathrm{H}/2},\quad \text{for } \ell=1,2,...,L. \tag{14}$$

The estimate $\hat{\Pi}_{\mathrm{S}}$ of the spatial projector $\Pi_{\mathrm{S}}=\tilde{\mathbf{U}}_{\mathrm{S}}\tilde{\mathbf{U}}_{\mathrm{S}}^{\dagger}$ is obtained from the $r_{\mathrm{S}}(L)$ leading eigenvectors of the spatial correlation matrix

$$\tilde{\mathbf{R}}_{\mathrm{S}}(L)=\frac{1}{L}\sum_{\ell=1}^{L}\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\tilde{\mathbf{H}}_{\mathrm{u}}^{\mathrm{H}}(\ell), \tag{15}$$

similarly, $\hat{\Pi}_{\mathrm{T}}$ is the estimate of the projector $\Pi_{\mathrm{T}}=\tilde{\mathbf{U}}_{\mathrm{T}}\tilde{\mathbf{U}}_{\mathrm{T}}^{\dagger}$ evaluated from the first $r_{\mathrm{T}}(L)$ eigenvectors of the temporal correlation matrix

$$\tilde{\mathbf{R}}_{\mathrm{T}}(L)=\frac{1}{L}\sum_{\ell=1}^{L}\tilde{\mathbf{H}}_{\mathrm{u}}^{\mathrm{H}}(\ell)\tilde{\mathbf{H}}_{\mathrm{u}}(\ell). \tag{16}$$

We observe that for $L\rightarrow\infty$ the estimates $\hat{\Pi}_{\mathrm{S}}$ and $\hat{\Pi}_{\mathrm{T}}$ are the projection matrices onto the true subspaces of the whitened channel since $\mathcal{R}\{\tilde{\mathbf{R}}_{\mathrm{S}}(L)\}\rightarrow\mathcal{R}\{\mathbf{Q}^{-\mathrm{H}/2}\mathbf{A}\}$ and $\mathcal{R}\{\tilde{\mathbf{R}}_{\mathrm{T}}(L)\}\rightarrow\mathcal{R}\{\mathbf{R}_{xx}^{1/2}\mathbf{G}\}$, respectively [1].

B. MS-S and MS-T

In a dense multipath radio environment the degree of temporal diversity could be as large as the support of the channel. In this case the temporal order rises to $r_{\mathrm{T}}(L)\simeq W$ and $\mathcal{R}\{\mathbf{G}\}$ approaches $\mathbb{R}^{W}$. Under these conditions it might be convenient to neglect the temporal projection and let $\Pi_{\mathrm{T}}=\mathbf{I}_{W}$. The resulting channel estimate, that exploits the stationarity of the spatial subspace only, is referred to as *MS-S (Multi-Slot Space) estimator:*

$$\hat{\mathbf{H}}(\ell)=\mathbf{Q}_{\mathrm{u}}^{\mathrm{H}/2}\hat{\Pi}_{\mathrm{S}}\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\mathbf{R}_{xx}^{-\mathrm{H}/2}. \tag{17}$$

The projector $\hat{\Pi}_{\mathrm{S}}$ is defined by the span of the first $r_{\mathrm{S}}(L)$ eigenvectors of the spatial correlation matrix (15). Following the same demonstration described in [4], it can be shown that the MS-S solution coincides with the MLE of the channel matrix for the model $\mathbf{H}(\ell)=\mathbf{U}_{\mathrm{S}}\mathbf{F}^{\mathrm{H}}(\ell)$, where $\mathbf{F}(\ell)=\Gamma(\ell)\mathbf{U}_{\mathrm{T}}^{\mathrm{H}}$ is a $W\times r_{\mathrm{S}}(L)$ full rank matrix. This algorithm was proposed (without proof of optimality) in [11].

Dually, for a large angle spread and/or a small number of antennas M ($r_{\mathrm{S}}(L)\simeq M$), it could be advisable not to use the spatial projection and choose $\Pi_{\mathrm{S}}=\mathbf{I}_{M}$. The *MS-T (Multi-Slot Time) estimator* exploits the stationarity of the temporal subspace:

$$\hat{\mathbf{H}}(\ell)=\mathbf{Q}_{\mathrm{u}}^{\mathrm{H}/2}\tilde{\mathbf{H}}_{\mathrm{u}}(\ell)\hat{\Pi}_{\mathrm{T}}\mathbf{R}_{xx}^{-\mathrm{H}/2}, \tag{18}$$

where $\hat{\Pi}_{\mathrm{T}}$ is defined as usual. With the same considerations made above for the dual problem, it can be proved that the MS-T solution coincides with the MLE of the channel matrix given the model $\mathbf{H}(\ell)=\mathbf{C}(\ell)\mathbf{U}_{\mathrm{T}}^{\mathrm{H}}$, where $\mathbf{C}(\ell)=\mathbf{U}_{\mathrm{S}}\Gamma(\ell)$ is a $M\times r_{\mathrm{T}}(L)$ full rank matrix.

C. Example

The underlying idea of the multi-slot approach is to collect instantaneous estimates of the space-time channel as the user moves and then use these observations to estimate the slowly varying spatial-temporal basis by averaging over the fast-faded amplitudes. Let the LSEs be $\mathbf{H}_{\mathrm{u}}(\ell)$ for $\ell=1,\ldots,L$, the space-time observations can be used to form the sample correlation matrices $\hat{\mathbf{R}}_{\mathrm{S}}(L)=(1/L)\sum_{\ell=1}^{L}\mathbf{H}_{\mathrm{u}}(\ell)\mathbf{H}_{\mathrm{u}}^{\mathrm{H}}(\ell)$ and $\hat{\mathbf{R}}_{\mathrm{T}}(L)=$

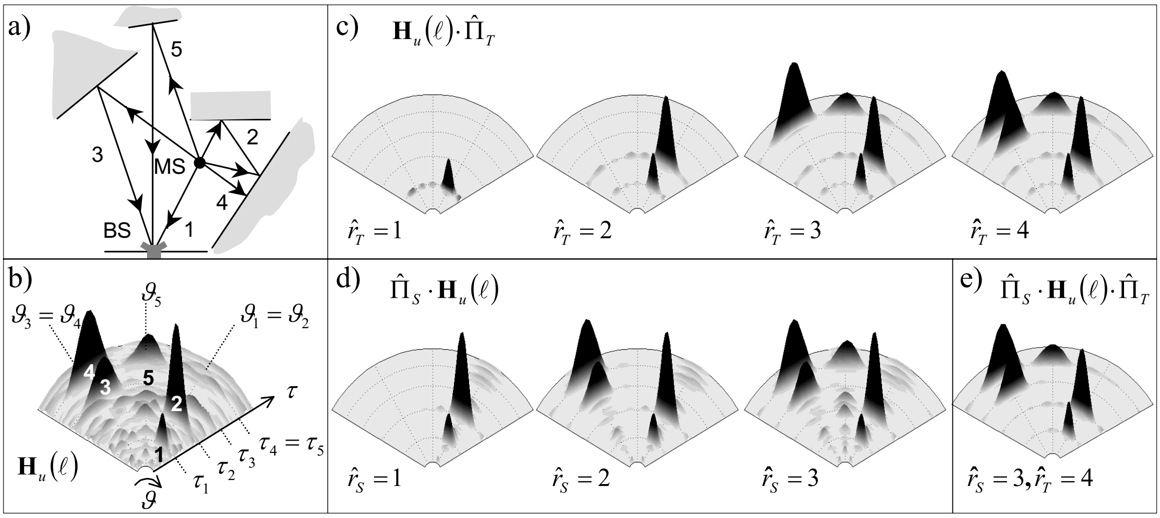

Fig. 2. Example of space and/or time projections for a channel with $P = 5$ paths and different degrees of space and time diversity: $r_S = 3$ and $r_T = 4$. a) multipath model; b) power-delay-angle diagram $\mathcal{P}(i, \vartheta)$ from the LSE; projection of $\mathbf{H}_u(\ell)$ onto temporal (c) or spatial (d) subspaces with increasing dimensions; e) projection onto spatial and temporal subspaces.

$(1/L)\sum_{\ell=1}^{L}\mathbf{H}_{\mathrm{u}}^{\mathrm{H}}(\ell)\mathbf{H}_{\mathrm{u}}(\ell)$ (here we assume $\mathbf{Q} = \sigma_n^2\mathbf{I}_M$ and $\mathbf{R}_{xx} = \sigma_x^2\mathbf{I}_W$ so that $\hat{\mathbf{R}}_{\mathrm{S}}(L) \propto \tilde{\mathbf{R}}_{\mathrm{S}}(L)$ and $\hat{\mathbf{R}}_{\mathrm{T}}(L) \propto \tilde{\mathbf{R}}_{\mathrm{T}}(L)$). Next, the invariant spatial/temporal subspaces can be evaluated from the analysis of the eigen-structure of $\hat{\mathbf{R}}_{\mathrm{S}}(L)$ and $\hat{\mathbf{R}}_{\mathrm{T}}(L)$. This is illustrated by an example in Fig. 2. The multipath channel is composed of $P = 5$ paths having $\mathrm{E}[\boldsymbol{\beta}(\ell)\boldsymbol{\beta}^{\mathrm{H}}(\ell)] = \mathrm{diag}\{0.33, 0.25, 0.19, 0.14, 0.083\}$.

The path pattern is described in Fig. 2-a, while Fig. 2-b shows the power-delay-angle diagram for a noisy estimate $\mathbf{H}_{\mathrm{u}}(\ell)$. This diagram can be obtained as $\mathcal{P}(i, \vartheta) = |[\mathbf{a}^{\mathrm{H}}(\vartheta)\mathbf{H}_{\mathrm{u}}(\ell)]_{1,i}|^2$, where $i = 1, .., W$ and $\vartheta \in [-\pi/3, \pi/3]$ span the delay and angle axes, respectively. Since $\alpha_1 = \alpha_2$, $\alpha_3 = \alpha_4$ and $\tau_4 = \tau_5$, the spatial and temporal diversity orders are, respectively, $r_{\mathrm{S}} = 3$ and $r_{\mathrm{T}} = 4$. The projector $\hat{\Pi}_{\mathrm{S}}$ onto the invariant spatial subspace is calculated from $\hat{\mathbf{R}}_{\mathrm{S}}(L)$ by using the diversity order $\hat{r}_{\mathrm{S}} = 1 \div 3$. The same holds for $\hat{\Pi}_{\mathrm{T}}$ with $\hat{r}_{\mathrm{T}} = 1 \div 4$. As shown in Fig. 2-c, d and e, by projecting the space-time matrix $\mathbf{H}_{\mathrm{u}}(\ell)$ the background noise is reduced.

The residual noise after the projection is that component that can no longer be eliminated as it belongs to the same subspace of the channel. By comparing the power-delay-angle diagram for the initial space-time matrix $\mathbf{H}_{\mathrm{u}}(\ell)$ in Fig. 2-a with the projected matrix $\hat{\Pi}_{\mathrm{S}}\mathbf{H}_{\mathrm{u}}(\ell)\hat{\Pi}_{\mathrm{T}}$ in Fig. 2-e, the visual inspection shows that the artifacts due to noise are reduced. It can be observed in Fig. 2-e that the double projection selects $r_{\mathrm{S}}r_{\mathrm{T}} = 12$ "intersections" in the space-time domain, i.e. the multipath components that have angles in $\{\alpha_1, \alpha_3, \alpha_5\}$ and delays in $\{\tau_1, \tau_2, \tau_3, \tau_4\}$. Since the initial LSE in Fig. 2-a contains all the MW channel samples, the noise-reduction after the double projection is $r_{\mathrm{S}}r_{\mathrm{T}}/MW$.

V. Performance analysis

A. Asymptotic performance $(L \to \infty)$

A lower bound on the mean square error MSE=$\mathrm{E}[||\hat{\mathbf{H}}(\ell) - \mathbf{H}(\ell)||^2]$ can be derived through the computation of the Cramér-Rao Bound (CRB) on the covariance for the channel estimate and then by taking the expectation with respect to the fading $\boldsymbol{\beta}(\ell)$. The CRB is largely simplified for $L \to \infty$ as the covariances of the temporal and spatial projectors are $O(1/L)$ and thus the MSE is dominated by the slot-dependent terms [1]. The lower bound of the MS-ST estimator for $L \to \infty$ is MSE$\geq$ $\mathrm{MSE}_{\text{MS-ST}}$ with

$$\mathrm{MSE}_{\text{MS-ST}} = \Phi(\Pi_{\mathrm{S}}, \mathbf{Q})\cdot\Phi(\Pi_{\mathrm{T}}, \mathbf{R}_{xx}^{-1}), \tag{19}$$

and $\Phi(\Pi, \mathbf{T}) = \mathrm{tr}\{\mathbf{T}^{\mathrm{H}/2}\Pi\mathbf{T}^{1/2}\}/\sqrt{N}$. Since from (4) the MSE for the LSE is known to be

$$\mathrm{MSE}_{\mathrm{u}} = \Phi(\mathbf{I}_W, \mathbf{R}_{xx}^{-1})\Phi(\mathbf{I}_M, \mathbf{Q}), \tag{20}$$

the bound (19) shows that the MS-ST method reduces the estimation error through the projection onto the spatial ($r_{\mathrm{S}} \leq M$) and temporal ($r_{\mathrm{T}} \leq W$) subspaces.

Recalling the derivation of the MS-S and the MS-T methods in Sec. IV-B, the MSE lower bound for the MS-S estimator can be inferred from (19) for $\Pi_{\mathrm{T}} = \mathbf{I}_W$:

$$\mathrm{MSE}_{\mathrm{MS\text{-}S}} = \Phi\left(\Pi_{\mathrm{S}}, \mathbf{Q}\right) \cdot \Phi\left(\mathbf{I}_W, \mathbf{R}_{xx}^{-1}\right). \quad (21)$$

Dually for the MS-T method it is $\Pi_{\mathrm{S}} = \mathbf{I}_M$:

$$\mathrm{MSE}_{\mathrm{MS\text{-}T}} = \Phi\left(\mathbf{I}_M, \mathbf{Q}\right) \cdot \Phi\left(\Pi_{\mathrm{T}}, \mathbf{R}_{xx}^{-1}\right). \quad (22)$$

The MSE bounds (19)-(22) are largely simplified for spatially uncorrelated noise ($\mathbf{Q} = \sigma_n^2 \mathbf{I}_M$) and training sequence with ideal correlation properties ($\mathbf{R}_{xx} = \sigma_x^2 \mathbf{I}_W$). Under these conditions, the bounds are summarized as $\mathrm{MSE}_{\mathrm{AWGN}}$ in Table I and have a simple interpretation: asymptotically (or for a large L) the MSE depends on the number of parameters to be estimated on a slot-by-slot basis. For MS-ST it depends on the $r_{\mathrm{S}} r_{\mathrm{T}}$ entries of the matrix $\Gamma(\ell)$.

As a final remark we note that a lower MSE bound can be obtained by parametrizing the channel matrix as a function of the joint spatial and temporal subspace (see [2] for details).

B. Relationship with other techniques

For single-slot processing ($L = 1$) all the multi-slot methods proposed here coincide with the reduced-rank (RR) approach for the channel rank $r = \mathrm{rank}\{\mathbf{H}(1)\} \leq \min\{W, M\}$ [4], [6]. The RR method uses the same degree $r = \min\{r_{\mathrm{T}}, r_{\mathrm{S}}\}$ for the spatial and the temporal diversity as the rank of the channel matrix is dominated by the more restrictive of the two. On the other hand, a multi-slot processing with uncorrelated fading gives the needed redundancy to allow the estimation of both the spatial and temporal subspaces by relaxing the constraint of the RR approach and differentiating between space and time rank orders. The MS channel estimators can thus be considered as the natural generalization of the RR algorithm to a multi-slot framework.

To facilitate the comparison between the single-slot (LS, RR) and multi-slot (MS-ST, MS-S, MS-T) methods in terms of performance, Table I summarizes the MSE bounds under asymptotic conditions: $N \to \infty$ and $L = 1$ for the RR method [5]; $\{N, L\} \to \infty$ for the multi-slot techniques. Notice that $\Pi^{\perp} = \mathbf{I} - \Pi$ is the orthogonal projector onto the complement of $\mathcal{R}\{\Pi\}$. The performance comparison between the multi-slot and the RR method is carried out by simulation results in [16].

The following relation holds among the LS and the MS estimate performances (19)-(22) [1]:

$$\mathrm{MSE}_{\mathrm{u}} \geq \{\mathrm{MSE}_{\mathrm{MS\text{-}T}}, \mathrm{MSE}_{\mathrm{MS\text{-}S}}\} \geq \mathrm{MSE}_{\mathrm{MS\text{-}ST}}. \quad (23)$$

For the comparison with the single-slot RR method we make use of the MSE lower bound given in Table I, $\mathrm{MSE}_{\mathrm{RR}}(\ell) = \mathrm{E}[||\hat{\mathbf{H}}(\ell) - \mathbf{H}(\ell)||^2]$ [5]. This error depends on the fading amplitudes within the ℓth slot and, in particular, on the rank-r projectors $\Pi_{\mathrm{S}}(\ell) = \tilde{\mathbf{H}}(\ell)\tilde{\mathbf{H}}^{\dagger}(\ell)$ and $\Pi_{\mathrm{T}}(\ell) = \tilde{\mathbf{H}}^{\dagger}(\ell)\tilde{\mathbf{H}}(\ell)$, with $\tilde{\mathbf{H}}(\ell) = \mathbf{Q}^{-\mathrm{H}/2}\mathbf{H}(\ell)\mathbf{R}_{xx}^{\mathrm{H}/2}$. In order to compare the performance of the RR estimate with those of the multislot methods for $L \to \infty$, here the instantaneous fading $\mathrm{MSE}_{\mathrm{RR}}(\ell)$ needs to be averaged with respect to the fading amplitudes (or, equivalently, averaged over $L \to \infty$ slots): $\mathrm{MSE}_{\mathrm{RR}} = \mathrm{E}[\mathrm{MSE}_{\mathrm{RR}}(\ell)]$. The following inequalities hold [1]

$$\mathrm{MSE}_{\mathrm{u}} \geq \mathrm{MSE}_{\mathrm{RR}} \geq \begin{cases} \mathrm{MSE}_{\mathrm{MS\text{-}T}}, & \text{for } r = r_{\mathrm{T}} \leq r_{\mathrm{S}} \\ \mathrm{MSE}_{\mathrm{MS\text{-}S}}, & \text{for } r = r_{\mathrm{S}} \leq r_{\mathrm{T}} \end{cases}, \quad (24)$$

which imply also $\mathrm{MSE}_{\mathrm{RR}} \geq \mathrm{MSE}_{\mathrm{MS\text{-}ST}}$ for any r.

The inequalities (23)-(24) can be easily justified in the case of $\mathbf{Q} = \sigma_n^2 \mathbf{I}_M$ and $\mathbf{R}_{xx} = \sigma_x^2\ \mathbf{I}_W$ (column $\mathrm{MSE}_{\mathrm{AWGN}}$ in Table I). For all the methods, the bound $\mathrm{MSE}_{\mathrm{AWGN}}$ is proportional to $\rho = \sigma_n^2 / N\sigma_x^2$ and to

TABLE I

MSE BOUNDS FOR $N \to \infty$ AND $L \to \infty$ ($\Pi_{\mathrm{S}}(\ell) = \tilde{\mathbf{H}}(\ell)\tilde{\mathbf{H}}^{\dagger}(\ell), \Pi_{\mathrm{T}}(\ell) = \tilde{\mathbf{H}}^{\dagger}(\ell)\tilde{\mathbf{H}}(\ell)$).

Method	MSE	$\mathrm{MSE}_{\mathrm{AWGN}}$
LS	$\Phi(\mathbf{I}_W, \mathbf{R}_{xx}^{-1}) \cdot \Phi(\mathbf{I}_M, \mathbf{Q})$	$\rho \cdot MW$
RR	$\Phi(\Pi_{\mathrm{S}}(\ell), \mathbf{Q}) \cdot \Phi(\mathbf{I}_W, \mathbf{R}_{xx}^{-1}) + \Phi(\Pi_{\mathrm{S}}^{\perp}(\ell), \mathbf{Q}) \cdot \Phi(\Pi_{\mathrm{T}}(\ell), \mathbf{R}_{xx}^{-1})$	$\rho \cdot [r(M+W) - r^2]$
MS-ST	$\Phi(\Pi_{\mathrm{S}}, \mathbf{Q}) \cdot \Phi(\Pi_{\mathrm{T}}, \mathbf{R}_{xx}^{-1})$	$\rho \cdot r_{\mathrm{S}} r_{\mathrm{T}}$
MS-S	$\Phi(\mathbf{I}_W, \mathbf{R}_{xx}^{-1}) \cdot \Phi(\Pi_{\mathrm{S}}, \mathbf{Q})$	$\rho \cdot W r_{\mathrm{S}}$
MS-T	$\Phi(\mathbf{I}_M, \mathbf{Q}) \cdot \Phi(\Pi_{\mathrm{T}}, \mathbf{R}_{xx}^{-1})$	$\rho \cdot M r_{\mathrm{T}}$

the number of independent unknowns. For instance, the RR approach reduces the number of unknowns with respect to LSE by a factor of $MW/(r(M+W)-r^2)$, which yields a MSE gain $\mathrm{MSE_u}\,/\,\mathrm{MSE_{RR}} \geq 1$ for any $r \leq \min\{W, M\}$. Furthermore, the gain of MS-ST with respect to LSE depends on the spatial (M/r_S) and temporal (W/r_T) gains as $\mathrm{MSE_u}\,/\,\mathrm{MSE_{MS\text{-}ST}} = W/r_\mathrm{T} \cdot M/r_\mathrm{S}$. We conclude that the multi-slot techniques show a definite advantage with respect to any single-slot method due to their capability to estimate the spatial and/or temporal projectors with any degree of accuracy. Indeed, for $L \to \infty$ the MSE of the multi-slot methods depends only on the number of parameters to be estimated on each slot: $r_\mathrm{S} r_\mathrm{T}$ for MS-ST, $r_\mathrm{S} W$ for MS-S, and $M r_\mathrm{T}$ for MS-T.

It is worth noticing that, when the characteristics of the radio environment are such that $r_\mathrm{T} \simeq W$ (or $r_\mathrm{S} \simeq M$), the additional computational load required to evaluate the temporal (or spatial) subspace in the MS-ST method provides no performance advantages. In this case the MS-S (or the MS-T) estimator has to be preferred with respect to the MS-ST method.

VI. SUBSPACE TRACKING IMPLEMENTATION

The implementation of the multi-slot techniques MS-ST, MS-S and MS-T implies a certain latency in providing the channel estimate (approximately $L/2$ slots) and might turn out to be too demanding in terms of computational complexity. Moreover, for all the estimators we have adopted the assumption that angles and delays have to be recomputed every L slots. An alternative implementation, that allows to cancel the latency and alleviate the computational burden of the eigenvalue decomposition (EVD), consists in updating the spatial and temporal subspace on a slot-by-slot basis through a subspace tracking technique [17], [18]. In this way angles and delays are allowed to vary continuously (but still slowly). It is advisable to select the most convenient algorithm in terms of performance and complexity. For the multi-slot estimation we have adopted the fast subspace tracker proposed in [19] and summarized in Table II with some minor modifications. The variable ℓ runs over the slots. In order to obtain the MS-ST estimate, the tracking algorithm needs to be performed both on the spatial and the temporal subspace for each slot ℓ, the input of the algorithm is given by the channel matrix $\mathbf{B}(\ell) = \tilde{\mathbf{H}}_\mathrm{u}(\ell)$ for the spatial subspace tracking and by $\mathbf{B}(\ell) = \tilde{\mathbf{H}}_\mathrm{u}^\mathrm{H}(\ell)$ for the temporal one. $\mathbf{U}$ is the orthonormal basis of the (spatial or temporal) subspace and γ rules the effective memory (in terms of slots) of the algorithm. In Table II the dimension r of the (spatial or temporal) subspace is assumed to be fixed, nonetheless the tracking algorithm can be modified in order to adaptively estimate r. The order of complexity for each slot is the combination of the two tracking loops $O(Mr_\mathrm{S}^2(L)) + O(Wr_\mathrm{T}^2(L))$. Notice that the algorithm as presented above can be made even more efficient but still retaining the same order of complexity [19]. Numerical results show that the performance of the fast subspace tracking algorithm is very close to the EVD implementation. A thorough study of the transient behavior of the subspace tracking implementation of the MS technique is under way and it will be presented in future works.

TABLE II
SUBSPACE TRACKING ALGORITHM.

Initialize: $\mathbf{U}(0) = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0} \end{bmatrix}$; $\Theta(0) = \mathbf{I}_r$; $\mathbf{A}(0) = \mathbf{0}$; $0 \leq \gamma \leq 1$; r
For each slot ℓ:
input: $\mathbf{B}(\ell)$
$\mathbf{Z}(\ell) = \mathbf{U}(\ell-1)^H \mathbf{B}(\ell)$
$\mathbf{A}(\ell) = \gamma \mathbf{A}(\ell-1)\Theta(\ell-1) + \mathbf{B}(\ell)\,\mathbf{Z}(\ell)^H$
$\mathbf{A}(\ell) = \mathbf{U}(\ell)\mathbf{R}(\ell)$ (QR factorization)
$\Theta(\ell) = \mathbf{U}(\ell-1)^H\,\mathbf{U}(\ell)$

VII. SIMULATION RESULTS

In this section the performance are evaluated for uncorrelated fading amplitudes ($\varphi(m) = \delta(m)$) since this condition leads to the largest model order but, as discussed in Sec. V, to the worst asymptotic performances. We consider $P = 8$ paths with exponentially decreasing power $\sigma_p^2 = \sigma^2 \times 0.5^{(p-1)}$ (σ^2 is scaled to have $\mathrm{E}[||\mathbf{H}(\ell)||^2] = 1$), clustered into two angles with four delays on each angle: in the first set, $\alpha_p = \pi/3$ for $p = 1, \ldots, 4$, we have the delays $[\tau_1 \cdots \tau_4] = [3.2, 5.1, 6.2, 6.8]T$ and in the second, $\alpha_p = \pi/6$ for $p = 5, \ldots, 8$, we have $[\tau_5 \cdots \tau_8] = [9.8, 11.1, 11.9, 12.8]T$. The transmitted pulse $g(t)$ is a raised cosine with roll-off factor

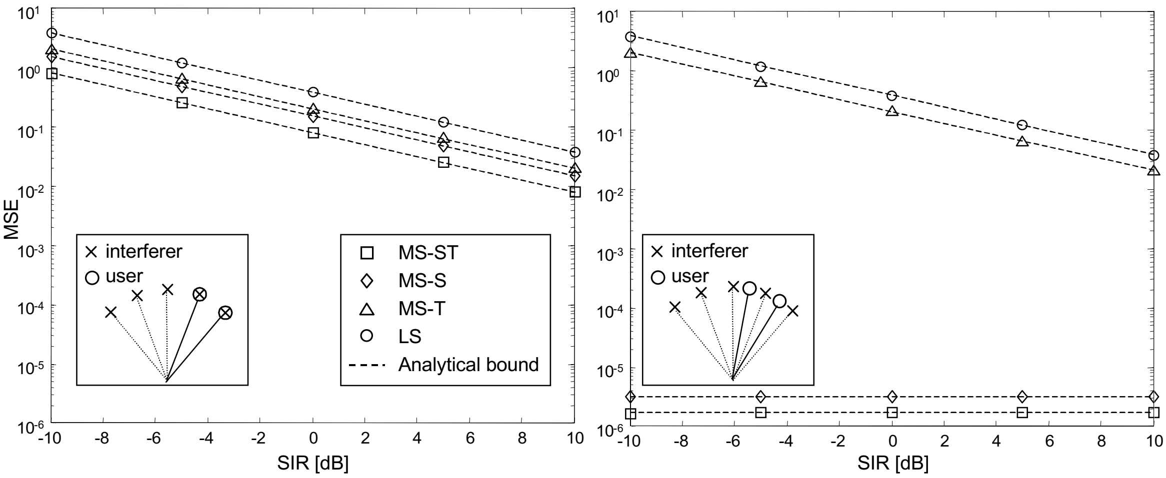

Fig. 3. MSE vs. SIR for $L \to \infty$: simulations (markers) and analytical bounds (19), (21), (22) (dahed lines). Angles of arrival of the user are aligned (left figure) or misaligned (right figure) with those of the interferers. ($M = 8$, $W = 15$, $N = 40$, SNR =50 dB).

0.2. The receiver is equipped with a uniform linear antenna array of $M = 8$ elements with half-wavelength inter-element spacing and the temporal support of the channel is $W = 15$ symbols. The training sequence, of length $N = 40$, is randomly generated such that $\mathrm{E}[\mathbf{X}^{\mathrm{H}}(\ell)\mathbf{X}(\ell)] = N\sigma_x^2\mathbf{I}_W$. Five interferers have directions of arrival $[\bar{\alpha}_1 \cdots \bar{\alpha}_5] = [-\pi/3, -\pi/6, 0, \pi/6, \pi/3]$ equally spaced within the angular support $[-60\ 60]\,\mathrm{deg}$. They are modelled as Gaussian disturbance with correlation matrix $\mathbf{R}_{\mathrm{i}} = (\sigma_{\mathrm{i}}^2/5)\sum_{k=1}^{5}\mathbf{a}(\bar{\alpha}_k)\mathbf{a}^{\mathrm{H}}(\bar{\alpha}_k)$ and $\mathrm{tr}\{\mathbf{R}_{\mathrm{i}}\} = M\sigma_{\mathrm{i}}^2$. The spatial correlation matrix is $\mathbf{Q} = \mathbf{R}_{\mathrm{i}} + \mathbf{R}_{\mathrm{b}}$ where $\mathbf{R}_{\mathrm{b}} = \sigma_{\mathrm{b}}^2\mathbf{I}_M$ accounts for the background AWGN. The signal to interference ratio is defined as $\mathrm{SIR} = \sigma_x^2\,\mathrm{E}[||\mathbf{H}(\ell)||^2]/(M\sigma_{\mathrm{i}}^2) = \sigma_x^2/(M\sigma_{\mathrm{i}}^2)$ and the signal to (background) noise ratio as $\mathrm{SNR} = \sigma_x^2\,\mathrm{E}[||\mathbf{H}(\ell)||^2]/(M\sigma_{\mathrm{b}}^2) = \sigma_x^2/(M\sigma_{\mathrm{b}}^2)$.

In Fig. 3 the simulations for the MSE on the channel estimate of MS-ST, MS-S and MS-T (markers) are compared with the MSE bounds (19), (21) and (22) (dashed lines), respectively. On the left, the angles of arrival of the user are aligned with those of the interferers: $\alpha_p = \bar{\alpha}_5 = \pi/3$ for $p = 1, \ldots, 4$, and $\alpha_p = \bar{\alpha}_4 = \pi/6$ for $p = 5, \ldots, 8$. On the right, the angles of arrival are slightly misaligned: $\alpha_p = \pi/4$ for $p = 1, \ldots, 4$, and $\alpha_p = \pi/8$ for $p = 5, \ldots, 8$ (see the box on both figures). The simulations are carried out for $L \to \infty$ so that $\hat{\Pi}_{\mathrm{S}} = \Pi_{\mathrm{S}}$ and/or $\hat{\Pi}_{\mathrm{T}} = \Pi_{\mathrm{T}}$, in addition $\hat{r}_{\mathrm{S}} = r_{\mathrm{S}} = 2$ and $\hat{r}_{\mathrm{T}} = r_{\mathrm{T}} = 8$. When the user angles are separated from those of the interferers (right figure), the spatial processing performed by the MS-ST and MS-S methods leads to a value of MSE that is independent on the interferer level, but it is ruled by the background noise (SNR=50dB). According to the analysis in Sec. V, here the MS-S method outperforms the MS-T because in this radio environment the spatial gain ($M/r_{\mathrm{S}} = 4$) exceeds the temporal gain ($W/r_{\mathrm{T}} = 1.875$). To be precise, this argument applies only to the ideal case ($\mathrm{MSE}_{\mathrm{AWGN}}$ in Table I), but it turns out to be a reliable rule of thumb to choose the multi-slot estimators according to their expected performance. In the following simulations we consider the angles of arrival for the user aligned with those of the interferers (Fig.3-left) as this corresponds to the worst case.

By now we have validated the asymptotic ($L \to \infty$) performance of the multi-slot estimators according to the theoretical bounds in Sec.V. In a practical situation, the number of processed slot L is finite for memory requirement and long-term angle-delay pattern variations. The former can be avoided by the subspace tracking method discussed in Sec.VI, the latter makes the angle-delay invariance hold only within a finite number of slots, thus limiting the values of L (or equivalently the memory of the subspace tracking). In Fig. 4 numerical results for varying L show that for a number of slots $L \geq 20$ the performance of the multi-slot estimators (solid lines) attains the asymptotic bounds (dashed line). As a simple rule of thumb, the convergence rate depends on the number of parameters to be estimated from the joint observation of the ensemble of slots, i.e., the number of entries of $\mathbf{U}_{\mathrm{S}}$ and/or $\mathbf{U}_{\mathrm{T}}$. This justifies the faster convergence of MS-S for this example. Moreover, recalling that the variance of any MLE approaches the CRB when the cardinality of the set of observations grows to infinity, we note that the convergence of the MSE of MS-ST to the bound computed from the CRB confirms numerically the optimality of MS-ST for large L, while MS-S and MS-T are close to the bound for any $L \geq 20$ (Sec.IV).

The performances of the subspace tracker proposed in [19] are evaluated in Fig. 5 in terms of MSE versus the number of slots L. The exact EVD implementation (dashed lines) and the subspace tracking implementation (bold lines) are compared for varying SIR's (forgetting factor $\gamma = 1$). The MSE corresponding to the LSE is also shown as a reference (dash-dotted lines) and confirms the accuracy of the subspace tracking methods for real time

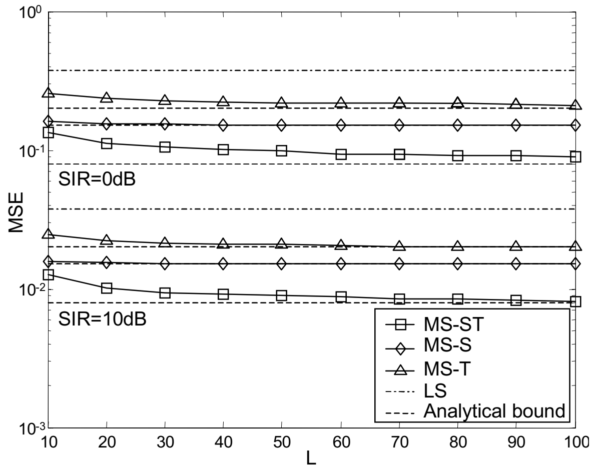

Fig. 4. Convergence properties of the multi-slot techniques MS-ST, MS-S and MS-T. For $L > 10$ the MSE attains the analytical bounds (19), (21) and (22) (dashed lines). As a reference, it is shown the MSE for LSE (dashed-dot lines) ($M = 8$, $W = 15$, $N = 40$, SNR= 50 dB).

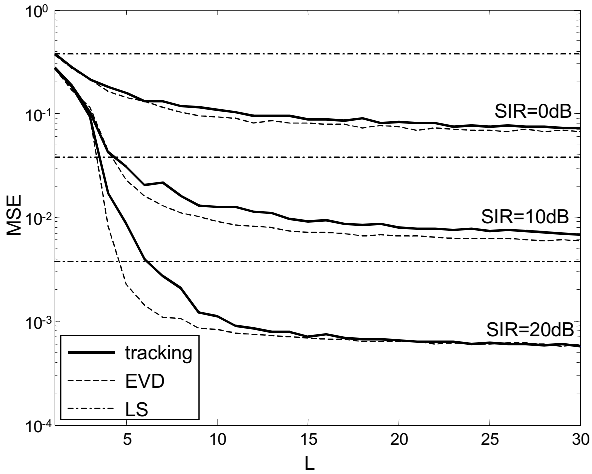

Fig. 5. MSE vs. the number of slot L for the exact EVD-implementation (dashed line) and for the subspace tracking implementation (solid line) for varying SIR's ($\gamma = 1$). The MSE for the LSE (dash-dot line) is shown for reference.

implementations.

VIII. Conclusion

The proposed multi-slot estimation methods exploit the invariance over the slots of both the spatial and the temporal subspaces, without estimating explicitly delays and angles of arrival. The paper has focused on the uplink of a single-user time-slotted system. Nonetheless, the approach (with straightforward modifications) appears to be a valid solution for a broader class of systems, such as TD-CDMA, FDMA, multicarrier or generically MIMO. Notice that, even when the receiving antennas are sufficiently spaced to cause the uncorrelation of the impinging wavefront, the MS-T method still retains its advantages. Analysis of the MSE shows that the asymptotic performance (i.e., for a large number of slots) depends on the number of parameters to be estimated on a slot-by-slot basis. Simulations proved the relevant advantage of the multi-slot approach compared to single-slot techniques as far as the quasi-static approximation of the space-time subspace holds true.

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