ANALYTIC FRAMEWORK FOR PERFORMANCE EVALUATION OF MULTI-ANTENNA WIMAX SYSTEMS OVER FADING CHANNEL

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ABSTRACT

This paper focuses on the uplink of multicell IEEE 802.16-d WiMax systems with OFDM modulation and antenna array at the base stations. We propose an analytical framework to assess the average error probability of all the different transmission modes over space-time dispersive Rayleigh fading channels. The proposed method takes into account the effects of the multilevel modulation, the error correction capability of the concatenated code, the interleaving scheme, the power-angle structure of the inter-cell interference and the array processing at the base station. Simulation results corroborate the proposed analysis for a IEEE 802.16-d cellular system over different propagation scenarios.

Index Terms— WiMax, Antenna arrays, Convolutional codes, Concatenated coding, Fading channel, Bit-interleaving.

1. INTRODUCTION

Worldwide Interoperability for Microwave Access (WiMax) is an OFDM-based technology [1][2] that provides last mile broadband wireless access. High data-rate, large spectral efficiency and improved coverage make WiMax one of the most promising technologies for the near future. The system is expected to operate in heterogeneous scenarios as possible applications range from wireless services in rural areas to intensive applications on notebooks and mobile devices. This makes mandatory to assess the WiMax performance in heterogeneous propagation environments.

The evaluation of the performance for bit-interleaved coded OFDM systems over frequency selective fading channels have recently attracted a great interest. In [3], the concept of *effective signal-to-noise ratio* (SNR) has been introduced as the SNR of an equivalent additive white Gaussian noise (AWGN) channel which would yield the same error probability as that of a given frequency-selective channel. The effective SNR can be used to adapt modulation and coding modes or to assess the average error rate for a given transmission mode [4]. Going one step further, in [5] a unified framework has been proposed to analytically assess the bit error rate (BER) of a multi-antenna WiMax system over space-time dispersive Rayleigh fading environments with spatially correlated intercell interference. The error rate is expressed as a function of the effective signal to interference plus noise ratio (SINR), redefined with respect to [4] so as to *jointly* account for the correlation of the inter-cell interference over the antennas and the correlation of the channel gains over *both* the antennas *and* the subcarriers.

The analysis in [5] accounts for a single transmission mode: BPSK modulation and convolutional code (CC). However, the pushing demand for high data rate applications makes mandatory the employment of more efficient transmission modes. In particular, WiMax prescribes M-QAM and concatenated coding schemes made by an inner CC and an outer Reed-Solomon (RS) code. In this paper, we extend the framework [5] to account for the whole set of standard-compliant transmission modes. The BER is first evaluated at the CC output by adapting the concept of effective SINR to M-QAM based on a set partitioning of the modulation mapping [6]. Then, the error rate of the overall concatenated code chain is derived by evaluating the error event correction ability of the outer block code. An efficient description of the long-term features of the multipath channel allows the analysis to cover heterogeneous propagation environments with varying angle-delay spreads. Numerical results show that the analytical model provides an accurate fitting of the BER over a large range of space-time multipath environments. The proposed model can also be extended for the evaluation of other performance metrics such as the frame error rate and the outage error probability, or it can be exploited for the design of channel aware scheduling algorithms.

2. MODELING THE WIMAX SYSTEM

We focus on the uplink of a IEEE 802.16-d multicell scenario [1][2] with fixed subscriber stations (SSs). We consider an hexagonal cell layout as in [5] where the transmission from the single-antenna subscriber station SS_0 to the L-antenna base station BS_0 is impaired by the N_I out-of-cell interferers $\{\mathrm{SS}_i\}_{i=1}^{N_I}$. The bit sequence $\{b_k\}$, to be transmitted by SS_0, is coded, bit-interleaved and mapped onto the sequence of complex symbols $\{x_k\}$. The standard prescribes M-QAM modulation coupled with a concatenated coding scheme made of an inner CC and an RS outer code. The modulation set of size $M = 2^m$ is here indicated by $\mathcal{X} = \{s_1, \ldots, s_M\}$, with $s_k = (s_k^Q + js_k^I)\sqrt{E_g}$ and $s_k^Q, s_k^I \in \{\pm 1, \pm 3, \ldots, \pm\sqrt{M} - 1\}$ (E_g is the transmitted waveform energy). The CC has generators (171,133) and code rate $R_{\mathrm{cc}} = k_{\mathrm{cc}}/n_{\mathrm{cc}}$ (a puncturing scheme is provided for some R_{cc} values), while the RS code is characterized by the parameters $(n_{\mathrm{rs}}, k_{\mathrm{rs}}, t_{\mathrm{rs}})$.

The modulated symbols $\{x_k\}$ are transmitted by SS_0 through OFDM signalling over a frequency selective fading channel. The $L \times 1$ baseband signal received by BS_0 on the kth subcarrier is:

$$\mathbf{y}_k = \mathbf{h}_k x_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{h}_k$ gathers the complex channel gains for the L receiving antennas, $x_k \in \mathcal{X}$ is the transmitted symbol with $E_s = \mathrm{E}[|x_k|^2] = \frac{2}{3}(M-1)E_g$, while $\mathbf{n}_k$ models the background noise and the intercell interference. The noise vector $\mathbf{n}_k$ is assumed to be zero-mean Gaussian, temporally uncorrelated but spatially correlated with spatial covariance matrix $\mathbf{Q} = \mathrm{E}[\mathbf{n}_k\mathbf{n}_k^{\mathrm{H}}] = \sigma_n^2\mathbf{I}_L + \mathbf{Q}_I$, where σ_n^2 is the variance of the background noise while $\mathbf{Q}_I = \sum_{i=1}^{N_I}\mathbf{Q}_{I,i}$ denotes the contribution from the N_I interferers $\{\mathrm{SS}_i\}_{i=1}^{N_I}$. The subcarrier index, $k \in \mathcal{K} = \{k_1, \ldots, k_K\}$, ranges over the K subcarriers used for data transmission, while the total number of subcarriers (which includes pilots, guard-bands and DC subcarrier) is indicated by N. See [5, Table 1] for a list of IEEE 802.16-d system parameters.

A minimum variance distortionless (MVDR) combiner [7] is applied to the L signals (1) in order to reduce the interference effects. The output is $\hat{x}_k = (\tilde{\mathbf{h}}_k^{\mathrm{H}}\tilde{\mathbf{h}}_k)^{-1}\tilde{\mathbf{h}}_k^{\mathrm{H}}\mathbf{y}_k$, where $\tilde{\mathbf{h}}_k = \mathbf{Q}^{-\mathrm{H}/2}\mathbf{h}_k$ is the channel vector weighted by the inverse Hermitian Cholesky factor of the noise spatial covariance $\mathbf{Q}$. As shown in [7], the resulting SINR after spatial filtering, defined as $\gamma_k = \mathrm{E}[|x_k|^2]/\mathrm{E}[|\hat{x}_k - x_k|]^2$, is given by $\gamma_k = E_s||\tilde{\mathbf{h}}_k||^2$. The sequence $\{\hat{x}_k\}$ obtained at the beamformer output is then used for Max-Log-MAP demodulation, bit deinterleaving, Viterbi and RS decoding.

The spatial channel vector is modelled as in [5] as:

$$\mathbf{h}_k = \sqrt{P_0}\sum_{r=1}^{N_R}\alpha_r \mathbf{a}(\theta_r) G_k \exp\left(-j2\pi\frac{k}{N}\frac{\tau_r}{T}\right). \tag{2}$$

This is the sum of N_R paths, each characterized by a direction of arrival (DOA) θ_r, a time of arrival (TOA) τ_r and a complex fading amplitude α_r. P_0 denotes the average power from all the paths, G_k is the frequency response of the cascade connection of the transmitter and receiver filters on the kth subcarrier, T is the sampling interval, while the vector $\mathbf{a}(\theta_r) = [a_1(\theta_r)\cdots a_L(\theta_r)]^{\mathrm{T}}$ represents the array response to the DOA θ_r. Amplitudes are assumed to be uncorrelated Rayleigh-faded, $\alpha_r \sim \mathcal{CN}(0, \rho(\tau_r, \theta_r))$, with variance $\rho(\tau_r, \theta_r)$ that depends on the power-delay-angle profile (PDAP) $\rho(\tau, \theta)$. We employ an exponential PDAP, $\rho(\tau, \theta) \propto \exp(-\tau/\sigma_\tau)\exp(-|\theta - \theta_0|/\sigma_\theta)$, with delay spread σ_τ and angle spread σ_θ. The angle θ_0 denotes the main DOA connecting SS_0 and BS_0, while $\propto$ stands for proportional. The PDAP is normalized so that $\sum_{r=1}^{N_R}\rho(\tau_r, \theta_r) = 1$.

We also adopt two simplified Rayleigh-faded channels as extreme cases of frequency selectivity: the frequency-flat (FF) channel where the channel gains are constant all over the bandwidth (as for $\sigma_\tau \to 0$) and the maximum frequency diversity (FD) channel where the channel gains are i.i.d. over the subcarriers (as for $\sigma_\tau \to \infty$).

A multipath model is employed also for the interference scenario. The ith interferer spatial covariance, for $i = 1, \ldots, N_I$, is indeed modelled as $\mathbf{Q}_{I,i} = P_i \sum_{r=1}^{N_R}\rho_i(\theta_{i,r})\mathbf{a}(\theta_{i,r})\mathbf{a}^{\mathrm{H}}(\theta_{i,r})$, depending on the DOAs $\{\theta_{i,r}\}_{r=1}^{N_R}$, the normalized power-angle-profile $\rho_i(\theta) \propto \exp(-|\theta - \theta_i|/\sigma_\theta)$ and the mean power P_i. The angle θ_i denotes the main DOA connecting SS_i and BS_0.

3. CONVOLUTIONAL CODE PERFORMANCE ANALYSIS

In this Section we analyze the performance at the output of the CC decoder. The extension to CC-RS concatenation will be carried in the following Section. Given a channel characterized by the SINR set $\gamma = [\gamma_1 \cdots \gamma_K]$, the bit error probability $P_b^{\mathrm{cc}}(\gamma) = P(b_k \neq b_k|\gamma)$ can be approximated using the union bound approach as [8]:

$$P_b^{\mathrm{cc}}(\gamma) \leq \frac{1}{k_{\mathrm{cc}}}\sum_{d \geq d_{\mathrm{free}}}\beta_d P_d(\gamma) \tag{3}$$

where d_{free} is the CC free Hamming distance, $P_d(\gamma)$ is the pairwise error probability (PEP) associated to the codeword c having Hamming distance d from the transmitted sequence, while β_d is the corresponding input weight. As in [5], we select as error event c the codeword that corresponds to the worst configuration of d error bits; such a configuration needs to be evaluated for the specific code and interleaver, as done in [5] for the IEEE 802.16-d standard.

For the derivation of $P_d(\gamma)$ we assume that the d erroneous bits in the error event are mapped by the interleaver onto d different subcarriers $\mathbf{f} = [f_1 \cdots f_d]$. Let the kth bit ($k = 1, \ldots, d$) belong to the QAM set element $x_k \in \mathcal{X}$ and be placed in the position $i_k \in \{1, \ldots, m\}$ of the modulation label set; let $\mathbf{x} = [x_1 \cdots x_d]$ and $\mathbf{i} = [i_1 \cdots i_d]$ be the sequences of the d QAM elements and the d bit positions in the QAM labels selected by the error event c. The PEP can be computed as [6]:

$$P_d(\gamma) \leq \sum_{\mathbf{i}} P(\mathbf{i}) \sum_{\mathbf{x}} P(\mathbf{x}) P_d(\gamma|\mathbf{i}, \mathbf{x}) \tag{4}$$

where $P_d(\gamma|\mathbf{i}, \mathbf{x})$ is the PEP conditioned to the set $(\mathbf{i}, \mathbf{x})$, while $P(\mathbf{i}) = 1/m^d$ is the probability of each label set. Notice that for any coded bit sequence to be modulated and any given label set $\mathbf{i}$, there are $2^{(m-1)d}$ possible symbol sequences $\mathbf{x}$ with equal probability $P(\mathbf{x}) = 1/2^{(m-1)d}$.

Similarly to [5], $P_d(\gamma|\mathbf{i}, \mathbf{x})$ is a function of the SINR at the decision variable (the effective SINR) that is a combination of the SINR values $\gamma_d = [\gamma_{f_1} \cdots \gamma_{f_d}]^{\mathrm{T}}$ observed over the subcarriers $\mathbf{f}_c$. However, here each SINR value needs to be scaled by a factor that takes into account the Euclidean distance between the transmitted QAM symbol and its nearest concurrent in the QAM constellation. This factor varies with x_k and i_k. More specifically, let $\mathcal{X}_0^i$ (and $\mathcal{X}_1^i$) be the subset of all symbols x_k whose QAM label has the value 0 (and 1) in position i_k, $i_k = 1, \ldots, m$. These subsets are shown in Fig. 1 for $M = 16$, $i = 1$ and Gray's mapping. If the transmitted bit is equal to 0 and it is mapped onto the ith label position, the transmitted symbol x_k belongs to the subset $\mathcal{X}_0^i$. The Max-Log-MAP demodulator decision is erroneous when the received symbol lies in $\mathcal{X}_1^i$. Thereby, the probability of error can be upper bounded by using the distance between x_k and the boundary of the area associated to the nearest neighbor in $\mathcal{X}_1^i$. For the 16-QAM example in Fig. 1 ($i = 1$), this distance is $\Delta_k\sqrt{E_g}$, with $\Delta_k = 1$ for x_k belonging to group 1 and $\Delta_k = 3$ for group 2. The same holds for the label position $i = 3$, while for $i = 2$ and $i = 4$ only the distance factor $\Delta_k = 1$ is experienced. As a result, the SINR at the decision variable is a function of the overall sequence $\mathbf{\Delta} = \mathbf{\Delta}(\mathbf{i}, \mathbf{x}) = [\Delta_{f_1} \cdots \Delta_{f_d}]$ of the d Euclidean distances (normalized by $\sqrt{E_g}$) that are selected by the d error bits in the codeword c:

$$\gamma_{\mathrm{eff}}(\gamma|\mathbf{\Delta}) = E_g \sum_{k \in \mathbf{f}_c} ||\tilde{\mathbf{h}}_k||^2 \Delta_k^2 = \frac{3}{2(M-1)}\sum_{k \in \mathbf{f}_c}\gamma_k \Delta_k^2. \tag{5}$$

The corresponding PEP is

$$P_d(\gamma, |\mathbf{i}, \mathbf{x}) = P_d(\gamma|\mathbf{\Delta}) = Q\left(\sqrt{2\gamma_{\mathrm{eff}}(\gamma|\mathbf{\Delta})}\right). \tag{6}$$

It is worth noticing that each distance Δ_k can assume only few values. In fact, for the 16-QAM case in Fig. 1 it is $\Delta_k = 1$ for three quarters of the sets (i_k, x_k) and $\Delta_k = 3$ for the remainder, i.e.: $P(\Delta_k = 1) = 3/4$ and $P(\Delta_k = 3) = 1/4$. Similar considerations holds for 64-QAM. Since only few distance values are observed, it is convenient to gather in (4) all the configurations of $(\mathbf{i}, \mathbf{x})$ that correspond to the same distance sequence $\mathbf{\Delta}$, yielding:

$$P_d(\gamma) \leq \sum_{\mathbf{\Delta}} P(\mathbf{\Delta}) Q\left(\sqrt{2\gamma_{\mathrm{eff}}(\gamma|\mathbf{\Delta})}\right) = \mathrm{E}_{\mathbf{\Delta}}\left[Q\left(\sqrt{2\gamma_{\mathrm{eff}}(\gamma|\mathbf{\Delta})}\right)\right], \tag{7}$$

where $P(\mathbf{\Delta})$ is the probability of the distance set $\mathbf{\Delta}$ and $\mathrm{E}_{\mathbf{\Delta}}[\cdot]$ indicates the average with respect to $\mathbf{\Delta}$. The average of the bit error probability with respect to fading, $P_b^{\mathrm{cc}} = \mathrm{E}_{\boldsymbol{\gamma}}[P_b^{\mathrm{cc}}(\gamma)]$, is finally obtained by plugging each PEP (7) into (3) and by integrating over $\gamma_{\mathrm{eff}}(\gamma|\mathbf{\Delta})$ using the pdf $p_d(\gamma_{\mathrm{eff}}|\Delta)$ of the effective SINR:

$$P_b^{\mathrm{cc}} = \frac{1}{k_{\mathrm{cc}}}\sum_{d \geq d_{\mathrm{free}}}\beta_d \mathrm{E}_{\mathbf{\Delta}}\left[\int Q\left(\sqrt{2\gamma_{\mathrm{eff}}(\gamma, c|\mathbf{\Delta})}\right) p_d(\gamma_{\mathrm{eff}}|\mathbf{\Delta}) d\gamma_{\mathrm{eff}}|\mathbf{\Delta}\right]. \tag{8}$$

Given a distance set $\mathbf{\Delta}$, the pdf $p_d(\gamma_{\mathrm{eff}}|\Delta)$ can be evaluated through the eigenvalue decomposition (EVD) of the space-frequency

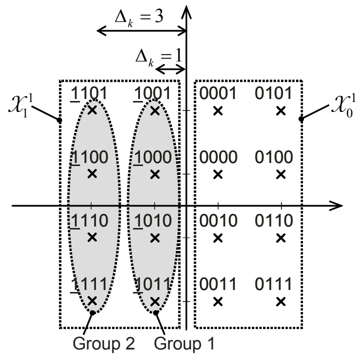

Fig. 1. The 16-QAM constellation with Gray's mapping. In the subset $\mathcal{X}_1^1$ there are two groups of bits with different minimum distances from the boundary of the subset $\mathcal{X}_0^1$.

channel correlation matrix as described in Sect. 3.1. However, in order to avoid the expensive EVD for each value of $\boldsymbol{\Delta}$, we propose to approximate the expectation $\mathrm{E}_{\boldsymbol{\Delta}}[\cdot]$ by means of a sample average: we simulate some values of $\boldsymbol{\Delta}$ as the outcomes of d i.i.d. random variables, having known pdf (see the pdf example above for 16-QAM); for each value, the effective SINR is obtained from (5) and its pdf is calculated according to the approach in Sect. 3.1; the estimate of the average bit error probability is then obtained by averaging over some realizations of $\boldsymbol{\Delta}$.

Remark. It can be easily observed that for QPSK ($M = 4$) it is $\Delta_k = 1, \forall k$, and the effective SINR simplifies to $\gamma_{\text{eff}}(\gamma, c) = \frac{1}{2}\sum_{k\in\mathbf{f}_c}\gamma_k$. For BPSK, it is $\gamma_{\text{eff}}(\gamma, c) = \sum_{k\in\mathbf{f}_c}\gamma_k$ as in [5].

3.1. Computation of the effective SINR pdf

Let us define $\tilde{\mathbf{h}} = \sqrt{E_g}[\Delta_1\tilde{\mathbf{h}}_{f_1}^{\mathrm{T}}, \cdots, \Delta_d\tilde{\mathbf{h}}_{f_d}^{\mathrm{T}}]^{\mathrm{T}}$ as the vector that gathers the scaled Ld channel gains associated to the d subcarriers of the error event c. This is a zero-mean complex Gaussian random vector with covariance matrix $\mathbf{R} = \mathrm{E}[\tilde{\mathbf{h}}\cdot\tilde{\mathbf{h}}^{\mathrm{H}}]$ given by

$$\mathbf{R} = E_g\begin{bmatrix} \Delta_1^2\mathbf{R}(f_1, f_1) & \cdots & \Delta_1\Delta_d\mathbf{R}(f_1, f_d) \\ \vdots & \ddots & \vdots \\ \Delta_d\Delta_1\mathbf{R}(f_d, f_1) & \cdots & \Delta_d^2\mathbf{R}(f_d, f_d) \end{bmatrix}, \quad (9)$$

where $\mathbf{R}(k, h) = \mathrm{E}[\tilde{\mathbf{h}}_k \cdot \tilde{\mathbf{h}}_h^{\mathrm{H}}]$ is the $L \times L$ channel crosscorrelation associated with the kth and hth subcarriers. According to the multipath model in Sect. 2, it is:

$$\mathbf{R}(k, h) = P_0\sum_{r=1}^{N_R}\rho(\tau_r, \theta_r)\tilde{\mathbf{a}}(\theta_r)\tilde{\mathbf{a}}^{\mathrm{H}}(\theta_r)G_kG_h\exp\left(-j2\pi\frac{k}{N}\frac{\tau_k - \tau_h}{T}\right) \quad (10)$$

with $\tilde{\mathbf{a}}(\theta_r) = \mathbf{Q}^{-\mathrm{H}/2}\mathbf{a}(\theta_r)$. The effective SINR (5) can be now rewritten as $\gamma_{\text{eff}}(\gamma|\boldsymbol{\Delta}) = ||\tilde{\mathbf{h}}||^2$, i.e. as the sum of the square magnitudes of the Ld correlated zero-mean Gaussian entries of $\tilde{\mathbf{h}}$. We can evaluate the pdf of such a sum following the same procedure as in [5]. This leads to the following effective SINR distribution:

$$p_d(\gamma_{\text{eff}}|\boldsymbol{\Delta}) = \sum\frac{A_i}{\lambda_i}\exp\left(-\gamma_{\text{eff}}/\lambda_i\right), \quad (11)$$

λ_i being the ith eigenvalue of $\mathbf{R}$ and $A_i = 1/\Pi_{k=1,k\neq i}^{d}(1 - \frac{\lambda_k}{\lambda_i})$.

4. CONCATENATED CODING EXTENSION

In this Section we extend the analytic framework to the concatenated coding schemes prescribed in the IEEE 802.16-d standard [1] [2].

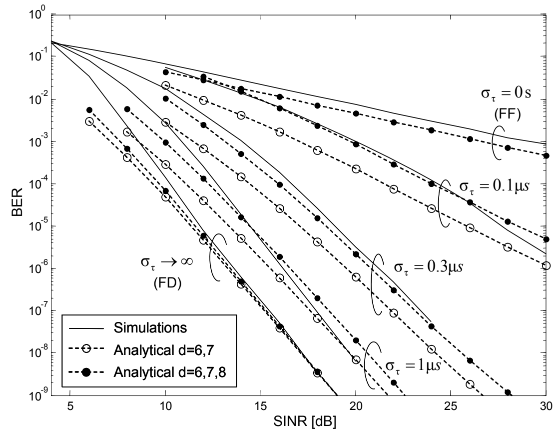

Fig. 2. Performance of a single-antenna WiMax system with QPSK modulation. The BER is evaluated at the output of the CC decoder.

For the QPSK and M-QAM modulations, the standard recommends an inner CC with rate R_{cc} and memory Q, and a RS $(n_{\text{rs}}, k_{\text{rs}}, t_{\text{rs}})$ as outer code. Conventional performance analysis [8] for concatenated codes accounts for the error occurrence in the RS codeword as a bit-isolated random process. However, the standard here does not prescribe an interleaver between the outer and inner codes thus making mandatory to model the error at the output of the CC decoder as bursty (any error event in the Viterbi trellis leads to a burst of errors that corrupts one or more consecutive symbols in the outer block).

Let D_c denote the maximum length (in coded bits) of the error event of distance d (the length is here evaluated after the possible puncturing inverse process). At the CC decoder output, the error burst has length $L_{\text{out},d} = D_cR_{\text{cc}} - Q$, as the error event always terminates with Q correct bits. To simplify the performance analysis, we divide the RS codeword (made by n_{rs} symbols of $B = 8$ bits each) into $S_d = n_{\text{rs}}B/(L_{\text{out},d} + Q)$ disjoint segments. In each segment, the error burst can occur with probability $p_{e,d} \leq (L_{\text{out},d} + Q)\,P_p$, where P_p denotes the probability for an error path to diverge along the Viterbi trellis. It can be easily shown that $P_p = P_b^{\text{cc}}z_d/\beta_d$, where z_d refers to the number of paths with Hamming distance d and $P_b^{\text{cc}} = \mathrm{E}_{\boldsymbol{\gamma}}[P_b^{\text{cc}}(\gamma)]$ from (7). The BER is eventually given by all the possible error event combinations in S_d segments:

$$P_b^{\text{cc+rs}} \approx \sum_d\frac{\beta_d}{z_d}\underbrace{\sum_{i=T_d+1}^{S_d}\frac{i}{S_d}\binom{S_d}{i}p_{e,d}^{i}\left(1 - p_{e,d}\right)^{S_d - i}}_{\text{Number of possible error paths, weighted by their probability}}. \quad (12)$$

Notice that the maximum number of symbols that can be corrupted by an error event of distance d in the RS codeword is $N_d = \lfloor L_{\text{out},d}/B\rfloor + 1$. It follows that the RS decoder can correct up to $T_d = \lfloor t_{\text{rs}}/N_d\rfloor$ error events.

Remark. In presence of flat or almost flat fading channels, the length of the error bursts can be longer than the error correction ability of the block code. In this case, the RS code does not provide any performance benefit and the BER in (12) simplifies to the CC performance in Sect. 3.

5. NUMERICAL RESULTS

In this Section we corroborate the analytic framework derived in Sect. 3 and 4 through numerical simulations of a IEEE 802.16-d multicell scenario. The performance is assessed as a function of the

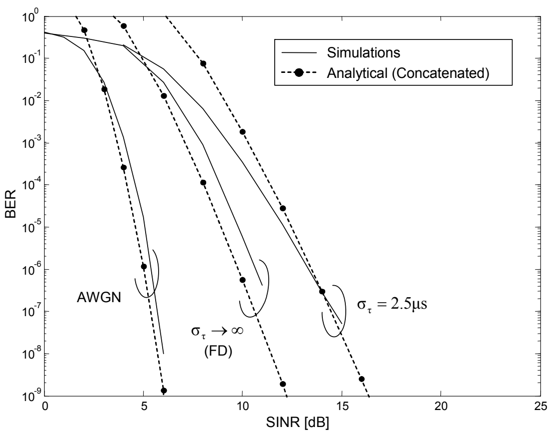

Fig. 3. Performance of a single-antenna WiMax system with QPSK modulation. The BER is evaluated at the output of the RS decoder.

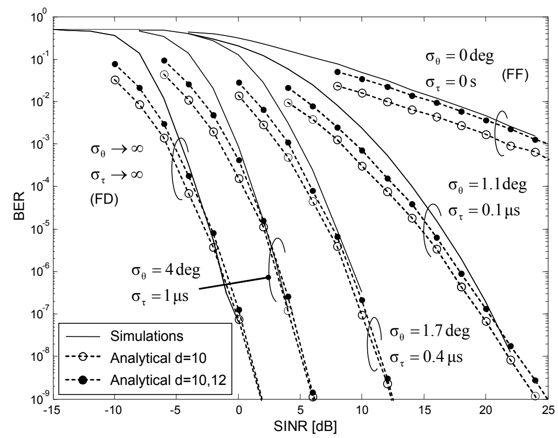

Fig. 4. Performance of a 4-antenna WiMax system with 16-QAM modulation. The BER is evaluated at the output of the CC decoder.

single-antenna SINR defined as $\text{SINR} = P_0/(\sigma_n^2 + \sum_i P_i)$ where $P_0/\sigma_n^2 = 20\text{dB}$. The active user SS_0 is transmitting from the main DOA $\theta_0 = 5\,\text{deg}$. All the N_I out-of-cell interferers are supposed to transmit at the same power level and their main DOAs are $\theta_1 = -46.1\,\text{deg}, \theta_2 = 0\,\text{deg}$ and $\theta_3 = 46.1\,\text{deg}$. For further details on the simulated radio environment please refer to [5].

We first consider a single-antenna ($L = 1$) link with $M = 4$ (QPSK), $R_{\text{cc}} = 2/3$ and RS $(32, 24, 4)$. In this case the background noise plus co-cell interference simplifies to AWGN. The channel is modelled as in Sect. 2 with exponential power-delay profile (PDP) and delay spread $\sigma_\tau \in \{0.1; 0.3; 1; 2.5\}\mu\text{s}$. The single path case (FF channel) and the maximum diversity case (FD channel) are simulated as well to provide upper/lower performance references. The simulated and the analytical BER at the output of the CC decoder are compared in Fig. 2 as a function of the SINR. The analytical performances are obtained using the union bound (8) truncated to the first two minimum distances $d = \{6, 7\}$ ($d = 6$ is the CC free distance), or to the first three $d = \{6, 7, 8\}$ (simulations show that this number is enough for a reasonable performance bound). As expected, the performance improves for increasing delay spread, from $\sigma_\tau = 0$ (FF) to $\sigma_\tau \to \infty$ (FD). The analytical BER is shown to be accurate all over the SINR domain. The BER at the output of the RS decoder is shown in Fig. 3. The analytical RS performance is obtained using (12) jointly with the union bound (8) truncated at the distances $d = \{6, 7, 8\}$. Since the RS encoder works on a codeword length equal to the OFDM symbol duration, the coding gain provided by the block encoder (as outer code) can be appreciated only for large σ_τ values. Thereby, we selected $\sigma_\tau = 2.5\mu\text{s}$ and the FD case for RS performance evaluation. The AWGN case is shown as a reference.

The multi-antenna case is considered in Fig. 4 with 16-QAM modulation and $R_{\text{cc}} = 1/2$. The performance is evaluated at the output of the CC decoder. The ULA at the base station BS_0 is composed of $L = 4$ antennas with inter-element spacing 1.4λ, where λ is the carrier wavelength (see [9] for antenna spacing selection). Multipath channels are simulated with delay spread $\sigma_\tau \in \{0.1; 0.4; 1\}\,\mu\text{s}$ and angular spread $\sigma_\theta \in \{1.1; 1.7; 4\}$ deg. In addition, the FD and FF cases are simulated. The interference covariance matrix is derived using as angular spread $\sigma_\theta = 4\,\text{deg}$ for all the three interferers. The union bound (8) is here truncated to the error events at either the free distance $d = 10$, or at the two first minimum distances $d = \{10, 12\}$. As already observed for the single-antenna case, the best performances are obtained with environments which provide an high grade of diversity. It can be easily seen that the analytical BER derivation permits an accurate fitting of the simulation results over the whole range of SINR and delay/angle spread.

6. CONCLUSIONS

In this paper an efficient analytical framework has been proposed to evaluate the performance of multi-antenna OFDM systems. The method allows to calculate, with moderate computational cost, the average bit error probability of WiMax systems without the need of simulating the whole transmission/propagation/receiving chain. Simulation results show that the proposed framework provides an accurate performance fitting for different modulations, coding schemes and frequency-selective radio scenarios.

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