Abstract— Since the demand for high-capacity in optical WDM networks keeps increasing, the realization of cheap and simple Optical Cross-Connects (OXC)s becomes more and more important. In this paper we propose new solutions for the construction of WDM optical switches employing only arrayed waveguide gratings and wavelength converters. Our approach defines single-stage OXC architectures and derives their non-blocking conditions, by considering two different connection request models for the lightpaths to be set through the switch. Depending on the degree of lightpath addressability, the OXC blocking is referred to either the wavelength-to-wavelength model or the fiber-to-fiber model. Further architectural considerations are added to identify different non-blocking solutions employing multiple switching/conversion stages.

I. INTRODUCTION

Telecommunications networks are still experiencing a tremendous increase in demand for capacity driven by some emerging networking and computing applications. The growth in the network traffic has stimulated the deployment of long-haul optical network systems which employ wavelength-division multiplexing (WDM) to achieve enormous transport capacity. Such systems, having tens of wavelengths per fiber with each wavelength modulated at 2.5 Gbit/s, 10 Gbit/s or more [1], rely mainly upon electronic devices to implement the switching functions. This leads to convert the optical signals to electrical form (O/E conversion) and then, after a suitable electronic switching, reconvert them to optical form (E/O conversion).

Although electronic switching is highly reliable, the dependence of switching hardware upon data bit-rate and transmission protocols becomes more and more a potential bottleneck. Furthermore the high costs due to the O/E/O conversions makes such a solution not viable for the perspectives foreseen with the evolution from WDM to DWDM scenarios, in which even more wavelengths per fiber will be carried. Consequently the transition of the switching functions from electronics to optics with the deployment of all-optical (OXC) cross-connects will potentially reduce the network-equipment complexity and increase its flexibility, provided that the OXC costs will be competitive with the cost of their OEO counterparts.

Non-blocking WDM Switches Based on Arrayed Waveguide Grating and Shared Wavelength Conversion

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GENERAL ASPECTS

and passive optical device (FWC): it changes the input signal wavelength. In Figure 2, the input port is routed to the fixed wavelength converter (FWC) as shown in Figure 2. In the FWM (FWM), where a connection can address more than one lightpath, it is assumed that all wavelengths are accepted on each output port. However, we can rearrange this in an OWB model, where every idle input and output port a lightpath with the same wavelength reference comb. Therefore, two different connection request models are adopted to study the blocking properties: 

• **fiber-based model (FBM),** where a connection can address just the output fiber but not the output wavelength; 
• **wavelength-based model (WBM),** where both the output fiber and the wavelength can be selected by the generic connection.

For both models two main classes of connecting networks can be identified: 

• **strict-sense non-blocking (SNB),** where every idle input channel can be connected to any idle output channel independently from the current network state, i.e., regardless of the connections already set-up and the internal network resources engaged for them; 
• **rearrangeable non-blocking (RNB),** where every idle input channel can be connected to any idle output channel, requiring in some cases the internal rearrangement of some of the input-output connections already set-up.

In evaluating the blocking properties for the two connection request models, we will have to consider that an idle output is represented by any idle wavelength in the addressed output fiber in the fiber-based model, while the availability of sufficient resources to reach the requested output wavelength on the addressed output fiber has to be verified to assess the non-blocking in the wavelength-based model.

**B. Optical components**

In this work, we focus our attention only on the case of AWG-based architectures where a limited number of optical components is used. In particular, the main characterizations and the relative notations of two largely used devices (the wavelength converter and the star coupler) are now specified. The wavelength converters (WCs) can be divided into two different categories:

• **fixed wavelength converter (FWC):** it changes the input lightpath wavelength onto a fixed value; 
• **tunable wavelength converter (TWC):** it is an active component able to convert the input lightpath onto any of the wavelengths which compose its tuning range. It can be realized in several ways, but it always needs a tunable laser source. For this reason it is more expensive than a simple FWC. Later on we will use \( R \) to indicate the number of wavelengths onto which the considered TWC is able to convert an arbitrary input lightpath.

The star coupler is a \( N \times N \) passive optical device that reproduces in every output the same set of lightpaths which derive from the combination of all the lightpaths entering its inputs. Its \( 1 \times N \) and \( N \times 1 \) versions represent respectively the optical splitter and combiner. In particular this second implementation can be exploited in order to realize operations of wavelength multiplexing.

The Arrayed Waveguide Grating (AWG) ([7], [8]) is a device able to perform a fixed permutation of the input lightpaths according to the wavelength and the inlet that each lightpath occupies. Although several types of connectivity for the AWG are assumed in literature, it is usually accepted that in an AWG with \( N \) input and output ports a lightpath with \( \lambda_j \) on the \( i_{th} \) input port is routed to the \( [(i + j) \mod N]_{th} \) output fiber [9]. However, apart from the particular considered connection, our conclusions hold for any AWG type.

There are two different operational modes in which this type of component can be used. In fact, considering the number of wavelengths which can be directed on a single AWG output port, two different cases are possible:

• each output port can be reached at least by one lightpath at time (we refer to this situation as the “mono-\( \lambda \)” environment); 
• more than one lightpath is accepted on each output port at the same time (this is the “multi-\( \lambda \)” environment).

In the following of this paper only the “multi-\( \lambda \)” scenario will be considered as it allows more flexibility in routing signals to the output ports. Anyway, also the “mono-\( \lambda \)” can be very useful mainly in multistage structures as shown in [10].

**III. KNOWN SINGLE STAGE CONSTRUCTIONS**

Even though some single-stage realizations are possible, several problems have not been solved yet. The main issue is represented by technological limitations in the used components.

In particular it seems very impractical to build a single-stage OXC using only an \( MW \times MW \) AWG and an array of \( MW \) TWCs with tuning range \( R = MW \) as shown in Figure 2. In fact, taking into account the future evolution of the DWDM protocol and hence in the increasing number of wavelengths carried by each fiber, it appears actually quite difficult to realize such an AWG. The same argument holds for the requirement in the converter tunability which still represents a very critical issue if we consider the current state of art of this component. Despite these important drawbacks, it is quite evident that any input signals can reach any AWG output ports irrespective of any previously established connection. After the AWG, a final array of FWCs is in charge for the final
wavelength conversion before multiplexing the lightpaths on the output fibers. What has just been stated is sufficient to prove the strictly non-blockingness (for the wavelength-based model) of the considered architecture.

Other implementations of blocking and non-blocking AWG-based switches have been introduced in [11]. The final conclusions of this work are that a single-stage strictly non-blocking structure can be obtained even with a W x W AWG, but the converter tunability cannot be reduced anyway. In fact all these non-blocking solutions show at least an array of TWCs with \( R = W M \).

In [10] and [9] a different approach is considered. In this case also the tunability requirement can be alleviated, but at the expense of the blocking properties. As a matter of fact, even if in [10] a well-defined architecture is not identified, some considerations show that only the rearrangeability in the fiber-based model can be obtained for such a construction and, additionally, this statement holds only for limited values of \( M \).

In the following sections, we will reconsider this last approach proposing two different implementations which can supply further indications about this type of construction. Before introducing our analyses, we believe that a better comprehension of the problem can be reached after reviewing the main issues of the original structure (first proposed in [9]).

The Ramamirtham-Turner design

A single-stage construction has been introduced in [9] (see Figure 3), which turns out to be blocking even in the fiber-based model. This design includes full-range TWCs and an array of W x W AWGs (one for each input fiber). Each AWG is connected with a different output fiber through \( W / M \) links.

Although the explanation of its blockingness has already provided in the relative paper, we believe that a formal proof can be useful to identify the most critical connection sets and so to better understand the blocking behavior itself. Moreover it provides the opportunity to underline the differences between such architecture and several others.

In this case and in general for each considered architecture, we ignore the useless case in which \( M = 1 \).

**Theorem 3.1:** The structure shown in Figure 3 is not RNB for the fiber-based model.

**Proof:** The main idea is to find the worst connection set and then to show that there are not enough available wavelengths to route the lightpaths on the right output.

First of all, let us assume \( W \) to be a multiple of \( M \) in order to have \( W = b \cdot M \) with \( b \) integer and \( b \geq 1 \). This condition must be set for construction reasons since there are \( W \) inputs and outputs in each AWG and only \( W / M \) of these latter ports are connected to the same final coupler.

From now on, without loss of generality, we consider exactly the final interconnection pattern shown in Figure 3 and the first output fiber as the reference to proceed with the following treatment.

Since there is only one conversion stage, each input lightpath addressing the considered output fiber has to select the AWG output links with a suitable wavelength in order to group at most \( W \) signals with \( W \) different wavelengths on the same final coupler. This means that at least \( W \) different wavelengths have to be available to route \( W \) arbitrary inlets to a particular output fiber.

Now, as we are using full-range TWCs, we cannot exploit more than \( W \) wavelengths to reach the final AWG output. On the contrary, it can be shown that for some connection sets, we do not have the availability of \( W \) wavelengths. For example, let \( Q(\lambda_i) \) be the set of wavelengths that the arbitrary input lightpath \( \lambda_i \) has to be converted to in order to reach the \( W / M \) links connected with the considered output fiber (with \( 0 \leq i \leq W - 1 \)). More precisely, we can write

\[
Q(\lambda_i) = \left\{ \lambda_j | 0 \leq ((i + j) \mod W) \leq \frac{W}{M} - 1 \right\}
\]

It is easy to note that \(|Q(\lambda_i)| = W / M \) for each \( i \).

However we are interested in identifying the total number of wavelengths suitable to realize an arbitrary connection set in which \( W \) entries share the same output fiber. If we define \( \Phi \) as the total number of different wavelengths available to serve these connection requests and \( q_i \) as the number of the first adjacent inputs of the AWG \( i \), because of the AWG cyclic
behave we obtain

$$\Phi = \left| \bigcup_{0 \leq i \leq W} Q(\lambda_i) \right| = \min \left\{ \left( \frac{W}{M} + q - 1 \right), W \right\}$$

where $q = \max_i \{ q_i \}$.

In the same way, it is easy to verify that, in general, if we consider groups of $q_i$ non adjacent inlets a wider number of available wavelengths is achieved, so it does not represent an interesting case in this proof. For this reason we will continue the treatment taking into account only connection sets with $q_i$ adjacent input lightpaths and consequently $\frac{W}{M} \leq q \leq W$.

Naturally, when $q = W$, we do not have any problem since $\Phi = W$. In fact all the inlets cross the same AWG and so even $W/M$ different wavelength assignments are possible. On the contrary, if $q < W$, it is not so sure that $W$ wavelengths can be used. In particular, it is now simple to identify the worst connection set, that is the one which minimizes $\Phi$. Focusing our attention, for example, on the case in which the first $W/M$ adjacent inlets in each of the $M$ different AWGs claim to reach the first output fiber, we obtain $q = \frac{W}{M}$ and

$$\Phi = \left| \bigcup_{0 \leq i \leq W/M} Q(\lambda_i) \right| = 2 \cdot \frac{W}{M} - 1$$

Since $\Phi < W$ for $M \geq 2$, this connection set cannot be realized because the available wavelengths are not enough to accomodate all the $W$ lightpaths that address the final output fiber. Finally, noting that we have only imposed the availability of a suitable number of wavelengths to reach the final output fiber and nothing about the final output wavelength, we can state that the structure shown in Figure 3 is blocking even for the less restrictive request model and for any value of $W$ and $M$.

The previous proof has put in evidence that the structure blocking is not due to a partial accessibility, rather to the lack of a suitable number of wavelengths to reach each output fiber. Then, since full-range TWCs are applied at the beginning of the structure, it is quite evident that the poor wavelength availability is above all a consequence of the limited number of converters used in the structure. This consideration suggests to improve the total number of the TWCs as proposed in [10]. At the same time it also appears evident that the introduction of a further conversion stage composed of $MW$ TWCs after the AWGs makes this type of architecture blocking (see Figure 4 where a further stage of full-range TWCs is inserted with respect to the Ramamirtham-Turner design). A formal proof is not strictly necessary in this case. It is enough to note that only $W/M$ lightpaths can be routed from each AWG to each output fiber. As a matter of fact, now only one wavelength can be accepted by each AWG output link because of the presence of the output TWCs.

Consequently, we can conclude that at least one link without TWC has to be inserted between each AWG and each final coupler. Taking into account this final consideration, we have identified two intermediate cases in which only part of the AWG outputs are provided with TWCs. In this sense, we consider the final conversion as shared. The new architectures and their properties are discussed in the next section.

IV. SINGLE STAGE CONSTRUCTIONS WITH SHARED CONVERSION

We follow here the approach sketched in [10] where a stage of AWGs is followed by a set of TWCs without further specifying the exact parameters of the structure. Here we propose two possible solutions for RNB switches using only an array of W x W AWGs and full-range TWCs. Unfortunately, these architectures result non-blocking only for the fiber-based model and for limited values of $M$. Although they are blocking for some values of $M$, it seems very useful to further study their implementations for several reasons. First of all they show simple and symmetrical designs which are realized with only one stage of AWGs. These features can be very important in the realization of switches under both the physical layer design and the cost issue, since they can result in relatively cheap components, reduced power consumption and low noise accumulation. Moreover these constructions could be very interesting if, even blocking, they show good probabilistic blocking performances, as pointed out in [9] with the Ramamirtham-Turner design.

A. Optical Switch with Shared Conversion version I (OSSC-I)

Figure 5 introduces a single-stage construction more powerful than the one proposed in [9]. It presents an initial array of $MW$ full-range TWCs feeding a stage of $MW$ W x W AWGs, followed by another stage of wavelength converters, in which only a limited number of full-range TWCs are used. In particular, we indicate with $k$ the number of links without TWCs between each AWG and each output fiber ($1 \leq k \leq W/M - 1$). Practically, for every AWG, there are again $W/M$ output ports available to reach each output fiber. But now only $W/M - k$ output ports (for each AWG) offer the possibility to change the lightpath wavelength providing an
additional full-range conversion after the AWG. Later on this structure will be referred to as “Optical switch with Shared Conversion-I” (OSSC-I), as the final TWCs are to be shared among the requiring inputs needing wavelength conversion.

Now we evaluate the properties of the structure in Figure 5, ignoring again the trivial case in which \( M = 1 \) and assuming \( 1 \leq k \leq W/M - 1 \) where \( k \) is a positive integer (\( W \) is always an integer multiple of \( M \)). Note that the cases in which \( k = 0 \) and \( k = W/M \) correspond respectively to those of Figure 4 and Figure 3.

**Theorem 4.1:** The architecture OSSC-I is RNB for the fiber-based model only when \( M < 4 \), depending on the value of \( k \). In particular, when \( M = 2 \), it must be verified that \( k \leq \frac{W}{2} - 1 \), while, for \( M = 3 \), the condition \( k \leq \frac{1}{2} \cdot \left( \frac{W}{3} - 1 \right) \) must hold.

**Proof:** As in the proof of Theorem 3.1, the main idea is identifying the worst connection set, that is the one in which the minimum number of wavelengths is available to reach a considered output. Differently from the previous proof in which the number of competing lightpaths is fixed and equal to \( W \), now the overall competitors are reduced because some lightpaths can be directed to the links provided with TWCs. Consequently, in this case, also the maximum number of competitors has to be identified. The combined search of these two different cases will determine the worst connection set which will provide the minimal conditions to grant non-blocking.

Also in this case, we will refer to the first output fiber without loss of generality, while the condition \( W = b \cdot M \) with \( b \) integer and \( b \geq 1 \) has to be imposed again for construction reasons.

The computation of the lowest number of available wavelengths can be realized in a way very similar to that exploited in the proof of Theorem 3.1. In fact, as it has already been stated, the worst case (from the wavelength availability point of view) occurs when the same \( q \) adjacent inputs of \( x \) different AWGs claim to address the same output fiber where the condition \( q \cdot x = W \) always holds (the importance of \( x \) will appear more clear later on). The condition \( W/M \leq q \leq W \) remains valid for \( q \), whereas \( 2 \leq x \leq M \) since the case in which all the lightpaths come from the same AWG is not critical.

So considering in particular the worst connection set we achieve

\[
\Phi = \min \left\{ \left( k + \left\lceil \frac{W}{x} \right\rceil - 1 \right), W \right\}
\]

where \( Q(\lambda_i) \) continues to represent the set of wavelengths an arbitrary input lightpath \( \lambda_i \) has to be converted to in order to reach the \( k \) links connected with the considered output fiber without any TWCs, while \( \Phi \) indicates the total number of the wavelengths available to satisfy the connection set requirement for the selected output fiber. Note that in the previous formula \( q \) has been replaced by \( \left\lceil \frac{W}{x} \right\rceil \) because, in general, it is not granted that \( W \) is a multiple of \( x \). In fact, when \( W/x \) is not integer, it should be considered the case in which some AWGs offer \( \left\lceil \frac{W}{x} \right\rceil \) lightpaths while the other only \( \left\lfloor \frac{W}{x} \right\rfloor \); however the total number of exploitable wavelengths has to be calculated assuming the maximum number of adjacent inputs, that is \( \left\lceil \frac{W}{x} \right\rceil \).

Since we are interested in considering the worst case, the previous formula reduces to

\[
\Phi = \min \left\{ \sum_{0 \leq i \leq M} Q(\lambda_i) \right\} = k + \left\lceil \frac{W}{x} \right\rceil - 1
\]

The identification of the total number of competing lightpaths \( \Psi \) in this situation is simple. As a matter of fact, \( W \) different signals can be routed on the same output fiber and, considering \( x \) different AWGs in the connection set, up to \( x \cdot \left( \frac{W}{M} - k \right) \) lightpaths can reach the requested output fiber without problems since they can exploit a further conversion after the AWGs. Consequently, it is possible to state that

\[
\Psi = W - x \cdot \left( \frac{W}{M} - k \right)
\]

Based on the reasoning that has been followed, \( \Psi \) represents also the maximum number of competing lightpaths with this type of structure and this number depends on \( x \).

Now, in order to grant the non-blockingness of the studied architecture, \( \Psi \leq \Phi \) has to be set. It means that we have to assume

\[
W - x \cdot \left( \frac{W}{M} - k \right) \leq k + \left\lceil \frac{W}{x} \right\rceil - 1
\]

Resolving this condition for \( M \), the following expression is obtained

\[
M \leq \frac{x \cdot W}{W + k \cdot x - \left\lceil \frac{W}{x} \right\rceil - (k - 1)} \tag{1}
\]
At this point, it is possible to recognize the worst connection set, finding the value of \( x \) that minimizes the maximum value of \( M \). In fact, we can study the last expression as a function of \( x \) since \( W \) and \( k \) are structural parameters that do not depend on the particular connection set. Assuming

\[
f(x) = \frac{x \cdot W}{W + k \cdot x - \left\lfloor \frac{W}{2} \right\rfloor - (k-1)}
\]

it is easy to grasp that the minimum value of \( f(x) \) is always achieved for \( x = 2 \). This means that the worst connection set is represented by the situation in which the same \( \lceil W/2 \rceil \) input lightpaths of two different AWGs address the same output fiber.

So considering (1) for \( x = 2 \), it is immediate to verify that we cannot have \( M \geq 4 \forall W, k \); so only the cases of \( M = 2 \) and of \( M = 3 \) are possible if non-blocking is requested and precise conditions about the values of \( W \) and \( k \) have to be detected for these cases. It will be taken into account the fact that \( \Psi \leq \Phi \), but now \( M \) and \( w \) assume known values. Assuming \( M = 2 \) and \( x = 2 \), it must be verified that \( k \leq W/2 - 1 \) (indeed, in this particular case, the considered architecture is non-blocking for every allowed value of \( k \)), while for \( M = 3 \), both the cases of \( W \) even and \( W \) odd have to be considered, but they converge in a single condition which sets \( k \leq 1/2 \cdot (W/3 - 1) \).

For a further check, consider the simple case in which \( M = 2 \). It should be quite clear that the conflicts are caused by the sharing of the same output wavelength between different lightpaths. In the same way, it is immediate to understand that the worst case happens when the group of competitors is composed of adjacent inlets directed to same output fiber. When \( M = 2 \) it is simple to show that the condition imposed previously is effectively due to the worst possible connection set. In fact assume that \( q_i \) adjacent inlets from the AWG \( i \) are requesting the first output fiber (where \( i = 1, 2 \)). Suppose that these input wavelengths belong to a set of wavelengths \( \Lambda_i \); in order to find the most critic situation, for example, the case in which \( \Lambda_i \subseteq \Lambda_j \). In this case we obtain that \( \Psi = 2k \), while \( \Phi = k + \max(q_1, q_2) - 1 \). It is now evident that the case in which \( q_1 = q_2 = W/2 \) is the worst one since it minimizes \( \Phi \). So we have shown that the connection set considered previously for arbitrary values of \( M \) represents effectively the most critical situation. The same conclusions can achieved for the case in which \( M = 3 \).

Finally, noting that only the right wavelength availability has been checked and that nothing has been imposed about the output wavelength values, it is possible to state that the achieved results can be exploited only for the fiber-based model. In the same way, the type of non-blocking ensured by the obtained conditions cannot be more general than the rearrangeable one, since it has not been verified that the number of paths available between each inlet and each output fiber is enough to eliminate the need for the rearrangement of even a single already existing connection.

The following two theorems are aimed at studying if the OSSC-I can be considered also SNB in FBM scenario or if it can be classified RNB also for the more restrictive WBM approach. They will prove that, in these two situations, OSSC-I is indeed blocking.

**Theorem 4.2:** The OSSC-I architecture is not SNB in the fiber-based model for any value of the parameter \( k \).

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

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\]

It is always possible to consider the case in which even other \( k - 1 \) inlets of the same AWG are already connected with the considered output fiber using the same group of wavelengths except one which is exclusive of the inlet \( \lambda_u \). In this case, the lightpath \( \lambda_u \) is obliged to use this only available wavelength. Suppose that a similar situation holds in an other arbitrary AWG \( j \) that is to say that also in this case the \( W/M - k \) output ports provided with TWCs are already occupied. If, for example, a new connection is requested between the inlet \( \lambda_u \) of the AWG \( j \) and the first output fiber, blocking occurs because the \( k \) addressable wavelengths are already used by the inlets of the AWG \( i \). In fact it is impossible to route two different lightpaths with the same wavelength to the same output fiber. So, since the rearrangement of the already existing connections is not allowed in this reference model, it is possible to state that this architecture is blocking.

**Theorem 4.3:** The OSSC-I architecture is not RNB in the wavelength-based model for any value of the parameter \( k \).

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

**Proof:** Considering an arbitrary AWG \( i (1 \leq i \leq M) \), assume that all the \( W/M - k \) output ports provided with TWCs and directed, e.g., to the first output fiber are already engaged. Now, if a connection has to be established between another inlet \( \lambda_u \) of the AWG \( i \) and the first output fiber, only the \( k \) output links without TWCs are available to route this lightpath and so \( k \) different wavelength values can be used \((0 \leq u \leq W - 1)\). Practically, it is possible to choose among the \( k \) wavelengths which compose the wavelength set \( \Delta(\lambda_u) \) where \( \Delta(\lambda_u) \) is defined as follows

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

Finally, noting that only the right wavelength availability has been checked and that nothing has been imposed about the output wavelength values, it is possible to state that the achieved results can be exploited only for the fiber-based model. In the same way, the type of non-blocking ensured by the obtained conditions cannot be more general than the rearrangeable one, since it has not been verified that the number of paths available between each inlet and each output fiber is enough to eliminate the need for the rearrangement of even a single already existing connection.

The following two theorems are aimed at studying if the OSSC-I can be considered also SNB in FBM scenario or if it can be classified RNB also for the more restrictive WBM approach. They will prove that, in these two situations, OSSC-I is indeed blocking.

**Theorem 4.2:** The OSSC-I architecture is not SNB in the fiber-based model for any value of the parameter \( k \).

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

**Proof:** Considering an arbitrary AWG \( i (1 \leq i \leq M) \), assume that all the \( W/M - k \) output ports provided with TWCs and directed, e.g., to the first output fiber are already engaged. Now, if a connection has to be established between another inlet \( \lambda_u \) of the AWG \( i \) and the first output fiber, only the \( k \) output links without TWCs are available to route this lightpath and so \( k \) different wavelength values can be used \((0 \leq u \leq W - 1)\). Practically, it is possible to choose among the \( k \) wavelengths which compose the wavelength set \( \Delta(\lambda_u) \) where \( \Delta(\lambda_u) \) is defined as follows

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

It is always possible to consider the case in which even other \( k - 1 \) inlets of the same AWG are already connected with the considered output fiber using the same group of wavelengths except one which is exclusive of the inlet \( \lambda_u \). In this case, the lightpath \( \lambda_u \) is obliged to use this only available wavelength. Suppose that a similar situation holds in an other arbitrary AWG \( j \) that is to say that also in this case the \( W/M - k \) output ports provided with TWCs are already occupied. If, for example, a new connection is requested between the inlet \( \lambda_u \) of the AWG \( j \) and the first output fiber, blocking occurs because the \( k \) addressable wavelengths are already used by the inlets of the AWG \( i \). In fact it is impossible to route two different lightpaths with the same wavelength to the same output fiber. So, since the rearrangement of the already existing connections is not allowed in this reference model, it is possible to state that this architecture is blocking.

**Theorem 4.3:** The OSSC-I architecture is not RNB in the wavelength-based model for any value of the parameter \( k \).

\[
\Delta(\lambda_u) = \left\{ \lambda_u \right\} \frac{W}{M} - k - (u + v \mod W) \leq \frac{W}{M} - 1
\]

**Proof:** It is sufficient to consider a simple connection set in which at least \( W/M - k + 1 \) lightpaths that cross the same AWG are directed towards a single output fiber. For construction, \( W/M \) output links are available for this routing. It is always possible to take as example the particular case in which all the \( W/M - k + 1 \) inlets can be directed to the output fiber with the requested wavelength using a single output link (and so without exploiting a further stage of wavelength conversion). In fact even \( W \) different inlets can select the same AWG output with \( W \) different wavelengths if, as in this case, a \( W \times W \) AWG is used. If the detected output link is provided with a TWC, only one of the lightpaths can be accommodated, while for the others it is necessary to provide a further conversion otherwise they cannot address the output fiber with the requested output wavelength. Now, since one output link connected to a TWC has been already occupied, only \( W/M - k - 1 \) TWCs for the considered output fiber are still available, while there are \( W/M - k \) lightpaths which are waiting for a second wavelength conversion. So it is impossible to provide the suitable conversion for any value of \( k \) and, on the contrary, we can point out that the examined structure is not RNB in the wavelength-based model.

Applying this last theorem also implies that the OSSC-I design cannot be SNB under the wavelength-based model.
B. Optical Switch with Shared Conversion version II (OSSC-II)

An alternative single-stage construction with shared output conversion is shown in Figure 6. In this case, the conversion capability is not symmetrical in the sense that for each AWG there is an output fiber which can be reached without the possibility of a further wavelength conversion. In fact, the design shows that each AWG is connected with \( M - 1 \) output fibers with \( W/M + 1 \) links. \( W/M \) of these links are equipped with full-range TWCs, while the last one can carry some different lightpaths, but without giving the possibility of a further wavelength conversion. The remaining output fibers can be reached with only one link which does not include any TWCs. Consequently only \( W - (W/M - M) \) AWG output ports are used and so, if we want to use \( WxW \) AWGs as in the previous case, the condition \( W \geq M^2 \) must be guaranteed (with this condition \( W/M \geq M \) and so the output port number is at least equal to \( W \)).

The fundamental aspect of this construction is represented by the fact, beyond the full accessibility of output fibers by all AWGs, there are not two or more AWGs connected to the same output coupler with only one link. For example, in Figure 6, the generic AWG \( i \) is connected with the output fiber \( j \) using only a single link (not equipped with TWC), while an arbitrary output fiber \( j \) (with \( j \neq i \)) can be reached still from the AWG \( i \) selecting \( W/M + 1 \) different AWG output ports: in this case the lightpaths have either the possibility to convert their wavelength choosing one of \( W/M \) links provided with TWCs or eventually to go onto the output fiber with their own wavelength passing on the last available link.

Note that, although the number of the second-stage interconnection links are now reduced with respect to the OSSC-I case, we cannot give any indications for what concerns the total number of TWCs. For this reason, it is better first to study the conditions that makes OSSC-II non-blocking and then to try a comparison between these two structures. The following theorem will help us in this aim.

**Theorem 4.4:** The OSSC-II structure is RNB for the fiber-based model only when \( M = 2 \) and \( W \geq 4 \).

**Proof:** The main idea is the same as that used in Theorem 4.1. Practically, first of all the worst connection set has to be recognized, then the number of the available wavelengths and the number of the competing lightpaths have to be compared. The only two differences with respect to the proof regarding the OSSC-I structure are represented by the fact that now there is a single link without TWCs that connects each AWG to each output fiber and additionally there are couples of AWGs and output fibers connected with only one link. Again we will proceed considering the first output fiber as the reference output fiber, without loss of generality.

Moreover, we suppose that all the AWGs have the same active output ports (remember that in general only \( W - W/M + M \) AWG output ports are used) and in general we refer to the implementation shown in Figure 6.

Using the same notation as in the previous proofs, we indicate with \( Q(\lambda_i) \) the set of wavelengths that enables inlet \( \lambda_i \) to select the links not provided with TWC and connected to the considered output fiber. In this case this link is unique, so \( |Q(\lambda_i)| = 1 \), while \( \Phi \) that is the total number of the wavelengths used to reach the first output fiber when, for example, the first \( q \) different lightpaths are routed on this link is equal to

\[
\Phi = \bigcup_{0 \leq i \leq q} Q(\lambda_i) = q
\]

Note that now it is not necessary to consider adjacent inlets to have the minimization in the used wavelengths. In fact, in this case only one output port can be addressed, so the relative position of the input lightpaths does not represent a significant aspect. Instead, it is still true that the most critical condition occurs when the lightpaths requesting the considered output fiber are divided in several AWG and additionally they have the same initial wavelengths. For this reason it is possible to state that in order to find the worst connection set, in general, we have to consider the case in which the \( W \) lightpaths addressed to the first output fiber are collocated as uniformly as possible in \( x \) different AWGs and moreover, in each AWG, they occupy the same group of inlets. In this situation it is possible to assume

\[
\Phi = \bigcup_{0 \leq i \leq \lceil \frac{W}{x} \rceil} Q(\lambda_i) = \lceil \frac{W}{x} \rceil
\]

Focusing on this particular type of connections, the maximum number of competing lightpaths \( \Psi \) has to be found. Taking into account the previous considerations and the fact that there are signals that crossing the first AWG do not have the possibility of a second wavelength conversion, the following result holds

\[
\Psi = W - (x - 1) \frac{W}{M}
\]

In fact, leaving out the first AWG, \( W/M \) represents the number of TWCs available after the remaining \( x - 1 \) AWGs to reach the first output fiber and so \((x-1)W/M\) is the number of
lightpaths that can be routed to the right output fiber, while Ψ summarizes the inlets needing to select a suitable wavelength just from the first conversion at the beginning of the structure. Now imposing Ψ ≤ Φ we ensure that an adequate number of wavelengths is available to route the competitor inlets in the connection set. Solving the last relation, the following result can be obtained

\[ M \leq \frac{(x - 1) \cdot W}{W - \left\lceil \frac{W}{x} \right\rceil} \]

At this point, the worst connection set has been found. Defining \( f(x) \) as follows

\[ f(x) = \frac{(x - 1) \cdot W}{W - \left\lceil \frac{W}{x} \right\rceil} \]

it is easy to verify that the minimum values of \( f(x) \) are always achieved for \( x = 2 \), independently from \( W \). Consequently, it can be stated that the worst connection set is that in which only two AWGs are considered and one of these AWGs is connected with only one link to the final coupler. Moreover, the competing inlets are divided in equal parts between these two AWGs and occupy the same AWG input ports. It is straightforward to see that, with \( x = 2 \), \( M < 3 \), \( \forall W \), so only the case \( M = 2 \) is valid when naturally \( W \geq M^2 \) for architectural reasons as already explained.

Finally, we show, with some simple considerations, that the structure is effectively RNB (see Figure 7). Defining \( q_i \) as the number of inlets from the \( i_{th} \) AWG claiming a connection, for example, with the first output fiber \( (i = 1, 2) \), with \( M = 2 \) two different situations are possible:

- \( q_1 \leq W/2; \)
- \( q_1 > W/2; \)

In the first case, we can first route the \( q_2 \geq W/2 \) inlets from the other AWG2 and then accommodate the lightpaths of the AWG1 which can exploit a full-wavelength conversion. In the other case, making an appropriate choice of the lightpaths, we can always direct \( q_2 < W/2 \) inlets from the first AWG to the outputs provided with TWCs and then route the remaining lightpaths from both the AWGs through the two links connected to the first output fiber without TWCs. So blocking does not occur in all the possible cases.

Since there are AWGs not followed by any TWCs in the connection with certain output fibers, it is immediate to realize that the OSSC-II structure is not RNB for the wavelength-based model. In fact an arbitrary inlet can select a particular output fiber with only one wavelength. In the same way it is possible to show that the examined structure is not SNB for the fiber-based model. Here we do not report a specific formal proof of this fact because its verification is simple and not particularly interesting.

On the other hand, it can be more useful to briefly compare the OSSC-I and the OSSC-II structures in order to draw some conclusions. The previous theorems have shown that they present the same blocking properties but in the case of the OSSC-II structure we can eliminate blocking only when \( M = 2 \), while for the OSSC-I design also the case \( M = 3 \) is valid. As far as the the number of used components is concerned, similar indications can be provided.

As it is evident from Figure 7 the second conversion stage is composed of \( W \) TWCs and we know that \( W \geq 4 \). On the contrary, Theorem 4.1 grants that \( k \) can be even equal to \( W/2 - 1 \). This means that for \( M = 2 \) just four TWCs can be used in the final conversion stage. On the contrary, while OSSC-II provides the opportunity of reducing the inlet to the final couplers to \( W/2 + 2 \), in OSSC-I the couplers have \( W \) input ports. This fact can turn out in a better performance from the physical layer point of view.

V. MORE POWERFUL CONSTRUCTIONS WITHOUT SHARED CONVERSION

As already discussed, the advantage of the architectures defined in the previous section can be detected mainly in the fact that they are essentially simple single-stage constructions. On the other hand, they could provide reasonably good blocking performance for network configurations that cannot prevent blocking. Nevertheless this issue has not been investigated yet.

Naturally there are some possibilities to achieve a more powerful construction that could guarantee non-blocking for arbitrary values of \( M \) and \( W \). In order to do so we start from the initial single-stage configuration and add one or more conversion stages. Unfortunately, this new approach forces us not to exploit sharing of wavelength converters.

A two-stage architecture has been proposed in [10], as that shown in Figure 8. It relies on two arrays of \( WxW \) AWGs and three stages of full-range TWCs. It has been proved that this architecture is RNB for the wavelength-based model. Effectively, this construction represents a particular case of a possible generalization of the Slepian-Duguid Theorem ([12],[13]) for three-stage networks with multiple links between switching elements in different stages.

The main advantage of this structure is that it permits to use similar components in each stage, that is to say AWG with the same number of input/output ports and TWCs with the same tuning range. This can be seen as a good feature since it allows to improve the structure modularity. On the contrary, the drawback is that it represents a deeper structure (more components must be crossed by each lightpath), so it would suffer higher noise accumulation and attenuation. Moreover it does not exploit probably the great implication of the multistage structures represented by the possibility to limit the tuning range requirement in the TWCs. Some indications about this opportunity are provided in [10] and [14].

Also a SNB (under the wavelength-based model) construction can be realized using essentially one stage of \( WxW \) AWGs. However this can be obtained only improving the internal paths of the structure. In fact the main principle that rules the functioning of this solution is based on the full
Fig. 8. A two-stage construction proposed in [10]

replication of the input lightpaths on each output port of the first AWG. This can be achieved allowing each inlet to select any AWG output port even if it has been already chosen by another lightpaths (this is what we have named the "multi-λ" environment). Figure 9 shows an example of this construction. All the AWGs at the first stage have only $M$ active output ports which can be arbitrarily chosen: for example, one can decide to use only the first $M$ outputs.

From Figure 9 it is quite straightforward to see that even all the inlets of the same AWG are allowed to reach together the same group of TWCs before the output wavelength multiplexers. In fact, each of the $M$ output links of the AWGs are connected to a demultiplexer able to separate the ingress signal into up to $W$ different lightpaths (naturally it is reasonable to suppose $W >> M$). After the demultiplexer an array of $W$ TWCs with tuning range $R = W$ makes any lightpath able to select the desired output wavelength before being multiplexed onto the requested output fiber. What we have just stated is sufficient to classify the structure as SNB in the wavelength-based model.

More in general, this structure can be exploited to further limit the tunability requirement of the TWC. As a matter of fact, within this structure the input lightpaths can be divided into groups made up of $n$ channels in order to have $W = bn$ where $b$ is a positive integer. These lightpaths will be afterwards connected, one by one, to the AWGs input ports. As these AWGs have to ensure a full connectivity, they should have at least $(WM)/n$ active output ports (this value represents the number of the demultiplexers to which each AWG is connected and also the number of wavelengths on which each TWC has to be able to convert the passing lightpaths). This means that we need AWGs with a number of ports at least equal to $\min\{n, (WM)/n\}$. As we have to guarantee the complete connectivity of the input signals towards each output, we use the interconnection links between the first and the second stage to carry even $n$ signals.

After the AWGs, we need to demultiplex each input link and afterwards to enclose, for each one of these, an array of $n$ TWCs. Here, all the possible $n$ lightpaths carried by each interconnection line can be converted on a suitable output wavelength, so that each TWC before the coupler requires a $R = n$ tuning range. Naturally, since $W = bn$ we can always configure the output TWCs in order to have the complete wavelength reference comb onto the output fiber.

Also in this more general case the architecture is SNB under the wavelength-based model. In fact each input lightpath has a private path towards each outlet of the structure. The construction shown in Figure 9 is effectively a particular case of the structure just explained in which $n = W$.

Once the blocking properties of this architecture have been proved and the conditions which could modify its construction have been explained, we can now take into consideration some details concerning the tuning range and the number of used components. We have already said that the converters tuning range is equal to $(WM)/n$ in the first stage, and to $n$ in the second one. If we set the condition $MW/n = n$ we find the value of $n$ that minimizes the tuning range of the TWCs which turns out to be equal to $\sqrt{MW}$. We obtain a value which is much smaller than the $MW$ calculated in the

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.
case of the monostage solution shown in the Figure 2.

The number of used components represents the price we have to pay for. As a matter of fact, if in the first stage \( M W \) converters are enough, in the final stage we need \( \left( \frac{W M}{n} \right)^2 \) TWCs for an overall number of \( W M (1 + \frac{W M}{n}) \) tunable converters. If we replace the \( n \) with the value obtained from the minimization of the tuning range, we obtain that this number is equal to \( MW(1 + \sqrt{MW}) \), that is to say an asymptotically \( O(MW^2) \) increasing value.

Another possible shortcoming of this architecture is represented by the high number of input ports in the final coupler which could turn out in poor physical layer performances, but these aspects have not been investigated yet.

VI. Comparison among the considered constructions

Some considerations about the considered structures have already been introduced throughout the paper. In particular our shared conversion switches have been compared each other and a quite precise conclusion has been reported. Also the more complex architectures of Section V have been analysed in order to catch advantages and drawbacks of the particular solution. Here a more complete comparison among architectures is carried out relying on the data reported in Table I.

The comparison can be subdivided into different parts considering that many of the designs considered here are, in general, blocking for both the connection request models, while the other, apart from the two-stage structure of Figure 8, are even SNB in the most restrictive request model.

Since the different solutions in each category have almost the same blocking property, the comparison can be realized mainly focusing on the number of used component or on their features. Taking into account the architectures that are blocking, we also consider our shared conversion switches. In this case the comparison can be made counting the number of ports used in the AWG because the other characteristics are almost the same in the different solutions. From this point of view, the more convenient construction is the Blocking Solution-II proposed in [11], as it uses only a \( W \times W \) AWG instead of either multiple AWGs (as in the case of the design studied in the previous sections) or an \( M W \times M W \) AWG (as for the case of the Blocking Solution-I present in the same paper). Apart from this simple consideration, further study about the real blocking behaviour of these structures has to be carried out in order to have a thorough comparison.

For what concerns the number of used components in the non-blocking solutions, it seems quite clear that, also in this case, the solutions proposed in [11] are the most convenient. In fact, mainly considering the non-blocking version of the Solution-II where more than one inlet is connected to the same AWG input port, we can appreciate the high exploitation of the internal paths available in the AWG. While our OSSC solutions share some TWCs, in this latter solution a single AWG with an acceptable number of input/output ports is responsible for the routing of even all the OXC input lightpaths. So, in other words, in this type of architecture the shared resources are represented by the input ports of the AWG which result to be much less with respect to all the other solutions. Moreover the “Multiport” configuration permits to replace the final TWCs with simple FWCs.
expedient gives furthermore the possibility of limiting the overall structure costs. This is probably the most important feature of this solution.

The main drawback of the solution proposed in [11] is represented by the tunability requirement in the TWCs. Our single stage architecture can be considered a better solution if we take into account this aspect. In fact even in the version proposed in Figure 9, the TWCs tuning range results to be equal to $M$ (instead of $R = MW$ of the other SNB solutions). In this particular situation it has been also used the same number of TWCs exploited in the “Single output port operation” structure. Furthermore it is possible to use an arbitrary value of $n$ and so reduce the TWC tuning range below $W$ (note that this is the only solution that allows such a feature). Unfortunately, choosing a small value for $n$, we increase directly the number of used components so reaching a number clearly much larger than the one achievable with other competing solutions. Another shortcoming (not put in evidence in Table I) of this last architecture is represented by the different set of wavelengths each TWC (interfaced with the same AWG) has to convert to. In fact, for each incoming arbitary value of $n$ and so reduce the TWC tuning range below $W$ (note that this is the only solution that allows such a feature). Unfortunately, choosing a small value for $n$, we increase directly the number of used components so reaching a number clearly much larger than the one achievable with other competing solutions. Another shortcoming (not put in evidence in Table I) of this last architecture is represented by the different set of wavelengths each TWC (interfaced with the same AWG) has to convert to. In fact, for each incoming lightpath we can exploit exactly $WM/n$ wavelengths to reach the $WM/n$ output ports connected with the final couplers. Since each channel enters the AWG from different input ports, we are obliged to use different TWCs.

VII. CONCLUSIONS

We have evaluated the possibilities of realizing an OXC using a single-stage solution. Mainly two different approaches have been followed. Initially we have focused our attention on the issues of a precise known construction. Starting from this design, we have defined two alternative solutions that exploit shared wavelength conversion. We have proved their blocking properties showing that only for particular cases they can be non-blocking. However we think that this solution brings interesting concepts which can be further analysed. A different approach has been then considered. In particular, adding a further full conversion stage at the end we have obtained a SNB solution which can also minimize the tunability requirement of the used tunable converter. Unfortunately, the main drawback of this construction is represented by the number of used components. Further study is needed to investigate the probabilistic blocking performance of single-stage solutions and the feasibility of alternative multi-stage architectures.

<table>
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<th>CONSTRUCTIONS</th>
<th>FBM</th>
<th>RB</th>
<th>SNB</th>
<th>WBM</th>
<th>SNB</th>
<th>#TWCs</th>
<th>USED COMPONENTS</th>
<th>Max. R</th>
<th>#FWCs</th>
<th>#WGs</th>
<th>AWG Dim.</th>
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<td>-</td>
<td>-</td>
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<td>-</td>
<td>MWxMWxW</td>
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<tr>
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<td>-</td>
<td>-</td>
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<td>W</td>
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<td>MW</td>
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<td>MWxMWxW</td>
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<td>X</td>
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<td>-</td>
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<td>WxW</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>MW + W - Mk</td>
<td>W</td>
<td>-</td>
<td>M</td>
<td>WXW</td>
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<td>-</td>
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<td>-</td>
<td>3W</td>
<td>W</td>
<td>-</td>
<td>M</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>MW + M$^2W$</td>
<td>W</td>
<td>-</td>
<td>M</td>
<td>WxW</td>
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References