Dynamics and Control of Game Interactions on Complex Networks

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The talk presents an extension of the mathematical formulation of evolutionary game dynamics to networked populations. The model, grounded on the standard replicator equation, has been modified in order to account for the dynamics of a finite set of players organized in a network of connections (graph). In the proposed approach the players are located at the vertices of the graph and are modeled as subpopulations of a multipopulation game. This framework allowed us to obtain a system of equations describing the strategic interactions of a finite set of individuals organized in a network, without any assumptions on game payoff matrices and on adjacency matrix of the graph. The availability of equations of evolutionary games on networks provided the mathematical basis to analyze the system with 2 strategies and N players. In particular, stability of steady states, existence of Nash equilibria and evolutionary stable strategies are discussed. The main result of the work was to prove that the stability of equilibria in the linearized equations depend on the eigenvalues of graph’s adjacency matrix. This fact allowed us to analyze and control the system’s behavior at both global and individual levels. The obtained results are also confirmed by means of extended simulations. Finally, the model has been used to explain the mechanisms of bacterial aggregation leading to the formation of biofilms.
Motion control of Vicsek’s agents in an inhomogeneous space

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1 Abstract

In this work, we study a system of mobile agents moving on a planar space and interacting according to a modified version of the Vicsek’s model, which take into account the inhomogeneity of space. In particular, we consider the space divided in two regions: one where the agents’ motion is governed by the traditional Vicsek’s model; the other, called control region, where a control law, consisting of a preferential direction blended with the Vicsek’s rules, is imposed. The mix between the social criterion (that is, the average direction coming from the Vicsek model) and the preferential direction is governed through a single control parameter.

We show that, under particular conditions, by acting only on the parameters of the control, i.e. the magnitude of the control action and the linear size of the control region, a global control of the system can be attained, that is, agents will move along the preferential direction even outside the control region. In particular, it is revealed that in presence of a small control region, first agents self-organize towards a spontaneously emerging direction (as a result of the Vicsek’s model interactions) and, subsequently, when the information on the preferential direction is spread through the agents, they tend to align along the desired direction of motion.

Furthermore, the effect of noise on the onset of the ordered motion along the preferential direction is also investigated. When a strong control law is applied the system shows a coordinated behavior, while, when one of the two conditions (large area, high value of control action magnitude) does not hold, then global control is not attained.

Finally, a counter-intuitive behavior can be observed for high levels of noise, when a strong control action favor the depletion of the control region, and a new behavior emerges where all agents tend to avoid the control region and occupy for most of the time the remaining part of the space. This emergent behavior is due to the fact that, under these conditions, agents are subjected to two strong and opposite forces: the exogenous control action, and the tendency towards a disordered motion, due to the high noise level.
Bayesian estimation of simple models for networked oscillators based on experimental data

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We propose a statistical method to infer a simple mathematical model for a network of oscillators, called *phase model*, directly from experimentally observed rhythmic data [1]. In various real-world complex networks, each node is not static, but a dynamical element that evolves with time [2]. A well-known type of such dynamical systems is a network of interacting phase oscillators; for example, power grids composed of power generators and substations have been studied within this framework [3]. The quantitative description of the network dynamics is then essential to analyzing and controlling such systems. Using observed time-series data, our proposed method yields a phase model of the form

\[
\frac{d\phi_i}{dt} = \omega_i + \sum_{j \neq i}^{N} \Gamma_{ij} (\phi_j - \phi_i),
\]

where \(\omega_i\) represents the natural frequency of \(i\)th oscillator and \(\Gamma_{ij}\) is the coupling function from \(j\)th oscillator to \(i\)th oscillator. Here, the state of each node \(i\) is characterized by the phase of oscillation cycles \(\phi_i\). The estimated phase model can reproduce the temporal behavior and synchronization properties in the original network, and can then be used for quantitative analysis and control of the network. Theoretically, a phase model can be derived as a reduced model of a detailed high-dimensional model. Thus, obviously, the phase model is accurate only if the detailed model correctly captures the essential dynamical features of the target system. In many practical situations, however, constructing a quantitatively correct detailed model is quite difficult, because we can generally observe only a limited amount of data. As an alternative approach, our method enables us to obtain a phase model even from limited data, without a need of constructing a detailed model. This advantage makes our method useful in data-driven studies of interacting rhythmic phenomena on various types of complex networks. We have confirmed in numerical experiments that the estimated model equations are consistent with the theoretical ones. If possible, we would like to present some applications to real-world biological data.

References


Speed-gradient control of cluster synchronization by adaptation of network topology

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Adaptive networks are characterized by mutual interactions between dynamics of the nodes and co-evolution of the coupling topology: The topology evolves according to the state of the system, while at the same time the dynamics on the network is influenced by that changing topology. Here, we present an adaptive control scheme for the control of zero-lag and cluster synchronization in delay-coupled networks of Stuart-Landau oscillators [1]

\[ \dot{z}_j(t) = \left( \lambda + i\omega - |z_j|^2 \right) z_j + K \sum_{n=1}^{N} G_{jn}(t)[z_n(t-\tau) - z_j(t)] \] (1)

with the complex variables \( z_j = r_j e^{i\phi_j} \in \mathbb{C}, j = 1, \ldots, N \). It arises in a center manifold expansion close to a Hopf bifurcation with \( \lambda \) as the bifurcation parameter. Based on the speed gradient method [2], our scheme adapts the topology of a network such that the target state is realized. This is achieved by the introduction of an appropriate goal function \( Q \) as follows:

\[ \frac{du}{dt} = -\Gamma \nabla_u \dot{Q}(z, u, t), \] (2)

where \( Q(z, u, t) \) becomes minimum if the desired target state is reached. This procedure yields additional equations for the elements of the coupling matrix \( \{ G_{jn} \} \), which are used as input signals \( u(t) \). In particular, we control zero-lag and cluster synchronization (See automated realization of a network exhibiting an 8-cluster state in Fig. 1).

![FIG. 1: Evolution of the network topology with the goal to achieve an 8-cluster state. Black: positive weighted links; gray (blue) negative weighted links. Node colors denote phase differences with respect to the first node: \( \Delta \phi = \phi_j - \phi_1 \). Parameters: \( \lambda = 0.1, \omega = 1, K = 0.1, \tau = \pi, \Gamma = 10, N = 40. \)]

The emerging topology of the network is modulated by the delay. If the delay time is a multiple of the system’s eigenperiod, the coupling within a cluster and to neighboring clusters is on average positive (excitatory), while the coupling to clusters with a phase lag close to \( \pi \) is negative (inhibitory). For delay times equal to odd multiples of half of the eigenperiod, we find the opposite: Nodes within one cluster and of neighboring clusters are coupled by inhibitory links, while the coupling to clusters distant in phase state is excitatory.

References:
Local and global dynamics of complex systems: when does the network structure matter? *

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Abstract— Complex systems, or networked systems, as found in statistical physics, biology, chemistry, social sciences, power networks and control theory, among others, typically arise from the composition of many small systems or agents. A fundamental question is to characterize the dynamics of the global complex system from two fundamental ingredients: the interconnection network, and the individual agents’ dynamics. In this talk we review some related results and focus on random walks, epidemics and other stochastic diffusion processes on networks. We find a criterion to separate those networks where the individual dynamics dictate the global dynamics, and those networks where the network does. We deduce the implications for the decomposition of natural complex systems into dynamically coherent mesoscale modules, intimately related to timescale separations in the global dynamics. Various illustrations on real-life data will be given.

Many interesting complex systems arise as interconnection of linear systems, described by the generic evolution equation

\[ D x(t) = L x(t), \]

where \( x \) is a vector compiling the states \( x_i(t) \) of agent sitting at node \( i \), where \( D \) is a differential-like operator and \( L \) is a matrix describing the interconnection structure and intensity between the agents. An example is consensus between people who converge to a same opinion while discussing with each other (opinion dynamics), or robots that converge to a same position in space by moving towards a fixed set of other fellow robots (consensus problem)[FM04], [OSFM07], described by

\[ \tau \frac{d}{dt} x(t) = L x(t), \]

where \( L \) is the Laplacian of the interconnection graph and \( \tau \) a time constant. The same equation also describes the evolution of the probability of presence of a continuous-time random walker on the network, ie a random walker whose jumping times follow a Poisson process.

In this talk we focus on more general diffusion processes, in particular on random walkers that, once arrived on a node, wait for a time between \( t \) and \( t + \Delta t \) to jump to the next node, with probability \( \rho(t) \Delta t \). The waiting time distribution \( \rho(t) \) can be arbitrary, leading to a relatively complicated form for \( D \), that is best expressed in the Laplace domain.

We find that the modes of the diffusion dynamics of the walkers decay with a characteristic time that are determined, either by the eigenvalues of \( L \), or by \( \rho(t) \). In particular, we prove that the slowest characteristic time, called the mixing time of the random walker as it indicates the convergence time to stationary distribution, is the maximum of three quantities, the first of which is related to the spectral gap, ie the presence of bottlenecks in the network, the second is related to the variance (‘burstiness’) of the waiting times, and the third is related to the fat tail (if any) of the waiting time distribution. Bottlenecks, variance, and fat tails, therefore appear as three separate obstructions in competition, the most obstructing of which determines the mixing time[DLR13].

We find real-life dynamic social networks whose mixing is governed by the interconnection structure, and others where the temporal patterns of the links dominate and regulate the mixing time. In the former case, we prove and exemplify that the network can be decomposed into clusters, or communities, which are governed by temporal patterns, thus for which the internal structural details can be forgotten, as far as the description of the diffusion dynamics matters. This can be generalized to the description of other diffusion processes such as epidemics, and has important consequences of simplified description of such models.

References


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EVENT-TRIGGERED PINNING CONTROL OF SWITCHING NETWORKS

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Pinning control is one of the possible inflections of the problem of multi-agent coordination. Multi-agent systems have attracted a large amount of research in the past few decades [1], [2], since they provide a suitable model for a wide spectrum of distributed phenomena spanning the fields of biology, social sciences, physics, economics and engineering [3].

In the pinning control problem it is required that a set of interconnected dynamical systems synchronize onto a certain reference trajectory [4]. A small fraction of the agents in the network, called pins or pinned agents, can be selected to receive a direct feedback from the reference. As opposed to more traditional consensus problems in which the possible synchronization solution is not known a priori, in the pinning control problem the reference trajectory is supposed to be a solution of the uncoupled agents’ dynamics, a priori known, and corresponding to a control objective.

In many scenarios of multi-agent coordination, the assumption of the topology of the interactions among the agents being constant over time turns out to be unrealistic because of possible failures in the communication channel between two or more agents [5]. Therefore, a pinning control algorithm which is intended to be robust against such failures should be designed by modeling the controlled network as a switched system.

Pinning control algorithms have been traditionally designed in a continuous-time fashion. However, continuous-time distributed control laws are typically not possible to be implemented for large-scale networks, not only because in several scenarios the systems are supposed to communicate over a wireless medium, which represents a shared resource with limited capacity, but also because of the limited switching frequency of the actuators. Event-triggered approaches have been successfully developed in order to cope with such matter for network synchronization [6], but not for pinning control problems.

The main contribution of this work is to address the problem of multi-agent coordination with a network model that includes at the same time nonlinear dynamics of the individual agents, pinning control, event-triggered signals and time-varying topologies. We design a model-based, distributed and event-triggered pinning control law which drives the states of all the systems onto a priori specified common reference trajectory. We derive a set of sufficient conditions under which accumulation points in the sequences of events are excluded - that is, absence of Zeno behavior is guaranteed [7] - and the agents achieve asymptotical convergence onto the reference trajectory.

References:

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