1. Let us consider the autonomous Lur’ء system in Figure 1

where \( \varphi(\cdot) \) is a sector nonlinearity and \( G(s) \) is the transfer function of a reachable and observable linear system.

1.1 Define the notion of absolute stability of the autonomous Lur’ء system in sector \([0,k]\), where \( k>0 \).

1.2 Write the statements of Popov criterion and the circle criterion for the absolute stability of the autonomous Lur’ء system in sector \([0,k]\), where \( k>0 \), with a graphical interpretation of both criteria. What is the relation between these criteria?

1.3 Discuss differences and similarities in terms of necessary and sufficient conditions for the absolute stability with the case when the sector nonlinearity in \([0, k]\) is time-varying, i.e., \( \varphi \) is a function of time \( \varphi(\cdot,t) \).

2. Consider the following feedback scheme
where

\[ G(s) = \frac{100}{(0.005s + 1)(0.5s + 1)(1 + 0.0005s)} \]

is the transfer function of an observable and reachable linear system and the nonlinear component is the relay with deadband reported in the figure below.

Set \( y^*(t) = 0, \; t \geq 0. \)

2.1 Determine how parameter \( a \) should be set so that the describing function method predicts a limit cycle.

2.2 Fix a value for parameter \( a \) such that the describing function method predicts a limit cycle and analyse its stability properties.

To this purpose, please recall that the sinusoidal-input describing function of the considered relay with deadband is

\[ D(E) = \begin{cases} 
0, & E < a \\
\frac{4}{\pi E} \sqrt{1 - \frac{a^2}{E^2}}, & E \geq a 
\end{cases} \]

3. Consider the following Lur’e system.
where

i) \( \varphi(\cdot) \) is the saturation function in the plot below

\[ u \]
\[ 1 \]
\[ 0 \]
\[ e \]

ii) \( F(s) \) is the transfer function of a SISO system of order 4 and is given by

\[
F(s) = \frac{k(1 + 100s)}{(s + 1)(1 + 0.01s)^2(1 + 0.0001s)}
\]

3.1 define the notion of \( L_2 \)-stability for the operator \( H \) with input \( u \) and output \( y \);
3.2 by using the small gain theorem, determine the values of \( k > 0 \) such that the operator \( H \) with input \( u \) and output \( y \) is \( L_2 \)-stable with finite gain;
3.3 by using the circle criterion, determine the values of \( k > 0 \) such that the operator \( H \) with input \( u \) and output \( y \) is \( L_2 \)-stable with finite gain. Compare the obtained value with the previous one and motivate why one is more conservative than the other.

4. Describe in a clear and concise way the issue of high frequency oscillations of the control input in a variable structure controller and suggest a possible solution to such an issue.

5. Consider a second order nonlinear single input system

\[
\dot{x} = f(x) + g(x)u
\]

with
\[ f(x) = \begin{pmatrix} x_1^2 - x_2 \\ -2x_1 \end{pmatrix}, \quad g = \begin{pmatrix} x_1^2 + 1 \\ 0 \end{pmatrix} \]  

5.1 State the necessary and sufficient condition for the system to be fully linearizable by a static state feedback control law at the equilibrium \( x^* = 0 \) associated with zero input \( u \).

5.2 Verify that the above condition is satisfied.

5.3 Construct a (local) state feedback linearization in \( x^* = 0 \) that sets all poles of the closed loop system equal to -20.