1. Let us consider the autonomous Lur’e system in Figure 1

where \( \phi(\cdot) \) is a sector nonlinearity and \( G(s) \) is the transfer function of a reachable and observable linear system.

Provide clear and precise answers to the following requests:

1.1 Define the notion of absolute stability of the autonomous Lur’e system in sector \([0,k]\), where \( k>0 \)

1.2 Write the statements of Popov criterion and the circle criterion for the absolute stability of the autonomous Lur’e system in sector \([0,k]\), where \( k>0 \), with a graphical interpretation of both criteria. Is there any relation between these criteria?

2. Consider the Lur’e system in Figure 2

where
\[ G(s) = \frac{10}{(1 + 10s)^2(1 + 0.1s)} \]

is the transfer function of a reachable and observable system, and block N is the MB/2 relay with hysteresis in Figure 3.

Set \( B/2 = 1 \), and determine the values for \( M > 0 \) such that the describing function method predicts a permanent oscillation. Evaluate the stability properties of such an oscillation. To this purpose recall that the describing function of the MB/2 relay in Figure 3 is given by

\[ D(E) = \frac{2M}{\pi E^2} (\sqrt{4E^2 - B^2} - jB), \quad E \geq B/2 \]

3. Consider the Lur'e system in Figure 4.

where

i) \( \varphi(\cdot) \) satisfies \( 0 \leq \varphi(y) \leq 3y, \quad \forall y \in \mathbb{R} \)

ii) \( G(s) \) is the transfer function of a SISO system of order 2 with gain \( \mu > 0 \)
\[ G(s) = \frac{\mu (1 - s)}{(1 + s)(1 + 0.01s)} \]

1. define the notion of \( L_2 \) stability for the causal operator \( H \) with input \( u \) and output \( y \);
2. determine the values for \( \mu > 0 \) such that the operator \( H \) with input \( u \) and output \( y \) is \( L_2 \)-stable with finite gain by using
   a. the small gain theorem
   b. the circle criterion
3. provide an estimate of the gain of \( H \) by the small gain theorem.

4. Given the dynamical system
   \[
   S : \begin{cases} 
   \dot{x}(t) = f(x(t), u(t)), & x(0) = x_0 \in \mathbb{R}^n \\
   y(t) = g(x(t), u(t)) 
   \end{cases}
   \]
   \[
   f(0, 0) = 0, \quad g(0, 0) = 0
   \]
   Define the notion of passivity and strict passivity.
   Let \( A \) be a square matrix of size \( n \) and \( B \) a column vector with \( n \) elements. Suppose that there exists a symmetric positive definite matrix \( P \) that satisfies
   \[
   A^T P + PA = -I
   \]
   Analyze the passivity and strict passivity properties of the linear dynamical system with transfer function.

5. Describe in a clear and concise way the issue of high frequency oscillations of the control input in a variable structure controller and suggest a possible solution to such an issue.

6. Consider the regular nonlinear SISO system \( S \)
   \[
   S : \begin{cases} 
   \dot{x} = a(x) + b(x)u \\
   y = c(x) 
   \end{cases}
   \]
   1. Define the notion of relative degree of \( S \) in a state \( x^* \) and the role that it plays in the (local) state feedback linearization of \( S \) around \( x^* \).
   2. Explain what are the conditions to obtain a full state feedback linearization.