1. Let us consider the control system in Figure 1

![Control System Diagram](image)

where the nonlinear block is the saturation function plotted below, next to its sinusoidal input describing function

![Saturation Function Graph](image)

and the transfer function of the system and controller are given by

\[ R(s) = \frac{1}{1 + sT} \]

\[ G(s) = \frac{20}{(1 + 0.1s)^3} \]

1.1 Describe in a precise yet concise way the describing function method for predicting periodic oscillations.
1.2 Determine if there a value for the parameters $T$ and $k$ such that the describing function method predicts a stable limit cycle with $v(t) = 2 \cos(10t)$.

2. Let us consider the variable structure control strategy for the output regulation of a linear dynamical system. Explain clearly the issue of the high frequency input switching and a possible solution to such an issue.

3. Given the nonlinear regular SISO system governed by the

$$S_o : \quad \dot{x} = a(x) + b(x)u$$

with $a(0) = 0$, $b(0) \neq 0$, and fully measurable state:

3.1 Provide a necessary and sufficient condition for the system to be fully linearizable in $x^o=0$ via static state feedback.

3.2 Check that such a condition is satisfied for the second order system with

$$a(x) = \begin{bmatrix} x_2^3 - x_1 \\ -x_1 \end{bmatrix} \quad b(x) = \begin{bmatrix} x_1^2 + 1 \\ 0 \end{bmatrix}$$

(suggestion: try with $y=x_1$ and $y=x_2$ ) and derive the static state feedback control law that linearizes the system and assigns the poles to be $\{-1, -10\}$.

4. Let us consider the autonomous Lur’e system in Figure 2

![Figure 2: autonomous Lur’e system](image)

where $\varphi(\cdot)$ is a sector nonlinearity and $G(s)$ is the transfer function of a reachable and observable linear system.

4.1 Define the notion of absolute stability of the autonomous Lur’e system in a sector.
4.2 Write the statement of Popov criterion and the circle criterion for the absolute stability of the autonomous Lur’e system in a sector providing a graphical interpretation of both criteria.

4.3 Describe briefly how the circle criterion can be derived from Popov criterion.

5. With reference to a causal operator $H : \mathcal{L}_c \rightarrow \mathcal{L}_c$

5.1 define the notions of boundedness, and weakly boundedness, zero-bias gain and gain, and describe the connection between notions and gains.

5.2 define when an operator is affine and what are the key characteristics of an affine operator.