1. Consider the autonomous Lur’e system in Figure 1.

\[
\begin{array}{c}
\circ \quad e \\
\downarrow \\
\varphi(\cdot) \\
\downarrow \\
u \\
\downarrow \\
G(s) \\
\downarrow \\
y
\end{array}
\]

Figure 1

where \( \varphi(\cdot) \) is a sector nonlinearity in \([0, k]\) with \( k > 0 \), whereas \( G(s) \) is the transfer function of a SISO reachable and observable linear system.

1.1. Define the notion of absolute stability of the autonomous Lur’e in the sector \([0, k]\).

1.2. Explain Popov criterion and provide its graphical interpretation by describing how the Popov diagram is plotted.

1.3. Suppose that \( \varphi(\cdot) \) is the saturation function in Figure 2.

\[
\begin{array}{c}
\alpha \\
\downarrow \\
1
\end{array}
\]

with \( \alpha \) taking values in \([0.01, 0.1]\) and that

\[
G(s) = \frac{1 + s}{(1 + 0.1s)^2(1 + 0.01s)}
\]

is the transfer function of a linear system of order 3. Determine if the equilibrium \( x = 0 \) of the system in Figure 1 is globally asymptotically stable.
2. Consider the Lur’e system in Figure 3

![Lur'e system diagram](image)

where

\[ G(s) = \frac{10}{(1 + 10s)^2(1 + 0.1s)} \]

is the transfer function of a linear system of order 3 and the nonlinear part N is an MB/2 relay with hysteresis.

2.1 Define the notion of sinusoidal input describing function \( D(E) \) for the MB/2 relay with hysteresis.

2.2 Set \( B/2 = 1 \) and determine the values of \( M > 0 \) such that the describing function method predicts a stable limit cycle for the considered Lur’e system. To this purpose recall that

\[
D(E) = \frac{2M}{\pi E^2} (\sqrt{4E^2 - B^2} - jB), \quad E \geq B/2
\]

3. Consider the dynamic system S

\[
S : \begin{cases}
\dot{x}(t) = f(x(t), u(t)), & x(0) = x_0 \in \mathbb{R}^n \\
y(t) = g(x(t), u(t))
\end{cases}
\]

\[
f(0, 0) = 0, \quad g(0, 0) = 0
\]
3.1 state the results that relates its passivity properties with the stability of the equilibrium $x=0$ associated to the zero input

3.2 determine the stability properties of the equilibrium $x=0$ associated to the zero input when $S$ is given by

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= -x_1^4 - x_2 + u \\
y &= x_2
\end{align*}
\]

by using $V(x_1, x_2) = \frac{x_1^5}{5} + \frac{x_2^2}{2}$ as a candidate storage function

4. With reference to variable structure control for the output regulation of a linear system:

4.1 draw the variable structure control scheme and describe the role of the various components

4.2 discuss the robustness properties of the control scheme with respect to a bounded disturbance that acts additively on the system input.

5. Given the nonlinear regular SISO system describe by

\[
S : \begin{cases}
\dot{x} = a(x) + b(x)u \\
y = c(x)
\end{cases}
\]

5.1 define the notion of relative degree of $S$ in a state $x^*$

5.2 explain how to construct a (local) state feedback linearization with pole assignment.