1. Consider the Lur’ë time-varying system drawn in the following figure

where \( \varphi(\cdot, t) \) is a time-varying sector nonlinearity in \([k_1, k_2]\), whereas \( G(s) \) is the transfer function of a SISO reachable and observable linear system.

1.1. Define the notion of absolute stability for such a closed loop system.

1.2. State necessary and/or sufficient conditions for the absolute stability of the above system, pointing out the possible differences with the case of a Lur’ë system with time-invariant sector nonlinearity.

1.3. Discuss possible connections between absolute stability analysis of a Lur’ë time-varying system and switched linear systems stability analysis.

2. Let us consider the nonlinear system in the following figure

where \( \varphi(\cdot) \) is a sector nonlinearity in \([-k, 4k]\), with \( k > 0 \), and
\[ G(s) = \frac{2}{(1 + s)^2(1 + 0.001s)} \]
is the transfer function of a linear system of order \( n=3 \).

2.1 Define the \( L_2 \) stability notion for the operator \( H \) with input \( y^o \) and output \( y \).

2.2 State the small gain theorem and the circle criterion for the \( L_2 \) stability of the operator \( H \) with input \( y^o \) and output \( y \).

2.3 Determine the maximum value for \( k>0 \) such that the operator \( H \) with input \( y^o \) and output \( y \) is \( L_2 \) stable via (a) the small gain theorem and (b) the circle criterion for the \( L_2 \) stability of \( H \), specifying in a clear a precise way the adopted procedure and motivating the obtained result.

3. Consider the linear system \( S \) described by

\[
\begin{align*}
\dot{x}_1 &= -x_2 + u \\
\dot{x}_2 &= 2x_1
\end{align*}
\]

Set \( s(x) = 2x_1 + x_2 - 2 \)

(a) show that system \( S \) converges to a uniquely defined (pseudo-)equilibrium \( \bar{x} \) when constrained to evolve on the surface \( s(x) = 0 \), starting from an arbitrary point on that surface. Determine \( \bar{x} \).

(b) design a state-feedback variable structure controller that makes the system reach the sliding surface \( s(x) = 0 \) within time \( t=4 \) when \( x(0) = [1 \ 10]' \).

(c) draw the block diagram of the designed closed-loop control scheme.

4. Discuss in a precise yet concise way the describing function method: framework, goal, methodology.

5. Consider a fourth order nonlinear single input system

\[ \dot{x} = f(x) + g(x)u \]

with
\[ f(x) = \begin{pmatrix} x_2 \\ -\sin x_1 - 2(x_1 - x_3) \\ x_4 \\ x_1 - x_3 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ x_1 + 1 \end{pmatrix} \]

5.1 Determine an output transformation \( y = h(x) \) such that the relative degree of the system in the state \( x^* = 0 \) is \( r = 4 \).

5.2 Construct a (local) state feedback linearization in \( x^* = 0 \) that set all poles of the closed loop system equal to -10.