Wireless Internet Exercises

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1 WLAN

1.1 Exercise 1

A Wi-Fi network has the following features:

- Physical layer transmission rate: 54 Mbps
- MAC layer header: 28 bytes
- MAC layer payload: 1452 bytes
- ACK and CTS frames: 14 bytes
- RTS frames: 20 bytes
- Propagation delay $\tau = 1\mu$s
- Slot time: 9μs
- SIFS: 16μs
- DIFS: 34μs
- Physical layer overhead OH: 20μs

Define MAC layer throughput as the number of bits sent by the MAC layer in a given period of time.

1. Assuming that there are two stations exchanging data using 802.11 DCF but without using RTS/CTS transaction, what is MAC layer throughput?
2. What is the throughput when RTS/CTS transaction is used?

Answer

1. The amount of data transmitted in 1 frame is 1452 + 28 bytes = 1480 bytes. The time required for the transmission of such data $T = (1480 * 8bit)/(54 * 10^6 bit/s) = 219.26\mu$s. For transmitting one frame, the total time required in DCF is: DIFS + T + τ + OH + SIFS + T_{ACK} + τ + OH. Therefore the total time is: $34 + 219.26 + 1 + 20 + 16 + (14 * 8bit/54 * 10^6 bit)$+1+20 = 313.3 μs. The MAC layer throughput is therefore 1480*8/313.3 = 37.8 Mbps
2. In this case we have additional time due to the RTS/CTS data exchange. We have $T_{RTS} = 20*8/54*10^6$ $= 2.96\mu$s and $T_{CTS} = T_{ACK} = (14 * 8bit/54 * 10^6 bit) = 2.07\mu$s. The total time is therefore: DIFS + T_{RTS} + tau + OH + SIFS + T_{CTS} + tau + OH + SIFS + T + tau + OH + SIFS + T_{ACK} + τ + OH = 410μs. Therefore the MAC layer throughput is 28.33Mbps.

1.2 Exercise 2

In the same Wi-Fi network of Exercise 1, the RTS/CTS threshold is set at 1000 bytes. The overall distribution of the frame size at layer 2 X is the following:

$$P(X) = \begin{cases} 0.2 \ X \leq 500B \\ 0.4 \ 500B < X \leq 850B \\ 0.3 \ 850B < X \leq 1450B \\ 0.1 \ X > 1450B \end{cases} \tag{1}$$

Compute the average MAC layer throughput of a station transmitting such frames, under the assumption that the channel is always sensed free.

Answer

We know that the MAC layer throughput is 37.8 Mbps if no RTS/CTS is used and 28.33 Mbps if RTS/CTS is used. We need to compute the percentage of time RTS/CTS is used or not. We know that 30% of frames are uniformly distributed in the 600B interval between 850B and 1450B. Only for frames below 1000B (25% of the interval or 7.5% of the total frames), the RTS/CTS will be active. Therefore, RTS/CTS will be not active for 0.2+0.4+0.075 = 67.5% of the time and it will be active for 0.225+0.1 = 32.5% of time. The average MAC layer throughput will be: 37.8*0.675 + 28.33+0.225 = 31.88 Mbps

1.3 Exercise 3

Consider a Wi-Fi AP (A) located in position (0,0) and two transmitters (B and C) located in position (d, d) and $(-d, -d)$, respectively. Assuming a communication range of 100 meters, for which values of d the hidden terminal problem exists?

Answer

There is a hidden terminal problem when B and C are in range of the Access Point but not in each other's range. In details:

$$\begin{cases} dist(A, B) < R \\ dist(A, C) < R \\ dist(B, C) > R \end{cases} \tag{2}$$

that is

$$\begin{cases} d\sqrt{2} < R \\ d\sqrt{2} < R \\ d2\sqrt{2} > R \end{cases} \tag{3}$$

substituting $R = 100$ leads to $35.35 < d < 70.71$

1.4 Exercise 4

Two stations A and B wants to transmit a frame using Wi-Fi DCF at $t = 0$. For A, this is the first transmission. For B, this is the second retransmission of the frame.

- What is the probability that B transmits before A?
- What is the probability of colliding after sensing the carrier free?

Answer

- For A, since this is the first transmission, $CW_A = CW_{min} = 31$. For B, since this is the second retransmission, $CW_B = 2 * (2 * (CW_min + 1) - 1) + 1) - 1 = 127$. Therefore A will choose uniformly between (0,31) while B between (0,127). We need to compute P(B<A). It is easy to show that this probability is equal to 1/8 by drawing on a 2D graph the two ranges and identifying the area of valid solutions.
- They collide if A = B. The probability that this happens is 64/(128*64) = 1/128 = 0.78%

2 Multiple Access

2.1 Exercise 1

Consider a polling system with exhaustive service with the following characteristics:

- Transmission time $T = 3$ms
- Token passing time $h = 0.8$ ms
- Traffic load $\rho = 0.7$
- Number of stations = 10

Compute

1. Average waiting time
2. Average cycle duration
3. Average number of packets in the local queue
4. Average number of packets transmitted per cycle

Answer

We can start computing the waiting time using the formula derived during lectures:

$$E[W] = \frac{\rho}{2(1-\rho)} + \frac{M-\rho}{2(1-\rho)} = 15.9ms \tag{4}$$

The average cycle duration can be computed as

$$E[C] = \frac{Mh}{1-\rho} = 26.67ms \tag{5}$$

The number of packets in the local queue is:

$$E[N_c] = \frac{\lambda E[W]}{M} = \frac{\rho E[W]}{TM} = 0.371 \tag{6}$$

And finally the average number transmitted per cycle is:

$$Q_c = \frac{E[C] - Mh}{T} = 6.22 \tag{7}$$

2.2 Exercise 2

Compute:

1. Average waiting time
2. Average cycle duration
3. Average number of packets in the local queue
4. Average number of packets transmitted per cycle

in case the system of Exercise 1 is modified using gated polling.

Answer

$$E[W] = \frac{\rho}{2(1-\rho)} + \frac{M+\rho}{2(1-\rho)} = 19ms \tag{8}$$

The average cycle duration can be computed as

$$E[C] = \frac{Mh}{1-\rho} = 26.67ms \tag{9}$$

The number of packets in the local queue is:

$$E[N_c] = \frac{\lambda E[W]}{M} = \frac{\rho E[W]}{TM} = 0.44 \tag{10}$$

And finally the average number transmitted per cycle is:

$$Q_c = \frac{E[C] - Mh}{T} = 6.22 \tag{11}$$

2.3 Exercise 3

7 stations using slotted Aloha collide at slot 0. Describe the evolution of the collision resolution algorithm, slot by slot, in the case of 7 stations that collide together in slot 0, assuming that no other station transmit during collision resolution and that random number generators of different stations provides the following numbers at each function call:

Table 1: CRA

	1	2	3	4	5	6	7
A	0	0	1	0	1	0	1
C	1	0	1	0	1	0	1
D	0	0	1	1	1	0	0
E	1	0	0	1	1	0	1
F	1	0	1	0	0	1	0
G	1	1	0	1	1	0	1
H	0	1	1	0	0	1	1

Answer

Assuming that 0 transmits and 1 waits:

Slot 1: ADH
Slot 2: AD
Slot 3: AD
Slot 4: A
Slot 5: D
Slot 6: H
Slot 7: CEFG
Slot 8: CEF
Slot 9: E
Slot 10: CF
Slot 11: CF
Slot 12: F
Slot 13: C
Slot 14: G

2.4 Exercise 4

In a WiFi network, statistical indicator for an access points provide the following information related to a 5 minutes period:

Table 2: 5-min Statistics

MCS (Rate)	**Usage**	**Protocol Overhead**
1 Mbps	20	20%
2 Mbps	20	20%
5.5 Mbps	10	30%
11 Mbps	50	30%

The total data traffic in the 5-min period is 120 MB.

Calculate the average time necessary for a user using the 11Mbps MCS for transferring a 15 MB file using the processor sharing model.

Answer

Under the processor sharing model, the delay d_i can be computed as:

$$d_i(x) = \frac{x/C_i}{1-\rho} \tag{12}$$

We need to compute the channel load ρ. Removing protocol overhead, the effective capacities C_i are:

$$1 \times (1 - 0.2) = 0.8 Mbps \tag{13}$$
$$2 \times (1 - 0.2) = 1.6 Mbps \tag{14}$$
$$5.5 \times (1 - 0.3) = 3.85 Mbps \tag{15}$$
$$11 \times (1 - 0.3) = 7.7 Mbps \tag{16}$$

The average capacity on the channel is

$$C = 0.2 \times 0.8 + 0.2 \times 1.6 + 0.1 \times 3.85 + 0.5 \times 7.7 = 4.715 \tag{17}$$

The channel load rho is given by:

$$\rho = \frac{120 \times 8}{5 \times 60} \times \frac{1}{C} = \frac{3.2}{4.715} = 0.678 \tag{18}$$

Finally, we have:

$$d_i = \frac{15 \times 8}{7.7} \times 10.322 = 48.39s \quad (19)$$

3 TCP

3.1 Exercise 1

Calculate the average time for transferring a 10MB file assuming:

- A long-lived TCP connection
- Segment losses all recovered with dup-ACK detection and fast retransmit
- Losses only due to congestion
- Bottleneck link capacity: 10 Mbps
- RTT = 200 ms
- MSS = 1000 B

Answer In this case losses are only due to congestion and not due to channel errors. The only reason to have a loss in case of no errors is when the transmission rate (the number of MSS W per RTT) is higher than the capacity. That means:

$$\frac{WMSS}{RTT} = C \quad (20)$$

which gives $W = 250$. We know from the model that the number of MSS transmitted in one cycle (of length W/2 RTT) is $3/8W^2 = 23437.5$. The number of MSS to transmit in total is 10000, which can be transmitted in 10000/23437 250/2 RTT = 10.67s. (Alternatively one can note that the average rate during one cycle is (W/2+W)/(W/2)C = 3/4C. Therefore the file is transmitted in (10 $\times$ 8Mb)/(7.5 Mbps) = 10.67s)

3.2 Exercise 2

Estimate the transfer time for a file of $D = 5MB$ in case of a long-live TCP connection assuming:

- All losses recovered with fast retransmit
- Loss probability $p = 8 \times 10^{-3}$
- RTT = 300ms
- MSS = 1000B
- Channel capacity 2Mbps
- Delayed ACK policy (1 ACK every 2 MSS)

Answer

We can apply the long lived TCP model with C = $\sqrt{(3/4)}$. We have:

$$R = \frac{MSS \times C}{RTT \times \sqrt{p}} = 258\text{kbps} \quad (21)$$

and

$$T = D/R = 155.04\text{s} \quad (22)$$

3.3 Exercise 3

Is the transfer time computed in Exercise 2 still correct if two identical TCP connection are active on the channel? What happens in case of 10 connections?

Answer

The computation is correct until losses are only caused by channel errors and not by congestion. With two active long lived connection, the maximum bandwidth used is $2R = 512$kbps. Since this value is much less than C, the computation is still valid. In case of 10 connections, the total bandwidth is higher than the capacity: some congestion will be present and the transfer time will be higher.

3.4 Exercise 4

Estimate the average transfer time for transferring a file of 30kB in a short-lived TCP connection assuming:

- $p = 2\times10^{-2}$
- RTT = 200 ms
- T_0 = 2s (initial RTO value)
- MSS = 1000B
- $b = 1$ (1 ACK every MSS)

Answer

We will apply the model for short lived connection. The number of segments to be transmitted is

$$N_{MSS} = 30 \tag{23}$$

The number of packet losses is

$$l = \frac{30p}{1-p} = 0.6 \tag{24}$$

The probability that a loss leads to a RTO is:

$$Q(p) = min(1, \frac{3}{\sqrt{\frac{8}{3p}}}) = 0.26 \tag{25}$$

The number of consecutive losses is:

$$u = l \times Q(p) \times (1-p) = 0.153 \tag{26}$$

then, the total time spent on RTO is:

$$t_{RTO} = u \times T_0 = 0.306 \tag{27}$$

The number of phases of data transmission is $v = u + 1 = 1.153$, and the corresponding data sent per transmission phase is

$$e = \frac{data + l}{v} = 26.53 \tag{28}$$

Therefore, the total time for data transfer is (assuming the initial congestion window $w_0 = 1$ MSS):

$$t_{TX} = v \times RTT \times log_2(\frac{e}{w_0} + 1) = 1.103s \tag{29}$$

And finally, $T_{tot} = RTT + t_{TX} + t_{RTO} = 1.609$

3.5 Exercise 5

Compute the maximum file for which a connection can be assumed short-lived, assuming that the slow start threshold SSTHRES and the receiver window RCWND are much greater than $C \times RTT$ and that :

- RTT = 250ms
- MSS = 512B
- C = 4194.304 Kbps
- $W_0 = 1$

Answer

The connection will stay in slow start until we reach the maximum capacity. Then there will be a loss and the congestion window will be reset to 1. The maximum congestion window for which this happens is:

$$W = \frac{C \times RTT}{MSS} \tag{30}$$

We know that W starts at 1 and in slow start is increased exponentially for each RTT. Therefore, to know after how many RTTs we will reach the capacity we can write:

$$2^{i-1} = \frac{C \times RTT}{MSS} \tag{31}$$

which leads to

$$i = \log_2(\frac{C \times RTT}{MSS} + 1) = 9 \tag{32}$$

The amount of data sent in 9 RTT is:

$$D = \sum_{k=1}^{i} 2^{k-1} MSS = 511 MSS = 261632\text{KB} \tag{33}$$

3.6 Exercise 6

Considering the same connection of the previous exercise, compute the time for transferrring a 100 KB file assuming:

- $p = 0.05$
- $T_0 = 300$ ms
- $w_0 = 1$

(use the extended formula for the average RTO time $t_u = T_0 \frac{1+p+2p^2+4p^3+8p^4+16p^5+32p^6}{1-p}$)

Answer

The number of segments to transmit is:

$$data = 196 segments \tag{34}$$

The number of losses is:

$$l = \frac{data \times p}{1-p} = 10.32 \tag{35}$$

The probability that a loss leads to a RTO is:

$$Q(p) = min(1, \frac{3}{\sqrt{\frac{8}{3p}}}) = 0.41 \tag{36}$$

The number of consecutive losses is:

$$u = l \times Q(p) \times (1 - p) = 4.02 \tag{37}$$

The average RTO time is:

$$t_u = T_0 \frac{1 + p + 2p^2 + 4p^3 + 8p^4 + 16p^5 + 32p^6}{1 - p} = 333.3ms \tag{38}$$

And the total time spent during RTO is:

$$t_{RTO} = ut_u = 1.34s \tag{39}$$

The number of transmission phases is $v = u + 1 = 5.02$ and the number of segments transmitted per phase is:

$$e = \frac{data + l}{v} = 41.1 \tag{40}$$

The time for transfering those segments is:

$$t_{TX} = v \times RTT \times log_2(\frac{e}{w_0} + 1) = 6.77 \tag{41}$$

And the total time is

$$T_{tot} = RTT + t_{RTO} + t_{TX} = 0.25 + 1.34 + 6.77 = 8.36s \tag{42}$$