Corruption Dynamics in Democratic Societies

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This paper investigates the dynamics of corruption at the top (i.e., by politicians). For this purpose a dynamic, politico-economic framework is developed and analyzed. A particular feature of this investigation is an analytical characterization of the dynamic properties of the system. This allows one to interpret these properties in terms of economic and institutional parameters. As an example, the framework is applied to simulate Italian history from 1948 to today. © 1998 John Wiley & Sons, Inc.

Key Words: corruption, democratic, dynamic, model

1. INTRODUCTION

Corruption is a perennial phenomenon and presumably a necessary evil that accompanies inevitably all kinds of political processes. The political scientist Friedrich wrote that “politics needs all these dubious practices, it cannot be managed without corruption” [1]. In fact, these practices may help to mitigate the abuse of state authorities. The evolution of corruption within a society—its emergence and expansion but also its uncovering, prosecution, and deterrence—is often complex and apparently spontaneous. For example, compare the longtime rumors about the Italian politicians and the sudden indictment against ex-Prime Ministers Giulio Andreotti and Bettino Craxi. Since this problem of corruption seems particularly topical today in Italy, the following paper makes use of this recent evidence. Corruption is present, however, albeit on a smaller scale, in almost all other industrialized countries (e.g., Germany and France) and in Socialist economies (see [2]) and appears endemic in many developing countries (e.g., according to The Economist, May 8, 1993, African leaders hold some $20 billion in Swiss banks). Corruption has a long tradition in politics (see [1] for historical examples).

The economics of supply and demand of bribes was pioneered in Rose-Ackermann [3]. (For particular recent applications see [4] on illegal tax evasion and [5] on the distribution of bribes within hierarchical bureaucracies; see also the survey in [6].) A major conclusion resulting from these economic investigations is that, given equal conditions, bribery and thus corruption increase with the amount of goods traded through the political process and through bureaucracies as opposed to being traded in the market. More concisely, corruption is positively correlated with government’s ownership and interventions in the economy; for example, Noack links political corruption to the establishment of government bureaucracies [7]. The reason is that in a competitive market nobody holds monopoly power over certain property rights, allotments, and
privileges. This conclusion became obvious for centrally planned economies only close to the collapse of Communism (e.g., in the six-hour speech Gorbachev gave to the 27th Congress of the Communist Party in 1986) and afterward.

The focus of this paper is on the dynamic pattern of the evolution of corruption and, in particular, on cyclical patterns. Plato already reports on the myth of periods of corruption alternating with periods where “God himself steers the world for the half of a cycle of a great world period. When he lets go, then the world, which so far has moved forward, begins to roll back again.” That is, campaigns against corruption seem to alternate with open or tacit allowance of bribery. Indeed, sometimes people subjugate themselves to dictatorial rules in the hope of eliminating extensive corruption. Initially, central power may be created and directed against corruption, but sooner or later the new rulers become infected by the same virus and fall sick, sometimes more so than their predecessors. For example, the corruption of the Weimar Republic was a central topic for Nazi propaganda, but the Nazi regime ended up in endemic and unprecedented corruption.

Nevertheless, the dynamics of corruption cycles and, in particular, the existence of corruption cycles have been so far theoretically investigated only by a few authors. Lui considers the impact of exogenous corruption deterrence on the (stationary) corruption and views anti-corruption campaigns as efforts to shift, in an environment with multiple equilibria, from an unfavorable to a favorable equilibrium [8]. Feichtinger and Wirl attempt to endogenize these episodes of crusades against corruption [9]. Bicchieri and Rovelli model the exchange of bribes as a system in which there are two types of players who play a sequence of repeated prisoner’s dilemma games with randomly chosen opponents [10]. Assuming that corruption generates small but cumulative social costs and that population contains a small number of irreducibly honest individuals, they show that a stable equilibrium of corruption can become unstable, thus triggering a sudden transition to honest behavior. Finally, Bicchieri and Duffy demonstrate how corruption can become cyclical under the assumption that politicians, in order to be reelected, have to compensate voters through material incentives [11]. By contrast, in our analysis, the end of a period of corruption is driven by the public getting informed about the existence of illegal activities. Our investigation is descriptive to the extent that no explicit dynamic optimization problem is solved. This allows for a more complex model and is thus complementary to the above-mentioned studies based on optimization models.

2. THE MODEL

Any dynamic model must trade off realism against analytical tractability. Therefore, we start with a fairly general framework that is subsequently simplified to become amenable for analytical purposes. Indeed, this analytical characterization of the model appears to us as a sufficient compensation for occasionally simplifying assumptions.

We consider three state variables:

\[ x(t) = \text{the public support (popularity) of politicians at time } t \]
\[ y(t) = \text{the (hidden) assets that the corrupt politicians hold at time } t \]
\[ z(t) = \text{the investigation effort at time } t \text{ (i.e., the sum of activities by different institutions—police, courts, and the press—that aim to uncover and to reveal corruption)} \]

Different units could be used to measure these variables. For example, \( x \) could be the percentage of people attaching a positive value judgment to their political class, \( y \) could simply be the monetary value of the assets accumulated by politicians through illegal practices, and \( z \) could be the number of officials involved in investigations concerning political corruption.

In addition, we consider the following flows as opposed to the above stocks.

1. Positive actions, denoted \( A \), taken by the politicians in order to recover or maintain public goodwill, \( A = A(x) \), and public expectation of such actions, denoted \( A^*(x) \).

We postulate that the change in popularity depends on the innovation of the politicians (i.e., how far their actions exceed or fall below the current expectations of the public). Obviously, expectations increase with popularity (i.e., \( A^*_+ > 0 \)). By contrast, the dependence of real actions on popularity is not clear. For example, one reviewer argued that the less support politicians have, the harder they might work to gain support (i.e., \( A^*_+ < 0 \)). But one could also argue that power is positively correlated with popularity (popular politicians often have carte blanche and can do almost anything they like) and that this correlation allows popular politicians to realize their projects. Here we assume that the negative effect of \( x \) on actions is dominated by the positive effect of \( x \) in terms of power and scope to deliver actions (i.e., \( A^*_+ > 0 \)). Moreover, we assume that \( A^*_+ < 0 \) since, in any case, positive actions must be bounded.

2. Collection of bribes, denoted \( B \) and defined as the total flow of money that the politicians earn from illegal practices: \( B = B(x,y) \).

The sum of bribes paid and received constitutes the crucial demand-supply interaction in our framework. The following partial derivatives are expected to hold: \( B^+ > 0 \), popular and thus more powerful politicians tend to abuse their power for private interests (i.e., they demand larger bribes because they can secure higher returns for their client). In the language of business, the politicians can sell more products, at a higher quality, and they can deliver the goods more punctually, if their power, due to higher popularity, increases. The above argument lacks
the demand side (i.e., less people will bribe as it is getting more and more costly to “buy” politicians). Therefore, the impact of $x$ on $B$, the total amount of bribes rather than the bribe handed over to individual politicians, seems ambiguous. However, we show in the Appendix that $B > 0$ holds globally.

The property $B_i > 0$ can also be ascertained by focusing on the most frequent form of corruption, namely that of a contractor obtaining a contract to carry out public works or to furnish supplies to the public sector by paying a percentage of the value of the contract to the corrupt politician in charge of the decision (so-called kickbacks). If we call $g$ the fraction of public expenditure $G$ subject to corruption, we can write, for this form of bribe, $B = gG$. But the percentage $g$, usually fixed by current illegal practice, increases with power and arrogance of the politicians (hence with $x$ and $y$). This phenomenon has been clearly ascertained in Italy, as well as in other countries. Moreover, corruption has the additionally negative effect of increasing public expenditure $G$, so that, in the end, $B$ increases with $x$ and $y$.

3. Part of the private consumption of the politician, denoted $C$, depends on present cash flow, the flow of bribes $B$, and on the assets $y$: $C = C(B, y)$.

It is reasonable and quite standard to assume that higher bribes and larger accounts increase private consumption (i.e., $C_p > 0$ and $C_y > 0$). The impact of popularity on consumption is hardly imaginable and thus is not taken into account.

4. Discovery and unveiling of corruption, denoted $D$. For accounting purposes, $D$ will measure the amount of money confiscated by the police and courts. Thus, it is reasonable to assume $D = D(y, z)$.

The partial derivative $D_y > 0$ expresses the intuitively plausible relation that the discovery increases with the assets and bank accounts accumulated (i.e., the state variable $y$). Furthermore, $D_z > 0$, because discovery increases when the courts, the police, and journalists expand their investigation efforts.

Now that we have introduced the relevant stocks and flows, we can specify our model. The rate of change of popularity is described by the following differential equation:

$$\dot{x} = \mu'(A(x) - A^*(x)) - \mu \cdot xD(y, z) \quad (1)$$

The first term in this equation has already been discussed: It is a source or a sink of popularity depending on the sign of the innovation $(A - A^*)$. By contrast, the second term, related with the uncovering of corruption, is always a sink of popularity. The dependence of this second term on the state of popularity can be explained in two different ways, arithmetically and politically. Arithmetically the differential equation (1) postulates that the relative change in popularity $(\dot{x}/x)$ is proportional to the discovery (i.e., the parameter $\mu$ translates the seized money into a relative decline of the popularity ratings of the politicians). In political terms, the dependency of $\dot{x}$ on $x$ states that the impact of unveiling of corruption of popular politicians is much more damaging than the discovery that unpopular politicians are corrupt. The coefficients $\mu'$ and $\mu$ identify the reaction of the public to political innovations and to the uncovering of corruption (measured in terms of the dollars confiscated).

The change in the (aggregate) politicians’ assets and bank accounts $y$, which are publicly unknown and thus called hidden capital, is a simple accounting equation: The assets increase with respect to the present flow of bribes $B$ and with the earned interest ($r$ denotes the return on capital assets); obviously, $y$ decreases by the amount spent for consumption and by the amount confiscated, that is,

$$\dot{y} = ry + B(x, y) - C(B(x, y), y) - D(y, z) \quad (2)$$

In the Italian case, the bribes were largely transferred to the corresponding political party and only a fraction was put into the private accounts of the politicians. This requires either a reinterpretation of $B$ as the amount that the politicians put into their pockets or an additional parameter must be introduced to account for this observation. Obviously, this multiplication of $B$ by a constant coefficient would not affect the substance of the model; thus, we save on introducing an additional, yet mathematically redundant, parameter.

Finally, the investigation activities against corruption are given by:

$$\dot{z} = \sigma D(y, z) - \delta z \quad (3)$$

where $\sigma D$ characterizes the immediate reinforcement and $\delta$ the persistence of investigators; of course, $\sigma$ accounts for the different units of measurement of $D$ and $z$. The positive influence of discoveries on activities is plausible and typical for bureaucratic agencies such as the police and courts; that is, providing higher output justifies larger budgets.

This setup of the evolution of the investigations, differential equation (3), assumes implicitly that the judicial system and the press enjoy at least some independence from the rulers. This assumption, rather than the dependence on popularity, is crucial for the adjective “democratic.” Without this assumption, corruption may be pervasive and may yet remain uncovered and untried or at best selectively enforced (e.g., for members of the opposition); this description seems typical for many (but not all) dictatorships of today and the past. Hence, it is this assumption that excludes many dictatorships but holds more or less in all countries with democratic institutions and, in particular, when the executive and the legislative branch of governments are (at least to a certain extent) separated, for independent courts, and for a free press [12]. The parameter $\sigma$ is somehow a measure of this independence.
We now specify model (1-3) by introducing the following functional forms:

\[
A^*(x) = kx \\
A(x) = \frac{\alpha x}{\beta + x} \\
B(x,y) = \alpha xy \\
C(B,y) = \omega B(x,y) + \theta y \\
D(y,z) = \gamma yz
\]

where all Greek letters are positive and constant parameters that capture institutional, individual, and national characteristics. The choice of these forms (mainly linear or bilinear) is not supported by a particular rationale. Nevertheless, these functional forms satisfy the sign constraints on the partial derivatives and are convenient because they help analytical tractability.

The specification of the actions according to (5) represents a simple concave functional form: \( \alpha \) is the maximum action that is achievable only with very high popularity, while \( \beta \) is the so-called half-saturation constant, namely the popularity at which positive actions are half maximum. This specification, together with the linear expectation (4), implies, in the absence of discoveries, \( D = 0 \), that the popularity expands up to a maximum \( x^* \) in a manner similar to the logistic law, if the parameters are such that \( A^* \) and \( A \) intersect at a positive \( x \). This implies \( \alpha/\beta > k \) and excludes quite reasonably the origin as a stable steady state.

The bribes \( B \) are specified as the product of \( x \) and \( y \) where the parameter \( \omega \) is a crucial characterization of the political system, namely, how far it is susceptible to the temptation of bribes. In practice, the magnitude of the parameter \( \omega \) depends on personal characteristics of politicians and the people within a country, as well as the demand-supply situation (e.g., the stakes the politicians can offer, large in a country with strong government ownership and interference but small in a “pure” market economy).

Equation (7) allows for different propensities to consume from the flow of bribes \( \omega \) and the stock of assets \( \theta \), similar to Friedman’s differentiation between transitory and permanent income. The value of the parameters \( \omega \) and \( \theta \) indicate whether the politicians are thrifty \( (\omega \) and \( \theta \) small) or use bribes and assets to finance their lifestyle \( (\omega \) and \( \theta \) large) rather than to accumulate a nest egg. Observe that a thrifty class of politicians is penalized because it is the accumulation of the assets that increases the risk of being discovered as corrupt.

Finally, the bilinear form (8) for the rate at which hidden assets are discovered and confiscated is justified if one models discoveries as random encounters of investigators \( z \) with traces of illegal practices (proportional to \( y \)).

With these specifications, we obtain the following dynamic system, written in terms of relative changes:

\[
(\dot{x}/x) = \mu^*\left[\frac{\alpha l}{(\beta + x) + k} - k\right] - \mu^*\gamma yz
\]

(9)

\[
\dot{y} = -\rho y \quad (10)
\]

\[
\dot{z} = \sigma y - \delta
\]

(11)

where some economically relevant but mathematically redundant parameters have been eliminated by defining the new parameters:

\[
\rho = \theta - r
\]

(12)

\[
\epsilon = (1 - \omega)/\theta
\]

(13)

The parameter \( \rho \) is positive if the return \( r \) on illegal assets is low compared with the marginal propensity \( \theta \) to use the assets for private consumption. This is quite plausible, in particular because the anonymity of the accounts exclude investments offering high returns (some Italian politicians simply kept the cash at home).

This write-up highlights the different dynamics, in particular, the delayed reaction of investigations \( z \). The reason is very simple and intuitively plausible. Corruption remains uncovered in the dark as long as the hidden capital is small. However, investigations are expanded for the reasons given above by success, and thus both \( z \) and discovery \( D \) trail behind all the other variables. In fact, one may conjecture an asymmetry in the build-up and the elimination of corruption. As long as corruption remains moderate, it is hard (or impossible) to discover, so that it grows slowly but continuously. Once corruption is prevalent, the incentives to report and strike a deal with the investigators, who depend on confessions, increase. The reason is that if the risk of being accused of corruption is sufficiently high, it may be advisable for some participants (politicians or managers) to report to the police in exchange for preferential treatment by the courts.

3. ANALYSIS OF THE MODEL

Model (9-11) is of the form

\[
(\dot{x}/x) = F(x, y, z)
\]

(\dot{y}/y) = G(x, z)

(\dot{z}/z) = H(y)

which is typical of positive systems. Indeed, if the three state variables are non-negative at time \( t = 0 \), then they remain non-negative forever. This is in agreement with the meaning of the state variables. Obviously, in the following we only consider the behavior of the system in the positive octant of the state space. Notice, in addition, that the three axes of the state space are invariant, since a trajectory starting on one of the axes develops entirely on it. The three corresponding dynamics are described by the equations

\[
\dot{x} = \alpha l \left[\frac{\alpha l}{(\beta + x) + k} - k\right] - \mu^*\gamma yz
\]

\[
\dot{y} = -\rho y
\]

\[
\dot{z} = \sigma y - \delta
\]
Thus, trajectories on the $y$ and $z$ axes tend to the origin, while under the assumption $(\alpha/\beta > k)$ there are two equilibria on the $x$ axis, namely the origin (which is unstable) and point

$$x^* = \frac{\alpha - \beta k}{k}$$

(14)

which is stable because $\dot{x} > 0$ for $x < x^*$ and $\dot{x} < 0$ for $x > x^*$ (see Figure 1). Moreover, it is easy to check (because $G$ and $H$ are linear) that the origin (0,0,0) and point $(x^*,0,0)$ are the only two equilibria on the faces of the positive octant (with a single exception corresponding to the particular setting of parameters separating the cases depicted in Figure 1a and 1b for which there are infinite equilibria of the kind $x = x^*$, $y = \text{any value}$, $z = 0$). Finally, one can notice that $x = x^*$ implies $\dot{x} \leq 0$. This means that the popularity is a bounded variable, that is

$$0 \leq x(t) \leq x^*$$

(15)

so that we should (and we will) be interested in discussing the behavior of the system only in region (15) of the positive octant. Actually, the intersections (straight lines) of the plane $x = x^*$ with the faces of the positive octant are also trajectories of the system, because $x = x^*$ and $y = 0$ imply $\dot{x} = \dot{y} = 0$ and $\dot{z} < 0$, while $x = x^*$ and $z = 0$ imply $\dot{x} = \dot{z} = 0$ and $\dot{y} \geq 0$ depending on the sign of $(x^* - p)$. More precisely, if $x^* > 0$, that is, if

$$\varepsilon < \frac{\rho k}{\alpha - \beta k}$$

(16)

the equilibrium $(x^*,0,0)$ is a stable node, as shown in Figure 1a, while if the opposite inequality holds, i.e.

$$\varepsilon > \frac{\rho k}{\alpha - \beta k}$$

(17)

the equilibrium is a saddle, as pointed out in Figure 1b.

**Uncorrupted systems**

If (16) holds (i.e., if $(x^* - p) < 0$), then $\dot{y} < 0$ for all $x(t)$ satisfying (15) (see equation (10)). This implies that $y$ tends to zero for $t$ tending to infinity, which, in turn, implies that $z(t)$ also tends to zero (see (11)). Hence, both $y(t)$ and $z(t)$ tend to zero and therefore $x(t)$ tends to $x^*$. This means that condition (16) guarantees that there are no equilibria or cycles or strange attractors inside region (15), and that all trajectories starting in this region asymptotically tend to the equilibrium $(x^*,0,0)$, which is therefore globally stable.

Conversely, assume that the equilibrium $(x^*,0,0)$ is globally stable in region (15) and consider the trajectory starting from point $(x^*,y(0),0)$. From equations (9-11) it follows that such a trajectory is characterized by

$$\dot{x}(t) = x^*$$

$$\dot{y}(t) = (x^* - \rho)y(t)$$

$$\dot{z}(t) = 0$$

so that the global stability of $(x^*,0,0)$ implies $\dot{y}(t) < 0$ (i.e., $(x^* - \rho) < 0$), which is again condition (16). Thus, inequality (16) is a necessary and sufficient condition for the global stability of $(x^*,0,0)$.

**Systems satisfying condition (16), namely systems characterized by the fact that $y(t)$ tends to zero, are called **uncorrupted**, because they settle down to an equilibrium at which bribing is impossible. It is interesting to notice that a system can be uncorrupted even if $\varepsilon > 0$ (i.e., even if the politicians are looking for bribes). Moreover, one should notice that the parameters $\sigma$, $\gamma$ and $\delta$, characterizing the dynamics of the investigation effort, have absolutely no role in making a system uncorrupted. On the contrary, the parameter $k$ is definitely very important. If people are quite demanding (i.e., if $k$ is sufficiently high), the system is uncorrupted. Finally, inequality (16) is also satisfied if the politicians consume large fractions of their assets and bribes ($\varepsilon$ small and $\rho$ high, see (12) and (13)). Nevertheless, the final consequence of this high tendency to consumption is that corruption is asymptotically ruled out so that bribes and their consumption annihilate.

![Figure 1](image_url)

Equilibria and trajectories of system (9-11) on the faces of the positive octant: (a) uncorrupted systems $\varepsilon < \rho k(\alpha - \beta k)$; (b) corrupted systems $\varepsilon > \rho k(\alpha - \beta k)$. 
Corrupted Systems

We now turn our attention to corrupted systems, namely to systems identified by inequality (17), with the aim of classifying their asymptotic behavior. Although we have no formal proof that our analysis is complete, we have identified three meaningful classes of corrupted systems called, respectively, strongly controlled, weakly controlled, and uncontrolled. While the discrimination between corrupted and uncorrupted systems can be done by looking at only five parameters (see (16), (17)), the discrimination between the various forms of corrupted systems is much more complex because it involves all parameters.

In the first class we have systems that settle to a positive equilibrium: A certain level of corruption is present in the system, and popularity and investigation effort are constant. Nevertheless, popularity is not at its maximum level, so that politicians cannot exert their best actions (some kind of social inefficiency). Systems in the second class behave periodically: Periods of high corruption alternate with periods of lower corruption, and popularity goes smoothly up and down following a kind of internal clock. On the contrary, systems in the third class have a much wilder behavior. A long and socially dangerous phase of political stagnation (low popularity) systematically alternates with a long phase of legality and efficiency (high popularity and low corruption). The transition from the first phase to the second is simply due to the raising of popularity, while the transition from the second phase to the first is composed of a period of increasing corruption followed by a sudden discovery of the hidden capital and a crash of popularity. This behavior resembles a cycle but is not really a cycle because the time between subsequent crashes of popularity increases.

The analysis has been carried out by finding the bifurcations of system (9-11), namely the combinations of parameters at which the system behavior qualitatively changes. Only two bifurcations are involved, namely the Hopf bifurcation and the so-called tangent bifurcation of cycles. The first can be fully discussed analytically. It corresponds either to the case in which a stable equilibrium surrounded by an unstable limit cycle becomes unstable, while the cycle shrinks to a point and disappears (subcritical Hopf bifurcation $H_s$) or to the case in which an unstable equilibrium surrounded by a stable limit cycle becomes stable while the cycle shrinks and disappears (supercritical Hopf bifurcation $H_p$). The tangent bifurcation of cycles, on the contrary, has been detected numerically, by means of the package LOCBIF implementing a powerful continuation technique [13].

There are two facts, which hold under condition (17), that are worth noticing before describing the details of the bifurcation analysis. The first (easy to check) is the existence of a unique strictly positive equilibrium

$$E^* = (x^*, y^*, z^*)$$

in region (15). This means that the equilibrium $E^*$ can only undergo Hopf bifurcations, because all other bifurcations would require the existence of multiple equilibria. The second important fact is the existence of an heteroclinic loop through infinity composed (see Figure 1b) by the concatenation of three trajectories: The first is the $z$ axis starting from infinity and ending into the origin; the second is the

FIGURE 2

A trajectory $P^* \rightarrow P^{*'}$ close to the heteroclinic loop through infinity.
heteroclinic connection starting from the origin and ending into the saddle \((x^*,0,0)\); and the third is the unstable manifold of such a saddle starting from \((x^*0,0)\) and ending at infinity. By continuity, trajectories starting close to this loop follow it closely (see Figure 2). For example, if for \(t = 0\) the state of the system is close to the origin (see point \(P^*\) of Figure 2), the system will remain practically there for a long time because the origin is an equilibrium. This is the phase of stagnation we have mentioned a moment ago. Then, the trajectory will leave the origin roughly along its unstable manifold (the \(x\) axis) until the saddle \((x^*,0,0)\) is approached. Here the system will rest again for a long time (the phase of legality and efficiency) and then move upward, somehow parallel to the unstable manifold of the saddle \((x^*,0,0)\).

When \(y\) becomes greater than \(\delta \sigma y\) (see (11)), \(z\) becomes positive and, soon after this, hidden capital is discovered. This leads to the collapse of \(x\) and \(y\) so that after a while the trajectory is close to the \(z\) axis and therefore tends toward the origin, where it finally rests at point \(P^*\) for a new period of stagnation. Of course, the trajectory starting from point \(P^*\) and ending in a new point \(P''\) close to the origin will be roughly similar to the trajectory \(P^* \rightarrow P''\) and the process will be repeated again and again. In Figure 2 point \(P''\) is closer than point \(P^*\) to the origin. This means that the heteroclinic loop is attracting, in the sense that all trajectories starting in its neighborhood will asymptotically tend to it. We have reported this case in Figure 2 and not the opposite one of a repelling heteroclinic loop, because all the numerical experiments we have performed with model (9-11) have clearly indicated that the heteroclinic loop was attracting. A formal proof of the attractiveness of the heteroclinic loop is certainly not easy because the known results on this topic do not concern the singular case of heteroclinic loops through infinity.

We now are in the position of presenting the three types of corrupted systems we have identified by means of our bifurcation analysis. In order to describe them we should, in principle, illustrate their equilibria, cycles, and attracting sets in the state space. Nevertheless, it turns out that the geometry of the trajectories is not particularly easy to capture by looking at three dimensional figures. For this reason we present the projections of the trajectories on the \((x,y)\) plane. In doing so (see Figure 2) we loose the sudden explosions and the subsequent decay of the investigation effort. Moreover, for the sake of clarity we qualitatively sketch trajectories and attractors to emphasize some of their features. Figure 3 shows the three distinct possible portraits of corrupted systems. In all cases the attracting heteroclinic loop is composed of three trajectories delimiting the figure.

In the first class of systems (Figure 3a) the positive equilibrium \(E^+\) is stable and is therefore an interior point of a three-dimensional basin of attraction. Trajectories starting outside the basin of attraction tend toward the heteroclinic loop. There also exists an unstable limit cycle, the projection of which on the \((x,y)\) plane turns out to be a simple closed curve \(\Gamma_0\). For this reason we can somehow interpret the portrait of Figure 3a as the phase portrait of a second order system and imagine that the basin of attraction of \(E^+\) is the region delimited by the closed curve \(\Gamma_0\). Obviously, this is not really correct, since the projections of two different trajectories, the first starting from \((x(0),y(0),z'(0))\) and tending to \(E^+\), the second starting from \((x(0),y(0),z''(0))\) and tending to the heteroclinic loop, would be two lines starting from the same point \((x(0),y(0))\) in the space \((x,y)\). Systems of this kind can therefore settle to an equilibrium \((E^+)\) where the hidden capital \(y^*\) is simply given by \(\delta \sigma y^*\). In other words, corruption is stabilized. For this reason, we say that these corrupted systems are strongly controlled.

In the second class of systems (Figure 3b) there are two limit cycles, one stable \(\Gamma_0\) and one unstable \(\Gamma_2\), and the equilibrium \(E^+\) is unstable. Thus, the system cannot tend toward an equilibrium but can tend toward a cyclic behavior. In such a case we are guaranteed that corruption cannot indefinitely grow and that popularity cannot fall below a certain level. In other words, corruption is still under control, although it might become periodically high. For this reason, we have called these corrupted systems weakly controlled. On a simply intuitive ground, one could imagine that the case of weakly controlled corruption is rather unlike, because periodically recurrent rises and falls of corruption would sooner or later be identi-
fied by politicians who would then try to avoid them. This is in good agreement with the numerical analysis carried out in the next section, where corruption turns out to be weakly controlled only in a very narrow region of parameter space (see region (b) in Figures 5, 6). Nevertheless, we should also notice that the above argument against weakly controlled corruption would be fully acceptable if the period of the limit cycle would be short in comparison with the life of a politician or with the life of the democratic system. But, as we will see in the next section, the limit cycles of our model have a very long period so that the above learning process is hardly conceivable in practice.

Finally, in the third class of systems (Figure 3c), for obvious reasons called uncontrolled, there are no limit cycles and the equilibrium \( E^* \) is unstable, so that the only attracting set is the heteroclinic loop. Systems of this kind have a very wild behavior, with recurrent episodes of higher and higher corruption. They are particularly critical because they repeatedly go through long periods of stagnation (low popularity of politicians), which create the most favorable conditions for dramatic structural changes potentially involving the loss of democracy.

Understanding what happens to the attractors and repellors of the system when one or more parameters are varied is very important. In particular, one can understand what happens when a system changes the class to which it belongs. With reference to Figure 3, we can very easily interpret the transitions between any pair of phase portraits in terms of collisions of equilibria and cycles (local bifurcations). For example, the transition from strongly to weakly controlled systems is a supercritical Hopf bifurcation: When a strategic parameter is varied, the stable limit cycle \( \Gamma_s \) of Figure 3b shrinks and collides with the equilibrium \( E^* \) and, after the collision (see Figure 3a), the cycle has disappeared and the equilibrium \( E^* \) has become stable. On the contrary, the transition from strongly controlled to uncontrolled systems, namely the transition from portrait 3a to portrait 3c, can be explained by the shrinking of the unstable limit cycle \( \Gamma_u \) followed by the collision and disappearance of this cycle with the stable equilibrium \( E^* \) which, after the collision (see Figure 3c), becomes unstable. This is a subcritical Hopf bifurcation. Finally, the transition from weakly controlled to uncontrolled systems is a tangent bifurcation of cycles: The cycles \( \Gamma_s \) and \( \Gamma_u \) of Figure 3b get closer and closer until they collide and disappear, thus giving rise to the portrait of Figure 3c. In conclusion, with reference to Figure 3, we have the following bifurcations

\[
\begin{align*}
  a &\leftrightarrow b & \text{supercritical Hopf bifurcation} \\
  a &\leftrightarrow c & \text{subcritical Hopf bifurcation} \\
  b &\leftrightarrow c & \text{tangent bifurcation of cycles}.
\end{align*}
\]

The combination of parameters giving rise to the supercritical and subcritical Hopf bifurcations involve all parameters. They can be analytically specified by imposing that two eigenvalues of the Jacobian matrix of the system evaluated at \( E^* \) are purely imaginary [14]. Distinguishing between super- and subcritical Hopf bifurcations is much more difficult. Nevertheless, we do not report these analytical results in this paper because they are too cumbersome and do not allow for any meaningful insight into the problem.

4. A NUMERICAL EXAMPLE

We now show how the model can be used to interpret the past and forecast the future patterns of corruption in a given coun-

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**FIGURE 4**

Scenario for Italy in the period 1948–1994 corresponding to the parameter setting proposed in the text. Initial conditions are \( x(0) = 0.5, y(0) = z(0) = 10^{-5} \).
try. For this application we have selected Italy, because after the recent scandals and discoveries involving more or less all political parties, the history of corruption from the end of World War II until today is roughly known, in the sense that the patterns of popularity, hidden capital, and investigation effort are well identified. Nevertheless, this application should be viewed only as a naïve exercise.

Soon after the end of the war, Italian citizens were asked to select between a monarchy and a republic through a national referendum. The result was slightly in favor of the republic, which started to operate with a young and quite unknown political class elected in 1948. In terms of our model, we can consider 1948 as the origin of time, \( t = 0 \), and assume that initial popularity of the politicians \( x(0) \) was relatively low, say one half of the maximum achievable popularity \( x^* \), while hidden capital and investigation effort were practically absent. During a period of about ten years, politicians reinforced their popularity, which afterwards remained permanently high. Until 1990, reported episodes of corruption were relatively rare and of marginal impact. It was only in 1990 that a blitz initiated an extensive campaign of investigations that proved that corruption has been well established since the mid-1970s and was a common practice all over the country in all governmental agencies and parties. The accounting of bribes and hidden capital from 1975 to 1990 is still largely unknown, but surprisingly high anyway. The secretaries of almost all parties were personally involved, and their charisma obviously fell immediately after the discovery. There is no doubt that today Italy is the country in which politicians receive the lowest respect within the European Community.

The described patterns of popularity, hidden capital, and investigation compare favorably with those schematically shown in Figure 2. In the attempt to be consistent with our theory, we should therefore conclude that a model that pretends to interpret the history of the Italian corruption from 1948 to 1994 should belong to the class we have identified as "corrupted and uncontrolled." But how can we calibrate the ten parameters of the model? Certainly not by using formal parameter estimation techniques, which require a great amount of precise data, unfortunately not available in studies of this kind. We therefore have been forced to estimate the parameters in a much more naïve way, using very simple arguments.

The reference parameter setting we propose for Italy is the following:

\[
\begin{align*}
\mu^* &= 1 \quad \mu = 10 \\
\kappa &= 1 \quad \alpha = 1.5 \quad \beta = 0.5 \\
\rho &= 0.1 \quad \varepsilon = 0.4 \quad \delta = 2 \quad \gamma = 1 \quad \sigma = 2
\end{align*}
\]

The first parameter of our list, \( \mu^* \), has been fixed arbitrarily to 1, because it multiplies (see (9)) two other parameters (\( \alpha \) and \( \kappa \)) that do not appear elsewhere in the model. Since \( \mu^* \) and \( \mu \) express how intensely people react to positive actions and corruption, we should fix reasonably only the ratio \( \mu / \mu^* \). We have taken this ratio equal to ten in order to express the belief that the damaging effect of the discovery of corruption is much bigger than the reinforcement of popularity due to positive

![Bifurcation curves](image)

FIGURE 5

Bifurcation curves \( H^s \) (supercritical Hopf), \( H^s \) (subcritical Hopf), and \( T \) (tangent of cycles) of model (9-11) in the parameter space \((\varepsilon, \delta)\) for the reference parameter setting proposed for Italy (see Point I). The four regions correspond to: (0) uncorrupted systems; (a) corrupted and strongly controlled systems; (b) corrupted and weakly controlled systems; (c) corrupted and uncontrolled systems.
innovation. We have also fixed $k$ to unity, thus identifying popularity $x$ with expectation of positive actions $A^*$ (see (4)). 

As far as $\alpha$ and $\beta$ are concerned, we have fixed them in such a way that the maximum popularity $x^*$ given by (14) was equal to unity and politicians in 1948 were capable of performing about 50 percent better than expected. The first condition gives

$$\alpha - \beta = 1$$

while the second one (taking into account that $x(0) = (1/2) x^* = 1/2$) is equivalent to

$$\frac{\alpha}{\beta + 0.5} = 0.75$$

The solution of these two equations is, indeed, $\alpha = 1.5$, $\beta = 0.5$.

The parameter $\rho$ has been put equal to 0.1, thus implying that the time constant $1/\rho$ characterizing the exponential decay of the hidden capital in the absence of bribes and discoveries is equal to ten years. Since, in our opinion, the consumption rate $\rho y$ is at least one-fourth the bribing rate (which in the most favorable conditions ($x = x^* = 1$) is given by $\varepsilon y$), we have fixed $\varepsilon = 0.4$. The parameter $\delta$ has been fixed to 2 so that the time constant ($1/\delta$) of the investigation effort in the hypothetical case of no discoveries (see (11)) is six months. The parameter $\gamma$ has been put equal to unity, thus fixing the units of the investigation effort in such a way that discovery $D$ is the simple product of hidden capital $y$ and investigation effort $z$ (see (8)). This also means that the total rate of consumption of the hidden assets is $(\rho + z)$ (see (10)); that is, investigation effort is a kind of surplus of consumption rate. Finally, $\sigma$ has been put equal to 2. This means that the unit of $y$ has been fixed in such a way that the turning point for $z$, namely the minimum value of $y$ for which $z$ increases, is unity.

Now that we have fixed the parameters, we can use the model to check whether it fits the historical evidence, at least qualitatively. First of all, we immediately recognize that our values of $\alpha$, $\beta$, $k$, $\varepsilon$, and $\rho$ satisfy condition (17) (i.e., the model belongs to the class of corrupted models). In order to see if we are in the case of uncontrolled corruption, we can simply simulate the model and check whether its solution tends, for any initial condition, toward the heteroclinic loop discussed in the preceding section. Of course, this turned out to be the case. We can be even more precise, however, by simulating the model over the period 1948–1994 for a set of initial conditions that capture the state of Italian society in 1948. For this, we fixed $x(0) = 0.5$ and searched for the value of $y(0)$ and $z(0)$ for which exploitation of discoveries and crash of popularity occurred in 1990. This was accomplished by letting $y(0) = z(0) = 10^{-5}$, which is consistent with the fact that hidden capital and investigation effort were practically absent in 1948. The corresponding patterns of the three state variables are shown in Figure 4 and are perfectly in line with the historical patterns: Popularity reaches its maximum in 1955 and remains constant afterward, while corruption is practically absent till the early 1970s.

We can now show how the model can be used to infer something about the future. The first and obvious exercise is to extend the simulation for 50–100 years without changing the parameters. This makes sense only if we do not expect significant future changes in the behavioral parameters of the compartments of the model (people, poli-
ticians, police, courts, etc.). In the present case, the answer is obvious. Since our parameter setting corresponds to a corrupted and uncontrolled democratic system, the almost-zero initial conditions in 1994 (see the endpoint of Figure 4) give rise to a trajectory approaching the heteroclinic loop (see Figure 2). In other words, we should expect more or less the same things observed in the period 1948–1994 with even more spectacular explosions and crashes. If, more realistically, we expect some variations in some parameters, but these variations are largely unknown, we can perform a bifurcation analysis with respect to the uncertain parameters. In such a way we can detect under which conditions the system might become weakly or strongly controlled or even uncorrupted. For example, if parliamentary immunity would be revised or abrogated, politicians and firms would be discouraged from bribing, so that the parameter $\varepsilon$ would be consistently reduced. The effect of this reduction can be determined by our theory. Indeed, we can use the critical value of $\varepsilon$ separating uncorrupted systems from corrupted ones (see equations (16) and (17)), namely

$$\varepsilon_{\text{crit}} = \frac{\rho k}{a - bk}$$

which in our case turns out to be 0.1. Thus, the answer is quite clear: If the revision of the parliamentary immunity will decrease the acceptance of bribes of at least four times, then $\varepsilon$ will decrease below $\varepsilon_{\text{crit}}$ and the system will become uncorrupted.

Detecting the influences of the parameters responsible for a transition from one class of corrupted systems into another class of corrupted systems is not as easy, because instead of using equations (16) and (17) as we have just done, one must perform a real bifurcation analysis and detect Hopf and tangent bifurcations. Figure 5 shows, for example, the bifurcation analysis with respect to $\varepsilon$ and $\delta$ (the persistence of the judiciary system) when all other parameters are those of the reference setting. In this figure, the vertical straight line $\varepsilon_{\text{crit}} = 0.1$ separates the uncorrupted systems from the corrupted ones, while the Hopf bifurcation curves $H'$ and $H$, and the tangential bifurcation curve $T$ identify the boundaries of three regions (a), (b), and (c) where, in accordance with our preceding analysis (see also Figure 3), the corrupted systems are strongly controlled, weakly controlled, and uncontrolled. Point I with $\varepsilon = 0.4$ and $\delta = 2$ represents the past Italian system, while points with smaller $\varepsilon$ and $\delta$ (more persistent investigators) are improvements of such a system. From this figure it follows that an improvement of the judiciary system per se is not capable of avoiding corruption but only of keeping it steady (region (a)). Moreover, one could state that corruption could be kept under control if investigators would be three or more times more persistent than in the past.

Figure 6 shows another bifurcation diagram, where the two parameters are $\varepsilon$ (again) and $\mu^*$, namely the reactivity of people to politics. This diagram shows that the behavioral parameter $\mu^*$ is also very powerful in keeping corruption under control. The lesson is the following: If Italians, instead of being as skeptical as they are, would be three times more sensitive to innovations of politicians, they would have a greater chance to change the character of their society.

5. Conclusion

This paper introduced a dynamic description of corruption of politicians. The framework relates the actions—positive actions of the politicians, bribery, consumption, and unveiling of corruption—to the state variables—popularity (as a proxy for power), assets accumulated from bribes, and investigation. The adjective “democratic” is reflected by modeling investigation efforts with some independency of political pressure. That is, the separation of the judiciary system from the government and the existence of a free press characterize democracies better than the requirement for popularity. The highlight of this study is a nearly complete analytical characterization of the dynamic properties of the system (i.e., what parameters ensure stability and low levels of corruption and vice versa). We differentiate between corrupted and uncorrupted systems and in the class of corrupted systems between strongly controlled (steady behavior), weakly controlled (cyclic behavior), and uncontrolled (wild behavior). For example, increasing the persistence of investigators can control the system but cannot eliminate corruption, while institutional changes that lower the practice of giving and taking bribes can.

We think that this is a first step toward a dynamic politico-economic modeling of the topical phenomenon corruption. A straightforward extension of this framework is to add further nonlinearities, which might allow for more complex patterns but would possibly sacrifice analytical tractability. Another route we leave open for research is to modify some of the assumptions about the feedbacks in the dynamic system and find out, for example, what would happen if politicians were lazy and took actions only to replenish lost support. Extensions to multiparty government and coalitions would also be of interest, as well as the study of the impact of different electoral systems.

APPENDIX: PROOF THAT THE TOTAL FLOW OF Bribes Increases WITH RESPECT TO POLITICIANS’ Popularity

Obviously, a popular politician asks for large bribes, yet this demand decreases the supply of bribes as it is getting more and more costly to “buy” politicians and bureaucrats. Hence, the impact of $x$ on total corruption (and this is what $B_x$ measures) may be ambiguous, at least globally. Of course, the assumption $B_x > 0$ is plausible locally for $x$ not too large, but hardly globally.

To consider this argument in detail, let us assume that politicians “sell” an illegal service for a bribe $b$. This service includes a quality component $q$ that rises with popularity for
the reasons given in the paper: a more popular politician secures higher returns for the client either directly or indirectly because the probability that the client’s petition will be successful increases; however, quality q may deteriorate with the number of bribes paid (denoted n) because of congestion, less exclusive rights as more and more people ask the politician for the same favor, etc. Formally, q = Q(x,n), Q > 0, Q_n ≤ 0. The population at large consists of N people who are indexed i; this index is continuously distributed over [0,N] and ranked according to the willingness to bribe, u_i = u(Q(x,n),i), such that u'_i > u'_j for i < j and i, j ∈ [0,N]. Let the index m denote the man who pays the marginal bribe if the politicians ask each individual for bribe b; that is, the type m is indifferent between paying the bribe or refusing to pay:

\[ m: u(Q(x,m),m) = b \]  \hspace{1cm} (A1)

The basic logic behind the people’s decision and the determination of m according to (A1) is sketched in Figure A1.

Therefore, all people i < m will actually pay the bribe, and the total amount collected by the politicians, denoted B, is

\[ B = bm \]  \hspace{1cm} (A2)

From the implicit function theorem, it follows that there exists a function M(x,b) that solves (A1), and its partial derivatives are given by:

\[ M_i = -(u_Q Q_i)/(u_Q Q_m + u_m), \hspace{0.5cm} M_b = 1/(u_Q Q_m + u_m) \]  \hspace{1cm} (A3)

Hence, M_i < 0 (i.e., higher bribes reduce the number of people who pay bribes) and M_b > 0 (i.e., popularity increases the number of people willing to bribe).

From the perspective of the model, the dependence of on popularity x is crucial. In order to investigate this aspect, further behavioral assumptions are needed, in particular, whether politicians collude or compete when setting the bribe. Let’s consider the extreme case of collusive and unscrupulous politicians. Unscrupulous means that the moral costs to the politician are negligible (or constant, which will not affect the decisions taken). Moreover, no other costs incur to the politician according to the framework in Section 2. Hence, collusive politicians act like a cartel and choose b such that B in (A2) is maximized. Let B denote the associated maximum revenues to the politicians that depend on popularity:

\[ B(x) := \max_b bM(x,b) \]  \hspace{1cm} (A4)

Application of the envelope theorem yields

\[ B_x = bM'_x \]  \hspace{1cm} (A5)

so that B_x > 0.

REFERENCES