Control of industrial robots

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• Solutions and answers can be given either in English or in Italian.
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Exercise 1
Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:

1.1 Find the expression of the inertia matrix $B(q)$ of the manipulator.

Denavit-Hartenberg frames can be defined as sketched in the picture.

Computations of the Jacobians:

**Link 1**

$$J_{p1}^{(l_1)} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad J_{P2}^{(l_1)} = \begin{bmatrix} a_1 c_1 - d_2 s_1 \\ a_1 s_1 + d_2 c_1 \\ 0 \end{bmatrix}, \quad p_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

**Link 2**

$$J_{p2}^{(l_2)} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad J_{p_2}^{(l_2)} = \begin{bmatrix} a_1 c_1 - d_2 s_1 \\ a_1 s_1 + d_2 c_1 \\ 0 \end{bmatrix}, \quad p_2 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad z_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

Auxiliary vectors for the above computations:

$$p_{l1} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, \quad p_{l2} = \begin{bmatrix} a_1 c_1 - d_2 s_1 \\ a_1 s_1 + d_2 c_1 \\ 0 \end{bmatrix}, \quad p_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

Inertia matrix:

$$B(q) = m_1 J_{p1}^{(l_1)^T} J_{p1}^{(l_1)} + I_1 J_{O1}^{(l_1)} J_{O1}^{(l_1)^T} J_{P1}^{(l_1)^T} J_{p1}^{(l_1)} + m_2 J_{p2}^{(l_2)^T} J_{p2}^{(l_2)} + m_2 J_{p_2}^{(l_2)} J_{p_2}^{(l_2)^T} = m_1 \begin{bmatrix} l_1^2 & 0 \\ 0 & 0 \end{bmatrix} + I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} a_1^2 + d_2^2 & a_1 \\ a_1 & 1 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

with:
\[ b_{11} = m_i l_i^2 + I_1 + m_2 \left( a_i^2 + d_2^2 \right) \]
\[ b_{12} = m_2 a_i \]
\[ b_{22} = m_2 \]

1.2 Compute the matrix \( C(q, \dot{q}) \) of the Coriolis and centrifugal terms \(^1\) for this manipulator.

The only derivative in the Christoffel symbols which is different from zero is:
\[ \frac{\partial b_{11}}{\partial q_2} = 2m_2 d_2 \]

Therefore:
\[ c_{111} = 0 \]
\[ c_{211} = -\frac{1}{2} \frac{\partial b_{11}}{\partial q_2} = m_2 d_2 \]
\[ c_{112} = c_{121} = \frac{1}{2} \frac{\partial b_{11}}{\partial q_2} = m_2 d_2 \]
\[ c_{122} = 0 \]
\[ c_{222} = 0 \]

The matrix of the Coriolis and centrifugal terms is thus:
\[
C = \begin{bmatrix}
  c_{11} & c_{12} \\
  c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
  c_{11} = c_{111} q_1 + c_{112} \dot{q}_2 = m_2 d_2 \dot{d}_2 \\
  c_{12} = c_{121} q_1 + c_{122} \dot{q}_2 = m_2 d_2 \dot{\theta}_1 \\
  c_{21} = c_{211} q_1 + c_{212} \dot{q}_2 = -m_2 d_2 \dot{\theta}_1 \\
  c_{22} = c_{221} q_1 + c_{222} \dot{q}_2 = 0
\end{bmatrix}
\]

1.3 Show that matrix \( N(q, \dot{q}) = B(q) - 2C(q, \dot{q}) \) is skew symmetric.

\[
N(q, \dot{q}) = B(q) - 2C(q, \dot{q}) = \begin{bmatrix}
  2m_2 d_2 \dot{d}_2 & 0 \\
  0 & 0
\end{bmatrix} - 2 \begin{bmatrix}
  m_2 d_2 \dot{d}_2 & m_2 d_2 \dot{\theta}_1 \\
  -m_2 d_2 \dot{\theta}_1 & 0
\end{bmatrix} = \begin{bmatrix}
  0 & -2m_2 d_2 \dot{\theta}_1 \\
  2m_2 d_2 \dot{\theta}_1 & 0
\end{bmatrix}
\]

which is a skew-symmetric matrix.

1.4 Explain where the property that matrix \( N(q, \dot{q}) \) is skew symmetric is used in robotics.

This property is mainly used in the proof of stability of some centralized control schemes, like the PD + gravity compensation controller.

\(^1\) The general expression of the Christoffel symbols is: \( c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right) \)
**Exercise 2**

2.1 Write the parametric expression of a segment in space, used for planning a linear path.

\[ p(s) = p_i + \frac{s}{\|p_f - p_i\|} (p_f - p_i) \]

where \( p_i, p_f \) are the initial and final points, respectively, and \( s \) is the natural coordinate along the path.

2.2 Write the parametric expression of a circumference in space, used for planning a circular path.

\[ p(s) = c + R p'(s) \]

where:

\[ p'(s) = \begin{bmatrix} \rho \cos(s/\rho) \\ \rho \sin(s/\rho) \\ 0 \end{bmatrix} \]

\( c \) is the position of the center of the circumference, \( \rho \) its radius, \( R \) the rotation matrix characterizing a frame with origin in the center of the circumference and such that the circumference is in the \((x, y)\) plane.

2.3 Define the problem of path planning with obstacle avoidance, making reference to the concept of configuration space of the robot.

The configuration space (C-space) is a space where the robot is at each time instant represented as a mobile point. Usually the joint space is used as a C-space. We then call \( C_{\text{free}} \) the subset of C-space that does not cause collisions with the obstacles.

Assume now that the initial and final postures of the robot in the workspace are mapped into corresponding configurations in the C-space, respectively a start configuration \( q_s \) and a goal configuration \( q_g \). Planning a collision-free motion for the robot means to generate a path from \( q_s \) to \( q_g \) if they belong to the same connected component of \( C_{\text{free}} \), and to report a failure otherwise.

2.4 With reference to the following pictures, briefly explain what are the most popular probabilistic methods and how they work.
Probabilistic planners belong to the general family of sampling-based methods, where the basic idea is to randomly select a finite set of collision-free configurations that adequately represent how $C_{\text{free}}$ is connected, and using these configurations to build a roadmap.

At each iteration a sample configuration is chosen and it is checked whether it entails a collision between the robot and the obstacles: if yes, the configuration is discarded, otherwise it is added to the current roadmap.

Two versions of such randomized sampling-based approach are:

- **PRM (Probabilistic Roadmap):** it corresponds to the first pair of pictures. The idea is to build a roadmap randomly sampling the $C$-space and checking whether the sampled configuration is collision free and can be connected to a roadmap which is progressively being formed. After a certain number of iterations we try to verify whether the path planning problem can be satisfied by connecting $q_s$ and $q_g$ to the same connected component of the PRM by free local paths.

- **RRT (Rapidly-exploring Random Tree):** it corresponds to the second pair of pictures. The method makes use of a data structure called tree. A similar method as with the PRM to sample the $C$-space is used, but only the segment between the configuration $q_{\text{near}}$ in the tree (which is progressively formed) closest to the sampled one and a candidate configuration at a certain distance from $q_{\text{near}}$ is checked to be collision free and in case added to the tree. To speed up the search, two trees are expanded, rooted at $q_s$ and $q_g$, respectively. After a certain number of expansion steps, the algorithm tries to connect the two trees and thus to complete the path.

### Exercise 3

3.1 Sketch the block diagram of the current control of a DC motor.

![Block Diagram](image)

3.2 Explain why the electrical dynamics and the electrical parameters of the model can be neglected in the design of the speed controller for a servomechanism.

Since the electrical dynamics is quite fast (compared with the mechanical one) we can design the current controller $R(s)$ so as to achieve a wide bandwidth (thousands rad/s). In the design of the current controller the e.m.f. can be considered as a slowly varying disturbance, that the controller can effectively reject. Once the current control loop is closed, it can be seen as practically instantaneous from the external position/speed controller, so that:

$$\tau_m(t) = K I(t) \approx K I^o(t)$$

This is why in the design of the speed controller for a servomechanism the electrical dynamics and the electrical parameters of the model can be neglected.

3.3 Take now the following values of the physical parameters:

$$J_m = 0.02 \text{ Kg m}^2$$
\[ D_m = 0 \]
\[ J_l = 3 \text{ Kg m}^2 \]
\[ n = 10 \]

Compute the gain \( K_{pv} \) of the PI speed controller so as to obtain a crossover frequency \( \omega_{cv} \approx 200 \text{ rad/s} \).

The load inertia reflected at the motor axis is:

\[ J_{lr} = \frac{J_l}{n^2} = 0.03 \]

The gain of the system is:

\[ \mu = \frac{1}{\frac{J_m}{J_m + J_{lr}}} = 20 \]

We use the following tuning formula:

\[ K_{pv} \mu = \omega_{cv} \]

Then:

\[ K_{pv} = \frac{\omega_{cv}}{\mu} = 10 \]

3.4 Compute now (approximately) the minimum value of the stiffness constant of the joint (\( K_{el} \)) such that the previous controller design can be considered robust with respect to the joint elasticity.

A rule of thumb to design the speed controller for an elastic servomechanism is:

\[ \omega_{cv} \leq 0.7 \omega_c = 0.7 \sqrt{\frac{K_{el}}{J_{lr}}} \]

Then:

\[ K_{el} \geq \frac{\omega_{cv}^2}{0.7^2} J_{lr} = \frac{4 \cdot 10^4}{49 \cdot 10^{-2}} \cdot 3 \cdot 10^{-2} = \frac{12}{49} \cdot 10^4 = 2449 \]

Exercise 4

4.1 Explain what do we mean with “adaptive” control.

When the equations of the dynamic models are reasonably known, but there is uncertainty on the parameters of the model itself, it is possible to design an adaptive controller. This is a model-based controller where the estimates of the dynamic parameters are updated during the operation of the controller (online).

4.2 Consider now this sketch of an adaptive controller:
The block denoted with $Y$ stands for and what is the property of the dynamic model of the manipulator, related to $Y$, that is exploited in the adaptive control.

The block denoted with $Y$ denotes a regressor. It makes reference to a property of the dynamic model of the manipulator that can be expressed in a linear form with respect to a set of dynamic parameters. The regressor is the matrix (function of positions, velocities, and accelerations) that once multiplied by the set of dynamic parameters returns the joint torques:

$$B(q)\dot{q} + C(q, \dot{q})\ddot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\pi = \tau$$

4.3 Explain what is the adaptation rule of the estimates of the parameters in the adaptive controller.

The adaptation rule is the following differential equation:

$$\dot{\pi} = K_\pi^{-1} Y^T(q, \dot{q}, \ddot{q}, \dot{\pi})\sigma$$

where $K_\pi$ is a gain matrix while the definitions of $q$, $\dot{q}$, and $\sigma$, can be taken from the block diagram.

4.4 Consider now a manipulator described by the equations:

$$B(q)\dot{q} + C(q, \dot{q})\ddot{q} + g(q) = \tau$$

and a scalar function $V(q, \dot{q}) = \frac{1}{2} q^T B(q) \dot{q}.$

Write the expression of the derivative of $V$ with respect to time, $\dot{V}$, along the trajectories of the system.

$$\dot{V} = q^T B(q) \dddot{q} + \frac{1}{2} q^T B(q) \dot{q}$$

$$= \frac{1}{2} q^T (B(q) - 2C(q, \dot{q})) \dot{q} + q^T (\tau - g(q)) = q^T (\tau - g(q))$$

(thanks to a known property of the dynamic model of the robot).