Control of industrial robots

Centralized control

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Centralized control means that the controller, when determining the input (torque/force) to each joint, takes into account measurements and/or models related to the other joints of the manipulator.

In general a centralized controller requires knowledge, at least a partial one, of the mathematical model of the manipulator.

In the absence of the decoupling effect given by the high reduction ratios (for instance in case of direct drive motors), the use of a centralized control strategy might turn out to be the only viable solution.

In the following we will discuss a few strategies for centralized control, both under the assumption of perfect knowledge of the model and when some uncertainty has to be taken into account.
Clear improvements with respect to standard non model-based controllers can be achieved:
Taxonomy

Joint space
- Open loop $\Rightarrow$ Computed torque control
- Closed loop
  - Regulation problem $\Rightarrow$ PD + gravity compensation
  - Trajectory tracking problem
    - Nominal conditions $\Rightarrow$ Inverse dynamic control
    - With uncertainty $\Rightarrow$ Robust control & Adaptive control

Operational space
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Computed torque control

In this scheme, the decentralized controller is complemented by a controller that operates in open loop, computing the disturbance torques based on the mathematical model, fed by the position reference signal and its derivatives.

\[
d_d = N^{-1} \Delta B(q_d) N^{-1} \ddot{q}_{md} + N^{-1} C(q_d, \dot{q}_d) N^{-1} \dot{q}_{md} + N^{-1} g(q_d)
\]

- Compensation of nonlinearities can be only partial (for instance just the gravitational terms and the diagonal terms of the inertia matrix)
- Compensation terms can be computed offline, in case of trajectories repeated several times
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### Operational space
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PD plus gravity compensation

Let us consider a manipulator whose equations are:

\[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \]

Assume that a constant equilibrium posture \( q_d \) is assigned (regulation problem).

We want to find a control law which guarantees the global asymptotic stability of the equilibrium point using Lyapunov method.

Let the state be defined as:

\[
\begin{bmatrix}
\tilde{q}^T \\
\dot{q}^T
\end{bmatrix}, \quad \tilde{q} = q_d - q
\]

As a candidate Lyapunov function we take:

\[
V(\tilde{q}, \dot{q}) = \frac{1}{2} \dot{q}^T B(q)\dot{q} + \frac{1}{2} \tilde{q}^T K_P \tilde{q}
\]

If \( K_P \) is symmetric positive definite, function \( V \) is positive definite as well.
Let’s take the derivative with respect to time of function $V$, taking into account that $q_d$ is constant and using the dynamic model:

$$
\dot{V} = \dot{q}^T B(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{B}(q) \dot{q} - \dot{q}^T K_P \ddot{q}
$$

$$
= \frac{1}{2} \dot{q}^T \left( \ddot{B}(q) - 2C(q, \dot{q}) \right) \dot{q} + \dot{q}^T \left( \tau - g(q) - K_P \ddot{q} \right)
$$

The first term is zero due to a property of the dynamic model of the manipulator which we have already proven (skew symmetry of matrix $\dot{B} - 2C$).
Consider now the following choice for the control law:

$$
\tau = g(q) + K_P \ddot{q} - K_D \dot{q}
$$

with $K_D$ symmetric and positive definite. We obtain:

$$
\dot{V} = -\dot{q}^T K_D \dot{q} \leq 0
$$

Then the derivative of $V$ is negative semidefinite.
In order to draw a conclusion about the stability of the equilibrium state, we need to study the trajectories of the system when $V = 0$. In closed loop it is:

$$ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = g(q) + K_P \ddot{\tilde{q}} - K_D \dot{\tilde{q}} $$

Notice that when $V=0$ it results $\ddot{q} \equiv 0$, $\dot{\ddot{q}} \equiv 0$, and then the only trajectory which complies with the system’s equations is characterized by:

$$ K_P \ddot{\tilde{q}} = 0 $$

or:

$$ \ddot{\tilde{q}} = 0 $$

This is enough to prove that the equilibrium state characterized by:

$$ \ddot{\tilde{q}} = 0, \quad \dot{\tilde{q}} = 0 $$

(robot at rest with zero position error) is globally asymptotically stable.
The control law obtained with the Lyapunov method can be interpreted as a PD control in joint space plus a term that compensates for gravitational effects.

- The theoretical result is of course valid as long as perfect gravity compensation is achieved.
- For a gravity free (or gravity compensated) manipulator a joint space PD control yields an asymptotically stable equilibrium state: the proportional gains can be seen as virtual springs while the derivative gains as virtual dampers.
PD plus gravity compensation

The PD plus gravity compensation allows to demonstrate an asymptotic result. We do not have control on the time history with which the error goes to zero:

If we need to have a better control on the time evolution of the error, or if we want to track a time-varying reference position, we need to turn to more complicated schemes.
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Inverse dynamics control

Consider again the dynamic model of the manipulator:

\[ B(q) \ddot{q} + n(q, \dot{q}) = \tau \]

where centrifugal, Coriolis and gravitational terms have been conveniently gathered in vector \( n \):

\[ n(q, \dot{q}) = C(q, \dot{q}) \dot{q} + g(q) \]

Assuming perfect knowledge of the dynamic model, we can take into consideration the control law (inverse dynamics):

\[ \tau = B(q) \dot{y} + n(q, \dot{q}) \]

Since matrix \( B \) is full rank for every robot configuration, the application of the above control law yields:

\[ \ddot{q} = y \]

the system dynamics has then been made linear, completely decoupled and composed of \( n \) double integrators (\( n \) unitary masses!)
The decoupled system:

$$\ddot{q} = y$$

can be easily controlled through a **decoupled PD controller**, to which a feedforward acceleration term can be added:

$$y = K_P \ddot{q} + K_D \dot{q} + \ddot{q}_d$$

$q_d$ is time varying

The following closed loop equation is obtained:

$$\dddot{q} + K_D \dot{q} + K_P \ddot{q} = 0$$

Then the error is governed by a second order dynamics that can be arbitrarily assigned, on each joint, by suitably selecting the gains in the diagonal matrices $K_P$ and $K_D$.

The inverse dynamics control is a special case of a more general methodology for controlling nonlinear systems called **feedback linearization**.
Inverse dynamics control

With the inverse dynamics control we have full control on the time history with which the error goes to zero:

\[ \tilde{q}_i(0) \quad \tilde{q}_i \]

Assuming matrices \( K_P \) and \( K_D \) diagonal, the time evolution of the error is governed by eigenvalues that are roots of the polynomial:

\[ s^2 + K_{Di}s + K_{Pi} = 0 \]
Inverse dynamics control

Inverse dynamics control can be represented through the following block diagram:

- Compensation terms must be computed online, with small sampling times \((\leq 1 \text{ ms})\)
- The recursive NE method is used: \(\tau = \text{NE}(q, \dot{q}, y)\)
- The method requires perfect cancellation of the terms of the dynamic model, which might be difficult in practice
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Robust control

In a realistic scenario we have to assume that the compensation of the dynamic model is not perfect. The inverse dynamics control can then be expressed as:

$$\tau = \hat{B}(q)y + \hat{n}(q,\dot{q})$$

which yields:

$$B(q)\ddot{q} + n(q,\dot{q}) = \hat{B}(q)y + \hat{n}(q,\dot{q})$$

Let us express the uncertainty of the model in the following terms:

$$\tilde{B} = \hat{B} - B, \quad \tilde{n} = \hat{n} - n$$

Since matrix $B$ is invertible, we obtain:

$$\ddot{q} = y + \left(B^{-1}\hat{B} - I\right)y + B^{-1}\tilde{n} = y - \eta$$

with:

$$\eta = \left(I - B^{-1}\hat{B}\right)y - B^{-1}\tilde{n}$$
Adopting the same control law for $y$ as in the ideal case:

$$y = K_P \ddot{q} + K_D \dot{q} + \ddot{q}_d$$

we obtain that the error dynamics is governed by the equation:

$$\ddot{q} + K_D \dot{q} + K_P q = \eta$$

which means that the system is still nonlinear and coupled.

We need then to add to the PD linear control an additional nonlinear term, function of the error, conceived to yield robustness to the project.

This term will be designed based on Lyapunov theory.
Robust control

From the equation:
\[ \ddot{q} = y - \eta \]

the expression for the second derivative of the error is obtained:
\[ \dddot{q} = \ddot{q}_d - y + \eta \]

Defining the system state as:
\[ \xi = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}^T \]

we obtain the matrix first order differential equation:
\[ \dot{\xi} = H\xi + D(\ddot{q}_d - y + \eta) \]

\[
H = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \quad D = \begin{bmatrix} 0 \\ I \end{bmatrix} \in \mathbb{R}^{2n \times n}
\]

the system is nonlinear and time-varying.
In the following we will use the concept of norm of a vector and of a matrix. Given a vector $\mathbf{x}$ we define the Euclidean norm as:

$$\|\mathbf{x}\| = \sqrt{\mathbf{x}^T \mathbf{x}}$$

Consider now a matrix $\mathbf{A}$, in general a rectangular matrix. We define norm of $\mathbf{A}$, induced by the definition of the norm of the vector, the quantity:

$$\|\mathbf{A}\| = \sup_{\mathbf{x} \neq 0} \frac{\|\mathbf{A}\mathbf{x}\|}{\|\mathbf{x}\|} = \max_{\|\mathbf{x}\|=1} \|\mathbf{A}\mathbf{x}\|$$

It can be proven that the norm of the matrix coincides with the maximum singular value of the matrix itself, i.e.:

$$\|\mathbf{A}\| = \sqrt{\lambda_{\text{max}}(\mathbf{A}^T \mathbf{A})}$$
Robust control

We characterize the uncertainty as follows:

\[ a) \quad \sup_{t \geq 0} \| \ddot{q}_d \| < Q_M < \infty \quad \forall \ddot{q}_d \]
\[ b) \quad \left\| I - B^{-1}(q) \hat{B}(q) \right\| \leq \alpha < 1 \quad \forall q \]
\[ c) \quad \| \tilde{n} \| \leq \Phi(\| \xi \|) \quad \forall q, \dot{q} \]

Assumption \( a) \) is related to the fact that trajectory planning guarantees a limited acceleration. Assumption \( b) \) is consistent with the fact that matrix \( B \) (and then \( B^{-1} \) too) is upper and lower bounded. Then it results:

\[ 0 < B_m \leq \| B^{-1}(q) \| \leq B_M \leq \infty, \quad \forall q \]

If we set, for instance:

\[ \hat{B} = \frac{2}{B_M + B_m} I \]
\[ \text{we obtain:} \quad \| B^{-1} \hat{B} - I \| \leq \frac{B_M - B_m}{B_M + B_m} = \alpha < 1 \]

Assumption \( c) \) concerns centrifugal, Coriolis and gravitational terms. We can take for \( \Phi \) a quadratic function in the norm of the state:

\[ \Phi(\| \xi \|) = \alpha_0 + \alpha_1 \| \xi \| + \alpha_2 \| \xi \|^2 \]
Let’s take now as a control law the following expression:

\[ y = \ddot{q} + K_P \dot{q} + K_D \dot{q} + w \]

where the new term \( w \) will be designed in a way to counteract uncertainty. Letting \( K = [K_P \ K_D] \), we obtain:

\[ \dot{\xi} = \tilde{H} \xi + D(\eta - w) \]

where:

\[ \tilde{H} = (H - DK) = \begin{bmatrix} 0 & I \\ -K_P & -K_D \end{bmatrix} \]

is the matrix, with all eigenvalues in the left half plane, of the closed loop nominal dynamics. If \( \eta = 0 \), it is enough to set \( w = 0 \) in order to return to the nominal case.

For the definition of the vector \( w \) we proceed with the Lyapunov method.
Consider the following Lyapunov candidate:

\[ V(\xi) = \xi^T Q \xi > 0, \quad \forall \xi \neq 0 \]

where \( Q \) is a symmetric positive definite matrix. Taking the derivative along the trajectories of the system we obtain:

\[
\dot{V} = \dot{\xi}^T Q \xi + \xi^T Q \dot{\xi} = \\
= \xi^T (\tilde{H}^T Q + Q \tilde{H}) \xi + 2 \xi^T Q D (\eta - w) = \\
= -\xi^T P \xi + 2 z^T (\eta - w)
\]

where we have set:

\[ z = D^T Q \xi \]

and we have exploited the fact that, since \( \tilde{H} \) has all eigenvalues with negative real parts, whatever positive definite matrix \( P \) is chosen, the equation:

\[
\tilde{H}^T Q + Q \tilde{H} = -P
\]

has only one positive definite solution \( Q \).
Robust control

Let’s adopt for $$w$$ the expression:

$$w = \rho\left(\|\xi\|\right)\frac{z}{\|z\|}, \quad \rho > 0$$

$$\rho$$ is a positive function of the norm of the state, to be determined. With this choice of $$w$$ we obtain:

$$z^T(\eta - w) = z^T\eta - \rho(\|\xi\|)\frac{z^Tz}{\|z\|}$$

$$\leq \|z\|\|\eta\| - \rho(\|\xi\|)\|z\| =$$

$$= \|z\|(\|\eta\| - \rho(\|\xi\|))$$

If we then guarantee that:

$$\rho(\|\xi\|) > \|\eta\| \quad \forall q, \dot{q}, \ddot{q}_d$$

we obtain that this term, and then $$\dot{V}$$ too, is negative.
Robust control

Remember that:

\[ \eta = (I - B^{-1}\hat{B})y - B^{-1}\tilde{n} \]

\[ y = \ddot{q}_d + K_P \dot{q} + K_D \dot{q} + w \]

It follows that:

\[ \|\eta\| \leq \|I - B^{-1}\hat{B}\| (\|\ddot{q}_d\| + \|K\|\|\xi\| + \|w\|) + \|B^{-1}\|\|\tilde{n}\| \]

\[ \leq \alpha Q_M + \alpha \|K\|\|\xi\| + \alpha \rho(\|\xi\|) + B_M \Phi(\|\xi\|) \]

\[ < \rho(\|\xi\|) \]

We might then select function \( \rho \) in such a way that:

\[ \rho(\|\xi\|) \geq \frac{1}{1 - \alpha} \left( \alpha Q_M + \alpha \|K\|\|\xi\| + B_M \Phi(\|\xi\|) \right) \]
Robust control

Notice that since:

$$\Phi(\|\xi\|) = \alpha_0 + \alpha_1 \|\xi\| + \alpha_2 \|\xi\|^2$$

in order to satisfy the previous inequality it will be enough to select:

$$\rho(\|\xi\|) = \beta_0 + \beta_1 \|\xi\| + \beta_2 \|\xi\|^2$$

with:

$$\beta_0 \geq \frac{\alpha Q_M + \alpha_0 B_M}{1-\alpha}, \quad \beta_1 \geq \frac{\alpha \|K\| + \alpha_1 B_M}{1-\alpha}, \quad \beta_2 \geq \frac{\alpha_2 B_M}{1-\alpha}$$

In this way:

$$\dot{V} = -\xi^T P \xi + 2z^T \left( \eta - \rho(\|\xi\|) \frac{z}{\|z\|} \right) < 0, \quad \forall \xi \neq 0$$

\(\xi = 0\) is a globally asymptotically stable equilibrium state.
The resulting control scheme is the following one:
Robust control

The control law is then composed of three terms:

1. \( \hat{B}(q)y + \hat{n}(q, \dot{q}) \) approximately compensates for the nonlinear terms

2. \( \ddot{q}_d + K_P \ddot{q} + K_D \dot{q} \) stabilizes the nominal dynamic system in the error

3. \( w = \rho(\|\xi\|) \frac{z}{\|z\|} \) gives robustness, counteracting the uncertainty

In order to avoid high frequency switching of the control variable (so called chatter) the third term can be approximated as:

\[
\begin{cases}
\rho(\|\xi\|) \frac{z}{\|z\|} & \text{for } \|z\| \geq \varepsilon \\
\frac{\rho(\|\xi\|)}{\varepsilon} z & \text{for } \|z\| < \varepsilon
\end{cases}
\]
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Adaptive control

As an alternative to the robust control, of particular interest when the equations of the dynamic models are reasonably known, but there is uncertainty on the parameters of the model itself, it is possible to design an adaptive control.

The adaptive control is based on the linearity of the dynamic model of the manipulator with respect to dynamic parameters:

\[ B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})\pi = \tau \]

where \( \pi \) is a suitable constant vector of uncertain dynamic parameters (kinematic parameters are assumed to be known without uncertainty).

Apparently a technique based on this relation should make use of measures or estimates of joint accelerations in order to compute the regressor \( Y \), which of course would considerably limit its practical use.

The technique we are going to discuss in the following, elaborated by Slotine and Li, solves the problem without information on the accelerations and without the inversion of the inertia matrix of the manipulator.
Consider first the following control law, where we assume that the dynamic model is known without uncertainty:

$$\tau = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + g(q) + K_D \sigma$$

where $K_D$ is a positive definite matrix. Let us define:

$$\dot{q}_r = \dot{q}_d + \Lambda \ddot{q} \quad \ddot{q}_r = \ddot{q}_d + \Lambda \ddot{q}$$

with $\Lambda$ positive definite (can be chosen as a diagonal matrix).

If we now set:

$$\sigma = \dot{q}_r - \dot{q} = \ddot{q} + \Lambda \ddot{q}$$

the term $K_D \sigma$ corresponds to a PD action on the error. Altogether the application of the control law implies:

$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + K_D \sigma = 0$$
Assume as a candidate Lyapunov function the following expression:

\[ V(\sigma, \dot{q}) = \frac{1}{2} \sigma^T B(q) \sigma + \frac{1}{2} \dot{q}^T M \dot{q} > 0 \quad \forall \sigma, \dot{q} \neq 0 \]

with \( M \) positive definite matrix. Taking the derivative with respect to time:

\[ \dot{V} = \sigma^T B(q) \dot{\sigma} + \frac{1}{2} \sigma^T B(q) \dot{\sigma} + \dot{\sigma}^T M \dot{q} = \]

\[ = \frac{1}{2} \sigma^T \left[ -2C(q, \dot{q}) - 2K_D \right] \sigma + \frac{1}{2} \sigma^T \dot{B}(q) \sigma + \dot{\sigma}^T M \dot{q} = \]

\[ = -\sigma^T K_D \sigma + \dot{\sigma}^T M \dot{q} \]

where the skew-symmetric property of matrix \( B - 2C \) has been exploited. Setting \( M = 2\Lambda K_D \) we obtain:

\[ \dot{V} = -\dot{\sigma}^T K_D \dot{\sigma} - \dot{\sigma}^T \Lambda K_D \Lambda \dot{\sigma} < 0 \quad \forall \dot{\sigma}, \dot{\sigma} \neq 0 \]

The state \( [\dot{q}^T, \sigma^T]^T = 0 \) is then a globally asymptotically stable equilibrium.
Consider now a control law based on estimates of parameters:

\[
\tau = \hat{B}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{g}(q) + K_D\sigma = \\
= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\pi} + K_D\sigma
\]

Notice that \( Y \) does not depend on joint accelerations. By substituting in the dynamic model:

\[
B(q)\ddot{q} + C(q, \dot{q})\dot{q} + K_D\sigma = -\tilde{B}(q)\ddot{q}_r - \tilde{C}(q, \dot{q})\dot{q}_r - \tilde{g}(q) = \\
= -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\pi}
\]

where:

\[
\tilde{B} = \hat{B} - B \quad \tilde{C} = \hat{C} - C \quad \tilde{g} = \hat{g} - g \quad \tilde{\pi} = \hat{\pi} - \pi
\]
Let’s modify the candidate Lyapunov function:

\[ V(\sigma, \tilde{q}) = \frac{1}{2} \sigma^T B(q) \sigma + \tilde{q}^T K_D \tilde{q} + \frac{1}{2} \pi^T K_{\pi} \pi > 0 \quad \forall \sigma, \tilde{q}, \pi \neq 0 \]

where \( K_{\pi} \) is a positive definite matrix. Taking the derivative with respect to time:

\[ \dot{V} = -\tilde{q}^T K_D \dot{q} - \tilde{q}^T \Delta K_D \Delta \tilde{q} + \pi^T \left( K_{\pi} \dot{\pi} - Y^T (q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma \right) \]

from the previous case due to the uncertainty

If we adopt the following adaptation law of the parameters:

\[ \dot{\pi} = K_{\pi}^{-1} Y^T (q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma \]

it results, taking into account that \( \pi \) is constant:

\[ \dot{V} = -\tilde{q}^T K_D \dot{q} - \tilde{q}^T \Delta K_D \Delta \tilde{q} \]

Thus \( \tilde{q} \) and \( \dot{q} \) asymptotically (and globally) converge to zero.
From the relation:

\[ B(q)\dot{\sigma} + C(q,\dot{q})\dot{\sigma} + K_D \sigma = -Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{\pi} \]

we obtain that:

\[ Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\hat{\pi} - \pi) \rightarrow 0 \]

This does not necessarily imply that \( \hat{\pi} \) tends to \( \pi \), which depends on the structure of matrix \( Y \).

It is actually a **direct adaptive control**, aimed at finding an effective control law, rather than at identifying the model parameters.
Adaptive control

This is the resulting control scheme:
Adaptive control

We can identify three elements in the control law:

1. $Y\hat{\pi}$ can be interpreted as an approximate inverse dynamic control

2. $K_D\sigma$ PD-like stabilizing action on the error

3. The vector of the estimates of parameters $\pi$ is updated following a gradient technique. Matrix $K_\pi$ determines the speed of convergence of the estimates.

- Compared to the robust control scheme we have worse performance in the presence of model errors or of partial representation of the dynamic model.
- On the other hand we have more regular control actions than those obtained with the robust control, which are characterized by potentially dangerous high frequency switching.
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Operational space control

- In the operational space control, the error is directly formed on the operational (Cartesian) space coordinates.

- The trajectory generated in the operational space is not subject to kinematic inversion. On the other hand, the measures of the variables in the operational space are actually the result of direct kinematics computations on the only available measures, i.e., on the joint coordinates measures.

- We will consider only the case of non-redundant robots not incurring in singular configurations (full rank Jacobian)
PD plus gravity compensation

As with the joint space, assume that a constant posture $x_d$ is assigned. Our goal is to find a control law which ensures the global asymptotic stability of the equilibrium state, using the Lyapunov method.

Let’s define the error in the operational space:

$$\tilde{x} = x_d - x$$

As a candidate Lyapunov function we take:

$$V(\tilde{x}, \dot{q}) = \frac{1}{2} \dot{q}^T B(q) \dot{q} + \frac{1}{2} \tilde{x}^T K_P \tilde{x}$$

By selecting a symmetric positive definite matrix $K_P$, also function $V$ is positive definite.
PD plus gravity compensation

Taking the derivative of function $V$:

$$\dot{V} = \dot{q}^T B(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{B}(q) \dot{q} + \ddot{x}^T K_P \ddot{x}$$

As $x_d$ is constant we have:

$$\ddot{x} = -J_A(q) \dot{q}$$

and thus:

$$\dot{V} = \dot{q}^T B(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{B}(q) \dot{q} - \dot{q}^T J_A^T(q) K_P \ddot{x}$$

$$= \frac{1}{2} \dot{q}^T (\dot{B}(q) - 2C(q, \dot{q})) \dot{q} + \dot{q}^T (\tau - g(q) - J_A^T(q) K_P \ddot{x})$$

The first term is zero due to the skew-symmetric property of matrix $\dot{B} - 2C$. 
Consider now the following control law:

$$\tau = g(q) + J_A^T(q)K_P \ddot{x} - J_A^T(q)K_D J_A(q)q$$

where $K_D$ is positive definite. We obtain:

$$\dot{V} = -\dot{q}^T J_A^T(q)K_D J_A(q)q \leq 0$$

Thus the derivative of $V$ is negative semidefinite. With a similar argument as the one used in joint space, we conclude that the trajectories compatible with $\dot{V} = 0$ are characterized by:

$$J_A^T(q)K_P \ddot{x} = 0$$

If the Jacobian is full rank we then obtain:

$$\ddot{x} = x_d - x = 0$$

the equilibrium state characterized by zero error (in the operational space) is globally asymptotically stable.
The control law obtained with the Lyapunov method can be interpreted as a PD control in the operational space, to which a term is added in the joint space for compensation of gravitational effects:

\[ \begin{align*}
    K_D & \quad x \\
    J_A(q) & \quad \tau \\
    \text{MANIPULATOR} & \quad \dot{q}
\end{align*} \]

**PD plus gravity compensation**

- \( K_P \) and \( K_D \) diagonal
- \( x_d \) constant
Taxonomy

Joint space
- Open loop $\Rightarrow$ Computed torque control
- Closed loop
  - Regulation problem $\Rightarrow$ PD + gravity compensation
  - Trajectory tracking problem
    - Nominal conditions $\Rightarrow$ Inverse dynamic control
    - With uncertainty $\Rightarrow$ Robust control & Adaptive control

Operational space
- Closed loop
  - Regulation problem $\Rightarrow$ PD + gravity compensation
  - Trajectory tracking problem
    - Nominal conditions $\Rightarrow$ Inverse dynamic control
Inverse dynamics control

Consider again the dynamic model of the robot manipulator:

\[ B(q)\ddot{q} + n(q, \dot{q}) = \tau \]

where centrifugal, Coriolis, and gravitational terms have been gathered in \( n \), and remember that the inverse dynamics control law:

\[ \tau = B(q)y + n(q, \dot{q}) \]

transforms the system, assuming perfect knowledge of the model, in a system of double integrators:

\[ \ddot{q} = y \]

The problem is now to determine the new input \( y \) in such a way to allow tracking of a trajectory \( x_d(t) \), assigned in the operational space.

Notice that taking the derivative of the differential kinematics we have:

\[ \ddot{x} = \frac{d}{dt}\dot{x} = \frac{d}{dt}(J_A(q, \dot{q})\dot{q}) = J_A(q)\ddot{q} + J_A(q, \dot{q})\dot{q} \]
The previous relation suggests the following choice for $y$:

$$y = J_A^{-1}(q)\left(\ddot{x}_d + K_D \ddot{x} + K_P \ddot{x} - J_A(q, \dot{q})\dot{q}\right)$$

By substitution we obtain:

$$\dddot{x} + K_D \ddot{x} + K_P \dot{x} = 0$$

Then the error in the operational space is governed by a second order dynamics which can be arbitrarily assigned, on each Cartesian degree of freedom, by suitably selecting the elements of the diagonal matrices $K_P$ e $K_D$.

Notice that the method needs computation of the inverse of the Jacobian matrix. As such it can be applied only to non-redundant robots in nonsingular configurations.
The inverse dynamics in the operational space can be represented by the following block diagram: