Control of industrial robots

Decentralized control

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Introduction

- Once the desired motion of the robot has been computed, a real time controller ensures that this motion is tracked in the best possible way.
- All industrial robots are in fact endowed with such a real time motion controller.
- The industrial robot control follows the independent joint control approach, i.e. it is a purely decentralized, non model based, control system.
- We will discuss the rationale behind this approach and recognize it as the problem of controlling several independent servomechanisms.
The motion controller is one of the functional units of a robot controller:

We will now consider the lowest level part of the robot control system, directly interacting with the robot internal sensors and actuators.
What are possible criteria to evaluate the performance of a motion control system?

- **Quality** of motion in nominal conditions
  - accuracy/repeatability
  - speed of task execution
  - energy saving

- **Robustness** of motion in perturbed conditions
  - adaptation to the environment
  - high repeatability in spite of uncertainties in modelling errors

From “Robotics 2”, Prof. Alessandro De Luca, University of Rome “La Sapienza”
Precise and high-speed motion control

https://www.youtube.com/watch?v=5ndaQwn15ng
Let us consider the dynamic model of the manipulator:

\[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \]

Assume that a motor acts on each joint of the manipulator. A simplified way to account for the dynamics of such motors is to consider just the effect related to the spinning of the motor around its own axis.
Decentralized model

The dynamic model of the motor is thus simply:

\[ J_{mi} \ddot{q}_{mi} + D_{mi} \dot{q}_{mi} = \tau_{mi} - \tau_{lmi} \quad i = 1, \ldots, n \]

where \( J_{mi} \) and \( D_{mi} \) are the moment of inertia and the coefficient of viscous friction of the motor, respectively, while \( \tau_{lmi} \) is the load torque at the axis of motor \( i \), equal to:

\[ \tau_{lmi} = \tau_i / n_i \]

\( n_i \) is the **reduction ratio** of the transmission between the \( i \)_th motor and the joint.

Assuming that the transmission elements are **rigid**, we also have:

\[ q_{mi} = n_i q_i \]
We now decompose the inertia matrix into the sum of a diagonal and constant term ("average" inertias) and a residual term:

\[ B(q) = \bar{B} + \Delta B(q) \]

If we set:

\[
J_m = \text{diag}\{J_{mi}\}, \quad D_m = \text{diag}\{D_{mi}\}, \quad N = \text{diag}\{n_i\}, \quad \bar{B}_r = N^{-1}\bar{B}N^{-1}
\]

re-organizing the equations we have:

\[
(F_m + \bar{B}_r)m + D_m \dot{q}_m + d = \tau_m
\]

or:

\[
(J_{mi} + \bar{b}_{rii})\ddot{q}_{mi} + D_{mi} \dot{q}_{mi} + d_i = \tau_{mi}, \quad i = 1, \ldots, n
\]

where:

\[
d = N^{-1}\Delta B(q)N^{-1}\ddot{q}_m + N^{-1}C(q, \dot{q})N^{-1}\dot{q}_m + N^{-1}g(q)
\]

"average" inertias scaled by the squares of the transmission ratios:

\[
\bar{b}_{rii} = \frac{b_{ii}}{n_i^2}
\]
The equations we have obtained can be interpreted as those of a linear and completely decoupled system, subjected to a disturbance deriving from the nonlinear and coupled terms of the dynamic model.

The larger the reduction ratios $n_i$, the less relevant is the disturbance term.
Independent joint control

- The decentralized model of the robot dynamics is used in the independent joint control, a widely used approach in the industrial robot controllers.
- The control system is articulated in $n$ SISO control loops, ignoring the dynamic coupling effects which are dealt with as disturbances.
- The single control problems are assimilated to the control of a servomechanism.
- The method heavily relies on the large values of the reduction ratios adopted in robotics. Without this effect, neglecting the variability of the inertia and the mechanical couplings with the other joints would be poorly justified.
A **servomechanism** is a paradigmatic system composed of a motor, a mechanical load and whatever transmission system in between.

The problem is **to control the motion of the load**, suitably modulating the torque applied by the motor.

Different scenarios can be given as far as the **available sensors** are concerned: position/speed at the motor and/or load side.

However we usually have the motor current available as a measure too: we will see a clever way to make use of this measure, which greatly simplifies the motion control.
The electrical dynamics

Assume that a **DC motor** is used, whose electrical scheme can be simplified as follows:

- A voltage $V$ is applied which produces a current in the armature circuit through the dynamics induced by the resistance $R$ and the inductance $L$. A back e.m.f. $E$, proportional to the motor angular speed $\omega_m$, is also present.

- Thanks to the electro-mechanical conversion implemented by the brushes-commutator system, the motor delivers a torque $\tau_m$ which is proportional to the current $I$.

- In industrial robotics **brushless motors** are more commonly used, for which however a similar description can be given, referred to the quadrature axis.
The system is described by the following equations:

\[ V(t) = RI(t) + L\dot{I}(t) + E(t) \]

\[ E(t) = K\omega_m(t) \]

\[ \tau_m(t) = Kl(t) \]

\[ \tau_m(t) = J_m\dot{\omega}_m(t) \]

\[ \dot{q}_m(t) = \omega_m(t) \]

which can be expressed by this block diagram:

Notice that the back e.m.f. couples the electrical dynamics with the mechanical dynamics.
If a current measure is available, we might close a control loop around the current:

- since the electrical dynamics is quite fast (compared with the mechanical one) we will design the current controller $R_I(s)$ so as to achieve a wide bandwidth (thousands rad/s)
- in the design of the current controller the e.m.f. can be considered as a slowly varying disturbance, that the controller can effectively reject
- once the current control loop is closed, it can be seen as practically instantaneous from the external position/speed controller:
  \[
  \tau_m(t) = K I(t) \approx K I^o(t)
  \]
- this is why in the following we will assume that the motor torque $\tau_m$ is the control variable for the position/speed control problem
The rigid approximation

A first way to approach the motion control problem is to consider the servo-mechanism (motor, load, and transmission) as a rigid system. In this case the equations of the system are:

- **motor:** \( J_m \ddot{q}_m + D_m \dot{q}_m = \tau_m - \tau_{lm} \)
- **load:** \( J_l \ddot{q}_l = n\tau_{lm} - \tau_l \)
- **transmission:** \( q_m = nq_l \)

\( (D_m: \) motor viscous friction coefficient; \( J_l: \) load moment of inertia; \( n: \) transmission ratio; \( \tau_{lm}: \) transmitted torque, motor side; \( \tau_l: \) external torque, load side).

Eliminating \( q_l \) and \( \tau_{lm} \) from these equations, we obtain:

\[
(J_m + J_{lr}) \ddot{q}_m + D_m \dot{q}_m = \tau_m - \tau_{lr}
\]

with:

\[
J_{lr} = \frac{J_l}{n^2}, \quad \tau_{lr} = \frac{\tau_l}{n}
\]
The rigid system can be described in terms of transfer functions:

\[ G_v(s) = \frac{1}{D_m + s(J_m + J_{lr})} \]

If the friction coefficient \( D_m \) is neglected (which is the most conservative situation, since the friction gives a stabilizing effect but the value of this coefficient is uncertain) we have:

\[ G_v(s) = \frac{\mu}{s}, \quad \mu = \frac{1}{J_m + J_{lr}} \]
P/PI control

Let us close a PI controller on the speed and a proportional controller on the position:

\[
\begin{align*}
q^o_m & \rightarrow K_{pp} \rightarrow R_{PI}(s) \rightarrow G_v(s) \rightarrow \frac{1}{s} \rightarrow q_m \\
\end{align*}
\]

This control scheme needs two independent measurements, one for the position (encoder or resolver), the other one for the speed (tacho).

It is a cascaded control scheme:

- first the internal speed loop is designed for a large bandwidth, so as to ensure a good disturbance rejection, too
- the external position loop is then designed for a restricted bandwidth
- in fact there are three nested loops: current, speed, and position loops
Design of the PI speed controller

PI controller

\[ R_{pl}(s) = K_{pv} \left(1 + \frac{1}{sT_{iv}}\right) = K_{pv} \frac{1 + sT_{iv}}{sT_{iv}} \]

Loop transfer function:

\[ L_v(s) = R_{pl}(s)G_v(s) = K_{pv} \mu \frac{1 + sT_{iv}}{s} \frac{1}{sT_{iv}} \]

If \( T_{iv} \) is sufficiently large, i.e. if the zero of the PI is in a sufficiently low frequency range, the crossover frequency \( \omega_{cv} \) is well approximated by the high frequency approximation of \( L \):

\[ \frac{1}{T_{iv}} = (0.1 \div 0.3)\omega_{cv} \]

placement of the PI zero

\[ \omega_{cv} = K_{pv}\mu \]

selection of the PI gain
Design of the P position controller

The position controller “sees” the speed closed loop, whose transfer function can be approximated as:

\[ F_v(s) \approx \frac{1}{1 + s/\omega_{cv}} \]

The loop transfer function is thus:

\[ L_p(s) = K_{pp} F_v(s) \frac{1}{s} = \frac{K_{pp}}{s(1 + s/\omega_{cv})} \]

It is enough to take \( K_{pp} \ll \omega_{cv} \) in order to guarantee a crossover frequency \( \omega_{cp} \):

\[ \omega_{cp} = K_{pp} \quad \text{selection of the P gain} \]
In order to get a faster response to the position reference, it is possible to include a feedforward contribution, known as “speed feedforward”: the position reference is differentiated and the contribution is added to the speed loop.

Usually the speed feedforward is weighted by a coefficient $k_{ff}$ comprised between 0 and 1:
If only a position sensor is used and the speed is obtained differentiating the position measure, a scheme which is fully equivalent to a PID controller is obtained:

\[
\tau_m = \tau_{lr} \cdot \frac{1}{sG_v(s)} R_{PID}(s) + q^o_m \\
1/s \quad q_m
\]

\[
\begin{align*}
K_P & = K_{pv} \left( K_{pp} + \frac{1}{T_I} \right) \\
K_I & = K_{pv} T_{iv} \frac{K_p}{K_{pp} K_{pv}} \\
T_D & = \frac{K_{pv}}{K_p} \frac{1}{T_I} 
\end{align*}
\]
Limitations of the rigid model

- The rigid model does not entail any significant limitations to the bandwidth of the speed controller: in principle we might obtain an arbitrarily fast closed loop system.

- In practice limitations clearly emerge, in terms of vibrations, noise, oscillations, etc.

- Obviously the rigid model is not adequate to explain how a servomechanism behaves in case the required performance are increased.

- We need to complicate the model.
Two-mass approximation

The simplest way to account for non-rigid behaviour of the system is to include an elastic coupling between motor and load, which are still considered rigid systems. In this case the equations become:

**Motor:** \[ J_m \ddot{q}_m + D_m \dot{q}_m = \tau_m - \tau_{lm} \]

**Load:** \[ J_l \ddot{q}_l = n\tau_{lm} - \tau_l \]

**Transmission:** \[ \tau_{lm} = K_{el} (q_m - nq_l) + D_{el} (\dot{q}_m - n\dot{q}_l) \]

Block diagram:

Now we have a 4th order system.
Limitations in the bandwidth

If we revise the design of the speed controller accounting for the flexibility in the joint, as it is done in basic robotics courses (see background material), we come up with limitations in the bandwidth of the speed controller.

Specifically, if we set:

\[ \omega_z = \sqrt{\frac{K_{el}}{J_{lr}}} \]

we have as a reasonable rule to tune the controller:

\[ \omega_{cv} \approx 0.7 \omega_z \]

The bandwidth of the position controller has to be reduced accordingly.
Successful control of vibrations

https://www.youtube.com/watch?v=z8mqyH3rej4
Background material
A “SITO” system

Let us study the response of the two-mass system to the torque command $\tau_m$ (we then set $\tau_l = 0$).

The system can be interpreted as a SITO (Single Input Two Outputs) system:

![Block diagram of the SITO system]

Solving the block diagram we have:

$$G_{vm}(s) = \frac{J_{lr}s^2 + D_{el}s + K_{el}}{J_{lr}J_m s^3 + (JD_{el} + J_{lr}D_m)s^2 + (JK_{el} + D_mD_{el})s + D_m K_{el}}$$

$$G_{vl}(s) = \frac{D_{el}s + K_{el}}{J_{lr}J_m s^3 + (JD_{el} + J_{lr}D_m)s^2 + (JK_{el} + D_mD_{el})s + D_m K_{el}}$$

The numerators are different.

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Relevant parameters

As in the rigid case, we set $D_m=0$. The following parameters are defined:

\[ \rho = \frac{J_{lr}}{J_m} \]  

\[ \omega_z = \sqrt{\frac{K_{el}}{J_{lr}}} , \quad \zeta_z = \frac{D_{el}}{2} \frac{1}{\sqrt{J_{lr}K_{el}}} \]  

\[ \omega_p = \sqrt{1 + \rho} \omega_z \]  

\[ \zeta_p = \sqrt{1 + \rho} \zeta_z \]  

We then have:

\[
G_{vm}(s) = \frac{1 + 2\frac{\zeta_z s}{\omega_z} + s^2}{1 + 2\frac{\zeta_p s}{\omega_p} + s^2} \frac{\mu}{s}
\]

\[
G_{vl}(s) = \frac{1 + 2\frac{\zeta_z s}{\omega_z} + s^2}{1 + 2\frac{\zeta_p s}{\omega_p} + s^2} \frac{\mu}{s}
\]
The frequencies $\omega_p$ and $\omega_z$ have a very clear physical interpretation:

When the system is unconstrained, it vibrates at the frequency of the poles of $G_{vm}$, i.e. $\omega_p$: this is called **natural frequency** of the system.

When the motor is mechanically constrained, the system vibrates at the frequency of the zeros of $G_{vm}$, i.e. $\omega_z$: this is called **locked frequency** of the system.
Location of poles and zeros

Where are poles and zeros of $G_{vm}$ located in the complex plane?

$$G_{vm}(s) = \frac{1 + 2\frac{\zeta_z s}{\omega_z} + \frac{s^2}{\omega_z^2}}{\mu} \left( 1 + 2\frac{\zeta_p s}{\omega_p} + \frac{s^2}{\omega_p^2} \right)$$

$$\frac{\omega_p}{\omega_z} = \frac{\zeta_p}{\zeta_z} = \sqrt{1 + \rho} > 1$$

The poles are in a higher frequency range and are more damped than the zeros.
What is the shape of the frequency response of $G_{vm}$?

$$G_{vm}(s) = \frac{\frac{1 + 2\zeta z s}{\omega z} + \frac{s^2}{\omega^2}}{s + 2\zeta p s + \frac{s^2}{\omega^2}}$$

$\rho = 1$
$\zeta_z = 0.1$
If $\tau_I = 0$, we can conveniently represent the system with the following block diagram:

$$
\tau_m \xrightarrow{G_{vm}(s)} \dot{q}_m \xrightarrow{1/s} q_m \xrightarrow{G_{lm}(s)} nq_I
$$

where the motor position and the load position (referred to the motor shaft) are formally linked by this transfer function:

$$
G_{lm}(s) = \frac{1 + 2\frac{\zeta_s}{\omega_s} s}{1 + 2\frac{\zeta_s}{\omega_s} s + \frac{s^2}{\omega_s^2}}
$$
P/PI control on the motor

In industrial robotics, sensors are usually located only on motor side. Let us concentrate on the response to changes in the reference signal ($\tau_l = 0$):

If the speed is obtained as a derivative of position:
P/PI control on the motor

\[ R_{Pl}(s) = K_{pv} \left( 1 + \frac{1}{sT_{iv}} \right) = K_{pv} \frac{1+sT_{iv}}{sT_{iv}} \]

Loop transfer function:

\[ L_v(s) = R_{Pl}(s)G_{vm}(s) = \frac{K_{pv}\mu}{s} \frac{1+sT_{iv}}{sT_{iv}} \frac{1+2\frac{\zeta_z s}{\omega_z} + \frac{s^2}{\omega_z^2}}{1+2\frac{\zeta_p s}{\omega_p} + \frac{s^2}{\omega_p^2}} \]

Let us introduce the following normalized design parameter:

\[ \tilde{\omega}_{cv} = \frac{K_{pv}\mu}{\omega_z} \]

it is the design crossover frequency, evaluated on the rigid model \((K_{pv}\mu)\), normalized to the frequency \(\omega_z\):

- large \(\tilde{\omega}_{cv}\): “aggressive” design
- small \(\tilde{\omega}_{cv}\): “cautious” design
Frequency response analysis

Let us place the zero of the PI one decade before the antiresonance frequency $\omega_z$:

$$T_{iv} = \frac{10}{\omega_z}$$

We analyze Bode diagrams of the loop transfer function in two cases:

\[
\tilde{\omega}_{cv} = 0.5 \\
\tilde{\omega}_{cv} = 1.5
\]

The phase margin is large in both cases.
From the Bode stability criterion we cannot discriminate between the two projects. They both seem to yield control systems with a high stability margin. However if we take a look at the closed loop frequency response, from motor and load sides, in the “aggressive” design case:

$$\tilde{\omega}_{cv} = 1.5$$

There is a clear resonance at the load side, which is associated to oscillations.
Let’s draw the root locus at varying $\tilde{\omega}_{cv}$ (equivalently $K_{pv}$):

There are complex poles whose damping first increases and then decreases.

The maximum damping is obtained when:

$$\tilde{\omega}_{cv} \approx 0.7 \quad \left(\omega_{cv} \approx 0.7 \omega_z\right)$$

**Note:** in this locus and in the next ones, the axes are normalized to $\omega_z$ for increased generality.
Proportional position control

The loop transfer function for the position control is then:
\[ L_p(s) = K_{pp} \frac{F_v(s)}{s} \]

We introduce in this case, too, a normalized design parameter:
\[ \tilde{\omega}_{cp} = \frac{K_{pp}}{\omega_z} \]

It is the design crossover frequency, evaluated on the rigid model \((K_{pp})\), normalized to the frequency \(\omega_z\).
Position loop: root locus

\[ L_p(s) = K_{pp} \frac{F_v(s)}{s} \]

We draw the root locus at varying \( \tilde{\omega}_{cp} \) (equivalently \( K_{pp} \)) for different values of \( \tilde{\omega}_{cv} \):

\[ \tilde{\omega}_{cv} = 0.5 \]
\[ \tilde{\omega}_{cv} = 1 \]
\[ \tilde{\omega}_{cv} = 1.5 \]

Larger bandwidths of the speed loop generate lightly damped poles, which in turn entail poor performance of the position loop.
Simulations

We simulate in Simulink the complete system, including a step on the external torque on the load side (torque disturbance):
Simulations

System: $\omega_z = 200 \text{ rad/s}, \rho = 1, \zeta_z = 0.1$

Speed PI: $T_{iv} = 10/\omega_z$

Position P: $\omega_{cp} = 0.1$

$\tilde{\omega}_{cv} = 0.5$

$\tilde{\omega}_{cv} = 2.5$

Torque disturbance
P control on the load and PI control on the motor

In several applications, like with machine tools, the position loop is closed at the load side:

\[ \tau_m = K_{pp} R_{PI}(s) G_{vm}(s) q_m + \frac{1}{s} G_{lm}(s) nq_l \]

In case the motor speed is obtained differentiating the position:
P position control

Nothing changes as far as the speed loop is concerned

The loop transfer function for the position control is now:

\[
F_v(s) = \frac{L_v(s)}{1 + L_v(s)}
\]

\[
L_p(s) = K_{pp} \frac{F_v(s)}{s} G_{lm}(s), \quad G_{lm}(s) = \frac{1 + 2 \frac{\zeta \omega}{\omega} s}{1 + 2 \frac{\zeta \omega}{\omega} + \frac{s^2}{\omega^2}}
\]
Root locus

\[ L_p(s) = K_{pp} \frac{F_v(s)}{s} G_{lm}(s) \]

We draw the root locus at varying \( \tilde{\omega}_{cp} \) (equivalently \( K_{pp} \)) for different values of \( \tilde{\omega}_{cv} \):

- \( \tilde{\omega}_{cv} = 0.5 \)
- \( \tilde{\omega}_{cv} = 1 \)
- \( \tilde{\omega}_{cv} = 1.5 \)

At increasing bandwidths of the speed loop the design of the position loop becomes more complicated. Even for small values of \( K_{pp} \) the system can get unstable.
Simulations

We simulate again in Simulink the complete system, including a step on the external torque on the load side (torque disturbance):
Simulations

System: $\omega_z = 200 \text{ rad/s}, \rho = 1, \zeta_z = 0.1$

Speed PI: $T_{iv} = 10/\omega_z$

Position P: $\omega_{cp} = 0.1$

$\tilde{\omega}_{cv} = 0.5$

$\tilde{\omega}_{cv} = 2.5$

torque disturbance
Simulations

System: \( \omega_z = 200 \text{ rad/s}, \rho = 1, \zeta_z = 0.1 \)

Speed PI: \( T_{iv} = \frac{10}{\omega_z} \)

Position P: \( \omega_{cp} = 0.7 \)

\[ \tilde{\omega}_{cv} = 1.5 \]

The system is unstable