Control of industrial robots
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Position and force control of a manipulator with rigid joints

Consider the UR5 manipulator sketched in the picture. Assume the following values for the geometrical and physical parameters of the links:

- **DH parameters:**

<table>
<thead>
<tr>
<th>Link ( i )</th>
<th>( d_i ) [m]</th>
<th>( a_i ) [m]</th>
<th>( \alpha_i ) [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08916</td>
<td>0.0</td>
<td>( \pi/2 )</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>-0.425</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>-0.39225</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.10915</td>
<td>0.0</td>
<td>( \pi/2 )</td>
</tr>
<tr>
<td>5</td>
<td>0.09465</td>
<td>0.0</td>
<td>( -\pi/2 )</td>
</tr>
<tr>
<td>6</td>
<td>0.0823</td>
<td>0.0</td>
<td>0</td>
</tr>
</tbody>
</table>

- **Masses and positions of the centers of mass expressed w.r.t the reference frame attached to the associated link:**

<table>
<thead>
<tr>
<th>Link ( i )</th>
<th>( m ) [kg]</th>
<th>( p_{cm_i} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.7</td>
<td>[0 -25.61 1.93] * 10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>8.393</td>
<td>[212.5 0 113.36] * 10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>2.275</td>
<td>[119.93 0 26.5] * 10^{-3}</td>
</tr>
<tr>
<td>4</td>
<td>1.219</td>
<td>[0 -1.8 16.34] * 10^{-3}</td>
</tr>
<tr>
<td>5</td>
<td>1.219</td>
<td>[0 1.8 16.34] * 10^{-3}</td>
</tr>
<tr>
<td>6</td>
<td>0.1879</td>
<td>[0 0 -1.159] * 10^{-3}</td>
</tr>
</tbody>
</table>

- **Inertia tensors expressed w.r.t. the reference frame attached to the associated link:**

<table>
<thead>
<tr>
<th>Link ( i )</th>
<th>( I_i ) [Kg*m^2]</th>
<th>Link ( i )</th>
<th>( I_i ) [Kg*m^2]</th>
</tr>
</thead>
</table>
| 1           | \[
\begin{bmatrix}
84 & 0 & 0 \\
0 & 64 & 0 \\
0 & 0 & 84
\end{bmatrix} * 10^{-4}
\] | 4           | \[
\begin{bmatrix}
16 & 0 & 0 \\
0 & 16 & 0 \\
0 & 0 & 9
\end{bmatrix} * 10^{-4}
\] |
| 2           | \[
\begin{bmatrix}
78 & 0 & 0 \\
0 & 21 & 0 \\
0 & 0 & 21
\end{bmatrix} * 10^{-4}
\] | 5           | \[
\begin{bmatrix}
16 & 0 & 0 \\
0 & 16 & 0 \\
0 & 0 & 9
\end{bmatrix} * 10^{-4}
\] |
| 3           | \[
\begin{bmatrix}
16 & 0 & 0 \\
0 & 462 & 0 \\
0 & 0 & 462
\end{bmatrix} * 10^{-4}
\] | 6           | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} * 10^{-4}
\] |
Assume that the motion is actuated through six motors connected to the links by means of rigid joints. Use the following numeric values:

- Moments of inertia of the motors around their own axes *(indicative values)*:
  \[ J_{m1} = J_{m2} = J_{m3} = 10^{-5} \text{ kg m}^2 \]
  \[ J_{m4} = J_{m5} = J_{m6} = 5 \times 10^{-6} \text{ kg m}^2 \]
- Reduction ratios: \( n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 100 \)

Consider the robot initially at steady-state with the *end effector located in the point (0.2, 0, 0) and oriented as the base frame (frame 0)*. Assume that the manipulator interacts with the environment, consisting in a plane, parallel to the plane \( x_0 y_0 \) and located at a coordinate on axis \( z_0 \) equal to \( z_e = -10^{-6} \). The environment is considered perfectly frictionless and substantially rigid: for the sake of simulation, set the stiffness constant of the environment equal to \( K_e = 10^7 \text{ N/m} \).

Using the independent joint control system designed in the previous lab session, we want to design a controller of the contact force, closed around the position control loop. The force controller will act on the position reference values in the joint space.

The performance of the force controller will be validated both through the following tests:

- a test with a step variation of the force reference (initial value: \( F_{d, \text{init}} = 10 \text{ N} \), final value: \( F_{d, \text{fin}} = F_{d, \text{init}} + 10 \text{ N} \)), when the robot is motionless and in contact;
- a test with constant force reference and motion of the end-effector along the \( x_0 \) axis for a 0.4 m displacement in 0.5 s, with a maximum speed of 1 m/s, using a trapezoidal velocity profile.

**How to proceed**

1. The complete Simulink diagram that simulates the position/force control system is the following one, where new blocks, compared to the diagram used in the previous lab session, have been included: a block *Environment*, a block *Transpose Jacobian* and a block *Force controller*.

![Simulink Diagram](image)

2. The block *Environment* can be assembled as in the picture:

![Environment Block Assembly](image)

   The nonlinear block *Detachment* has zero output when its input is negative, i.e. when the \( z \) coordinate of the end-effector is below \( z_e \) (and then contact is not established).

   The normal unit vector pointing out from the contact surface is:
   \[
   n = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}
   \]

   The block *Normal unit vector* determines the component of the force vector along the axes \( x_0, y_0 \) and \( z_0 \).
3. The block **Transpose Jacobian** multiplies the contact force by the transpose of the geometrical Jacobian:

The **interpreted Matlab function** to be used is the following one:

\[
\text{UR5.jacob0(u(1:6)))'*[u(7:9);zeros(3,1)]}
\]

In fact we want to compute the torques, \(\tau_e\), due to the contact forces (we are neglecting the moments) applied by the environment to the end effector of the UR5 robot.

From the theory we know that these torques are equal to:

\[
\tau_e = J(q)^T \int f_{\mu=6}^{6}
\]

where, in the considered case:

\[
f = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}
\]

and \(\text{jacob0(u(1:6))}\) is the Robotics Toolbox function that returns the complete Jacobian with 6 rows.

4. The block **Force controller** computes the force control action:

The output of the PI force controller has to be projected onto the normal unit vector of the contact surface, then it has to be multiplied by the transpose of the Jacobian matrix (so as to take the meaning of equivalent control torque). The signal obtained must be filtered through the transfer functions that compensate for the presence of the zeros of the PID position regulators:

\[
C_i(s) = \frac{1}{K_{D_i}s^2 + K_{P_i}s + K_{I_i}} \quad i = 1, ..., 6
\]

where \(K_{D_i}, K_{P_i}, K_{I_i}\) are obtained from the formulas of the equivalence of the parameters of the P/PI controllers actually used, with the parameters of PID controllers. Notice that the gains obtained through this equivalence have to be multiplied by the square of the reduction ratios (as they refer to motor side quantities).

5. Taking into account that the transfer function from the output of the force controller to the controlled force is nominally \(-1/s\), design the PI force controller so as to achieve a bandwidth of 60 rad/s.

6. Notice that the block \(\text{taum0}\) in the Simulink diagram is a constant term representing the torque that the motors have to produce at steady-state, i.e when the robot is motionless and in contact (motor-side torques at steady-state, \(\tau_m\)).
To compute this term one has to refer to the dynamic model of the manipulator in interaction with the environment, which is given by:

\[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau_i - J(q)^T F \]

At steady-state:

\[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \bar{\tau} - J(q)\bar{F} \]

Hence the load-side torques at steady-state are:

\[ \bar{\tau}_i = g(q) - J(q)^T \bar{F} \]

To obtain the motor-side torques:

\[ \bar{\tau}_m = \frac{\bar{\tau}_i}{\pi} \quad \text{where} \quad \bar{q} \quad \text{is the initial joint configuration.} \]