

PERFORMANCE ANALYSIS OF DIGITAL RADIO LINKS WITH NONLINEAR TRANSMIT AMPLIFIER  
AND DATA PREDISTORTER WITH MEMORY

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**ABSTRACT.** A new adaptive data predistortion technique to combat nonlinearity effects in digital radio links is presented. The method is attractive for its simplicity (two digital FIR filters) and because of the inherent adaptability to any nonlinearity. The performance gain is measured in about 3dB for the 64QAM system with respect to a system without predistortion, and of 1dB with respect to memoryless data predistortion. The corresponding values for the 256QAM system are of 4dB and of 1.5dB, respectively.

I. INTRODUCTION

In recent years the increasing demand of high bit-rate digital radio systems forces to design spectrally efficient modulation formats, like 64 and 256QAM with bandlimited pulses to reduce adjacent channel interference (ACI). The nonlinear distortion introduced by the transmitter high power amplifier (HPA) has a twofold effects: i) increases of intersymbol interference, and ii) widening the spectrum of the transmitted signal to increase ACI. Both are more severe as the size of the alphabet increases.

Several techniques have been developed to combat these effects. They may be divided in two classes, one operating at the transmitter, the other at the receiver. Among those operating at the receiver we recall Maximum Likelihood Sequence Estimation (MLSE) proposed by Mesiya, McLane and Campbell [1] and by Van Etten and Van Vugt [2], and nonlinear equalization (NLE) proposed by Falconer [3],

and Benedetto and Biglieri [4]. Both of these methods suffer from high complexity and do not prevent spectrum broadening.

The technique operating at the transmitter is mainly related to distorting the data alphabet. The pioneering paper by Saleh and Salz [5] in which a memoryless data predistorter (MLP) has been proposed for rectangular pulses, has been followed by a lot of papers [6-9] in which has been shown that it works even for bandlimited pulse shaping. Recently Biglieri proposed a data predistortion scheme with memory [10] which improves the system performance with respect to the memoryless predistorter. In [11] MLSE, NLE, and MLP are compared.

Following [10] we present hereafter an adaptive data predistorter with memory with reduced complexity to be used for high speed digital radio transmission.

II. SYSTEM MODEL

The equivalent baseband model of a digital radio link is reported in Fig.1. There  $\{a_n\}$  is the T-spaced complex data message with symbols taken from an alphabet representing the signal constellation,  $\{b_n\}$  is the T-spaced sequence of distorted symbols driving the baseband pulse amplitude modulator (PAM) with a pulse shape  $g(t)$ , so that  $x(t)$  turns out to be:

$$x(t) = \sum_n b_n g(t-nT). \quad (1)$$

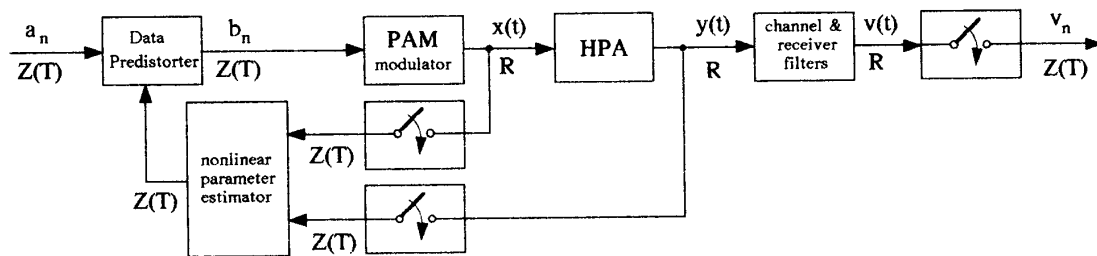


Fig.1. Model of a digital radio link with data predistorter.

9.6.1.

Due to the presence of a nonlinear HPA, the transmitted signal has the expression:

$$y(t) = x(t) \sum_{k=0}^{\infty} \alpha_{2k+1} |x(t)|^{2k} \quad (2)$$

$y(t)$  is finally filtered by the channel and the receiving filter whose global impulse response is  $h_R(t)$ , to get

$$v(t) = \int_R dt y(\tau) h_R(t-\tau). \quad (3)$$

The noisy samples of  $v(t)$  are used to estimate the transmitted data sequence  $\{a_n\}$ . In Fig.1  $Z(T)$  means a discrete time domain with a  $T$  spaced symbols, and  $R$  means a continuous time domain. The system we have outlined is nonlinear with memory, so that it may be represented by a Volterra system. As shown in [10], once we know the pulse shapes  $g(t)$  and  $h_R(t)$ , we are able to find a data predistortion algorithm so as to obtain:

$$v_n = a_n h_o. \quad (4)$$

In practice, it is impossible to build such a data predistorter because it requires a digital system with an infinitely large number of operations per symbol period. Then we limit our study to nonlinear compensation for the Volterra system up to the order  $p$ , so that the first nonlinear term appearing at the output is of order  $p+1$  ( $p$ -th order inverse). The conditions to get the  $p$ -th order inverse are ample and are satisfied by the system we are considering on [10,12,13].

We remark that the data predistorter works at the symbol rate. This means that we are linearizing the end-to-end digital channel, while at present we don't know what happens along the continuous time, or analog, part of the system. In particular we have not checked what happens for the power spectrum of  $y(t)$ . This is an important aspect of the problem because of the spectrum constraints in the real world. The required computation is not straightforward and it will be considered in the future.

### III. THE NONLINEAR PREDISTORTER

The design of a digital nonlinear predistorter has been proposed in [10]. There,

the Volterra system has been characterized by measurements done on a prototype to get the impulse responses of the kernels. Here we propose a new design approach, and we search for adaptive algorithms to adapt the coefficients of the nonlinear predistorter to the real one of the HPA. The motivation of the new approach is twofold: i) find a simple circuit implementation of the predistorter, and ii) to make it adaptive to compensate for drifts of the nonlinear coefficients related to aging, humidity, etc.

The main assumption we do in the following is that the shaping pulse  $g(t)$  is bandlimited, with bandwidth  $B$ . The  $p$ -th power of  $x(t)$  is bandlimited to  $pB$  (see [14]). We can obtain all the system parameters from the samples of  $y(t)$  taken at a rate  $F_c$  satisfying the sampling theorem conditions. Also, for our convenience we choose  $F_c$  as the least multiple of the symbol rate  $1/T$  such that  $F_c \geq 2pB$ . Hence, the system model we are working on is represented in Fig.2 with  $T_c = 1/F_c$ . We note that the discrete-time pulses involved are the sampled version of the corresponding analog pulses.

The data predistorter we are looking for is modeled as a discrete-time Volterra system having only the odd-order terms, i.e.

$$b_n = \sum_{l=1}^q \beta_{2l-1} \sum_{i_1, i_2, \dots, i_{2l-1}} h^{(2l-1)}(n-i_1, \dots, n-i_{2l-1}) \cdot a_{i_1} \dots a_{i_1} a_{i_1+1} \dots a_{i_{2l-1}} \quad (5)$$

The problem we have to solve is related to the computation of the kernels  $h^{(2l-1)}$ ,  $l=1, 2, \dots, q$ , and of the coefficients  $\beta_{2l-1}$ .

This can be done by a straightforward computation if we know exactly the filter shapes  $g$  and  $h_R$  and the nonlinear coefficients  $\alpha_1$  (see [10]). In practical system, however, we know exactly  $g$ .  $h_R$  is the impulse response of the channel cascaded with the receive filter and the linear adaptive equalizer used to combat selective fading. In [8] it has been shown that the equalizer compensate only for the ISI produced by the selective fading, so that we may consider  $h_R$  fixed and known. The

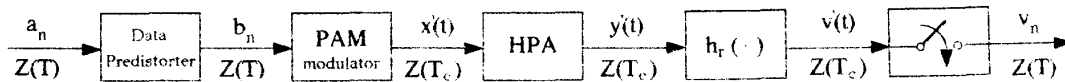


Fig.2. Discrete-time model of the radio link.  $T=NT_c$ .

nonlinear coefficients  $\alpha_1$  are generally not known with a sufficient accuracy since they depend on the particular device used and may be subject to change due to aging, operating conditions etc. Then we need to set up an adaptive scheme to track  $\alpha_1$ .

A final remark about eq. (5). Its complexity is larger than we can manage in practical system. Then we need to find some specific solutions with lower complexity for the scheme we are looking for.

#### IV. FILTER DESIGN AND ADAPTATION ALGORITHM

In this preliminary study we limit ourselves to the 3rd order inverse. Straightforward considerations using the results reported in [10] adapted with the notations used in this paper (eqs. 1 to 5) brings to the following results:

$$\beta_1 = 1/\alpha_1 ; h^{(1)}(0)=1 ; h^{(1)}(n)=0, n \neq 0 ;$$

$$\beta_3 = -\alpha_3 / (\alpha_1^3 \alpha_1^*) ; \quad (6)$$

$$h^{(3)}(n_1, n_2, n_3) = \int_R d\tau g(n_1 T - \tau) g(n_2 T - \tau) \cdot g(n_3 T - \tau) h_R(\tau)$$

Two problems have to be solved in the real world. First, to find a different expression for  $h^{(3)}$  than (6), which is not computable, and second to set-up an algorithm to estimate the nonlinear coefficients  $\alpha_1$ .

#### Filter design

An easy implementation of  $h^{(3)}$  is shown in Fig.3. It requires that  $g$  and  $h_R$  be bandlimited pulses. The third order kernel  $h^{(3)}$  is given by a cascade of a digital filter at a rate  $T_c = T/N$  with kernel  $g$ , a nonlinearity, a digital filter with kernel  $h_R$  and a sampler (decimator) to keep the output at a rate  $1/T$ . Limit of space prevent us to report the proof.

If the bandwidth of  $g(t)$  is  $B=(1+\delta)/2T$ , where  $\delta$  is the roll-off factor, for the third-order kernel we get  $N \geq 3(1+\delta)$  to meet the sampling theorem conditions. For example, with

classical system design where  $\delta=0.5$ , we get  $N=5$ .

We remark that for the considerations done in Section II  $h^{(3)}$  is known and fixed.

#### Nonlinear coefficient estimation

The adaptive algorithm to estimate the nonlinear coefficients  $\alpha_1$  and  $\alpha_3$  has the target to be based on signal samples taken at the transmitter.

With reference to Fig.1 we consider the samples of the signal after the PAM modulator, say:

$$x(kT) = \sum_n b_n g(kT-nT), \quad (7)$$

and the samples of the signal after the HPA, i.e.:  $y(kT)$ .

From  $x(kT)$  we build the estimated sample  $y'(kT)$  of the signal after the HPA, i.e.:

$$y'(kT) = \alpha_1' x(kT) + \alpha_3' x(kT)^2 x(kT)^*, \quad (8)$$

where  $\alpha_1'$  and  $\alpha_3'$  are the estimated values of  $\alpha_1$  and of  $\alpha_3$ .

The estimation error is given by:

$$\epsilon(kT) = y'(kT) - y(kT). \quad (9)$$

The mean squared error function is a quadratic convex function on  $\alpha_1'$  and  $\alpha_3'$ . Then, their optimum values are obtained by zeroing the partial derivatives of  $E[|\epsilon|^2]$ . This may be done recursively by using standard techniques as, e.g., the gradient algorithm.

The estimated values of  $\alpha_1$  and of  $\alpha_3$  are also used to compute the coefficients  $\beta_1$  and  $\beta_3$  of the predistorter by using (6).

#### V. EXAMPLES OF APPLICATIONS

The above predistorter design have been applied to compute the flat fade margin in a digital radio system. We recall that the flat fade margin  $F$  [8] is the signal reduction that can be sustained on the radio path before the error rate reaches some performance threshold

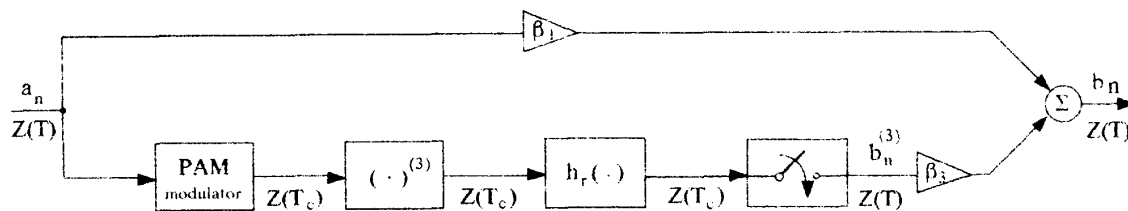


Fig.3. Possible implementation of the third-order data predistorter.

### 9.6.3.

$P_{e0}$ . In the numerical results below it has been set  $P_{e0}=10^{-4}$ . The computation has been done by simulation by considering 30.000 samples per run to evaluate the system performance and for testing the convergence of the gradient algorithm.

- The system parameters were:
- 64 and 256 QAM system;
  - raised cosine filter shaping with roll-off factor  $\delta=0.5$ ;
  - HPA nonlinear characteristic  $f(x)$ ;
  - standard design for  $g$  and  $h_R$ .

Curves of  $F$  are reported in Fig.4 for the following cases:

- a)  $f(x) = \alpha_1 x + \alpha_3 x^2 x^*$ ,  
with  $\alpha_1=1.73$ ,  $\alpha_3=-.866+j.5$ ;
- b)  $f(x) = 2x/(1+|x|^2)\exp[j(\pi/3)|x^2|/(1+|x|^2)]$ .

Both cases have been evaluated with and without the third-order predistorter.

In Table 1 the results obtained are compared with the ones obtained in [8] with a zero-memory data predistorter.

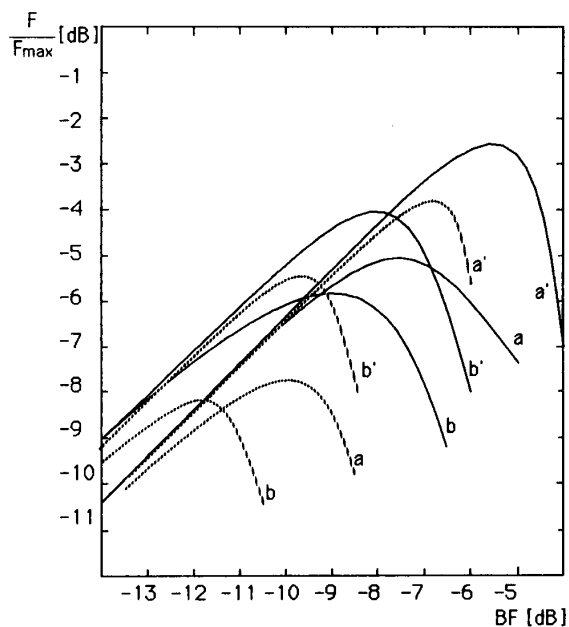


Fig.4. Curves of flat fade margin over its maximum value ( $F/F_{max}$ ) vs. input back-off (BF) for 64QAM (continuous lines) and 256QAM (dotted lines) with and without 3-rd order predistorter (prime denotes with 3-rd order predistorter). a) and b) refer to cases a) and b), respectively.

TABLE 1

FLAT FADE  $F/F_{max}$  in dB

Type of Predistortion	64 QAM		256 QAM	
	a)	b)	a)	b)
None	-5.8	-5.8	-7.8	-8.4
Tx Pred.	-3.6		-5.4	
Rx Pred.			-4.2	
with memory	-2.6	-4.0	-3.7	-5.4

With reference to Table 1 we recall that Tx predistortion means that the signal samples driving the predistorter are taken at the output of the HPA, while with Rx predistortion the signal samples are taken at the sampling point. The Rx technique is ideal and is used only for comparison purposes.

We note that in case a) the predistorter with memory exhibits an improvement of some tenth of dB with respect to the ideal Rx predistortion, and of the order of 1 dB with respect to Tx predistortion.

The performance in case b) are available only for the predistorter with memory. With respect to case a) we get a worsening of  $F$  of about 1.5 dB. This is mainly due to a residual nonlinear phase distortion in the received signal constellation that is hard to compensate with a low order Volterra system. The significant improvements on flat fade margin of the 3-rd order predistorter even in the presence of a strong nonlinearity, as considered in case b), shows that the adaptive method proposed works even in the "real" world and it may be used in practical applications.

## VI. CONCLUSIONS

A new method to introduce an adaptive digital predistortion circuit to compensate for the nonlinearity produced in digital radio system by the HPA has been proposed. The method gives a system improvement versus fading margin greater than 3 dB for the 256 QAM system with respect to uncompensated system.

It is worth noting the simplicity of the predistorter, it requires to estimate only two parameters ( $\alpha_1$  and  $\alpha_3$ ) in place of tens of the third order kernel coefficients as proposed in previous schemes.

Future work will be addressed to evaluate the predistorter effects on the transmitted power spectrum and in the optimum design of transmit and receive filters. Also the effects of a 5-th order term in the predistorter need to be taken into account to check the feasibility of further significant improvements.

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