3 GLISIC, S.G.: 'Design study for a CDMA-based LEO satellite network: downlink system level parameters', *IEEE J. Sel. Areas Commun.*, 1996, **SAC-14**, (9), pp. 1796–1808

4 KALET, I.: 'The multitone channel', *IEEE Trans.*, 1989, **COM-37**, pp. 119–124

5 EDFORS, O., and SANDELL, M.: 'OFDM channel estimation by singular value decomposition', *IEEE Trans.*, 1998, **COM-46**, (2), pp. 931–939

Block-wise physical model synthesis for musical acoustics

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A block-wise approach to sound synthesis based on nonlinear wave digital modelling is proposed. Starting from a semantic description of the physical model, the technique generates, validates and initialises appropriate simulation source code with time-varying parameters.

Introduction: Research into sound synthesis for acoustic and musical applications has mainly focused on physical modelling [1], with a renewed interest in wave digital filters (WDFs) [2] and in digital waveguides (DWGs) [3]. Such block-wise modelling methods, in fact, in addition to being compatible with each other, can now include all nonlinearities that are typical of acoustic musical instruments. Extended wave digital (WD) structures have been proposed which are able to incorporate a wide class of nonlinear elements with memory through the definition of non-energetic adaptors with memory [4]. In this Letter we further extend such results and propose an automatic procedure that generates, validates and initialises WD-based simulation source code, starting from a semantic description of a time-varying reference physical model. The method is general enough to include a wide variety of nonlinearities and, since principles of the conservation of pseudo-energy are satisfied, it enables us to choose a sampling rate just on the basis of the bandwidth of the signal flow.

Global approach: We built a physical WD model starting from the electrical equivalent of a mechanical or fluidodynamical analogue reference model. This model is partitioned into 'instantaneously decoupled' subsystems (which are characterised by a delayed mutual interaction), to be individually synthesised and initialised in the WD domain. Each subsystem is constructed by connecting together a number of individually discretised WD blocks through a non-energetic macro-adaptor, which models the subsystem's global constraints (Kirchhoff laws). Since our digital waves are defined as $A(z) = V(z) + R(z)I(z)$, $B(z) = V(z) - R(z)I(z)$, with $R(z)$ being a reference transfer function [4] (RTF), the macro-adaptor cannot be memoryless. A proper choice of the RTF enables us to eliminate the instantaneous input/output dependency in its WD implementation (instantaneous adaptation). The WD implementation of an 'adapted' bipole will thus be of the form $B(z) = z^{-1}K(z)A(z)$, where the delayed reflection filter $K(z)$ can also be zero. As for the macro-adaptor, only one of its ports is allowed to be adapted, which means that no local instantaneous reflection can occur. This port can thus be connected to a nonlinear element with instantaneous input-output dependency without computability problems. The nonlinearities that can be considered are of the form $p = g(q)$, $P(z) = H_v(z)V(z)$, $Q(z) = H_i(z)I(z)$, where p and q are related to v and i through finite difference equations. This includes a wide variety of dynamical elements, such as nonlinear reactances or whole portions of circuits containing lumped nonlinearities. A port from which it is possible to extract a delay element is said to be decoupled. Whereas decoupling corresponds to adaptation in the case of bipoles, for multi-port elements decoupling is a stronger condition. A set of macro-blocks is said to be decoupled when they are connected through a decoupling multi-port (which is an N-port element with at least $N - 1$ decoupling ports). As they do not instantaneously interact with each other, decoupled blocks can be individually synthesised and initialised. A WD structure will be allowed as many nonlinearities as there are decoupled subsystems. Decoupling blocks are quite frequent in musical acoustics, as reverberating structures are often implemented as a network of delay lines (DWGs).

Once an M-port decoupling block has been identified, we can temporarily replace it with a set of M independent reactances, obtaining M independent subsystems. A decoupled subsystem is characterised by a nonlinear element and a set of $N - 1$ reflection filters (linear macro-bipoles), all connected through an N-port non-energetic macro-adaptor with memory that models the circuit topology. The key point in the model synthesis is the construction and the initialisation of this macro-adaptor.

Synthesis of macro-adaptor: An N-port macro-adaptor can be automatically built through a tableau-based approach, specifically designed for WD structures. Its description is given in the form $\mathbf{S}(z)\mathbf{C}(z) = \mathbf{0}^T$, where $\mathbf{S}(z)$ is a $2N \times N$ matrix, $\mathbf{0}$ is a vector with N zero elements and $\mathbf{C}(z) = [A_1 \ldots AN\ B_1 \ldots B_2]^T$ is the vector of digital waves. A generic macro-adaptor can be thought of as a network of elementary (parallel or series) three-port adaptors with memory that belong to a pre-defined collection. This allows us to construct $\mathbf{S}(z)$ by 'pasting' a number of pre-defined 6×3 matrices into a larger sparse matrix. This matrix equation can be quite easily rearranged and inverted in order to obtain a state update equation, or else it can be solved iteratively using some efficient numerical method for sparse matrix equations.

Initialisation: As our macro-adaptors are not memoryless, they need to be properly initialised, which is a critical operation for WD models of mechanical systems as it usually affects the mutual position and contact conditions of mechanical elements. The determination of the state update equation can be seen as a direct form of the synthesis problem, as output signals are computed from input signals and memory content. Initialisation, on the other hand, can be seen as an inverse problem, as memory content must be derived from output and input signals. As the nonlinearity is 'lumped', this operation can be quite easily performed through nonlinearity inversion and matrix inversion.

Time-varying models: Changing any model parameters in a WD structure usually affects all the other parameters as they are bound to satisfy global adaptation conditions. Temporal variations of the nonlinearities are easily dealt with by employing special WD two-port elements that are able to perform a variety of transformations of the nonlinear characteristics (non-homogeneous scaling, rotation, etc.). Time-varying RTF changes, on the other hand, are dealt with through a global re-computation of all model parameters on the behalf of a process that works in parallel with the simulator. This operation requires the re-mapping of the nonlinearities as well. This parameter update, however, is not computationally intensive as it is performed at a rate that is normally only a fraction of the signal rate (e.g. 100 times slower). It is important to remember, however, that abrupt parameter changes must be carefully dealt with in order not to affect the global energy in an uncontrollable fashion.

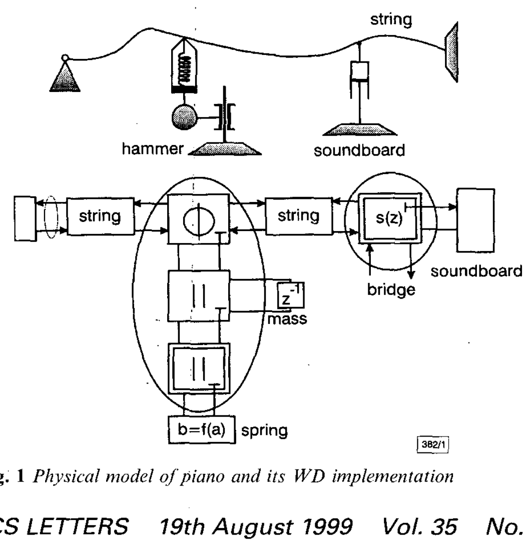

Fig. 1 *Physical model of piano and its WD implementation*

Implementation: Some methods are already available for synthesising linear macro-blocks [3], therefore the automatic synthesis procedure is based on the assumption that such elements are already available in the form of a collection of pre-synthesised structures. In its current state, the system that we developed is able to automatically construct WD structures based on standard WDF adaptors and new dynamic adaptors chosen from a reasonably wide collection. Currently, this family includes WD mutators [4] and other types of adaptors developed for modelling typical nonlinear elements using classical nonlinear circuit theory (both resistive and reactive). The available linear macro-blocks belong to the family of the DWGs [3], while the nonlinear maps are currently pointwise described in the Kirchhoff domain and then automatically converted in a piecewise continuous WD map. The parameters can be modified 'on the fly' in order to make the structure time-varying. A parallel process deals with the problem of re-computation of all WD parameters, depending on their changes expressed in the Kirchhoff domain.

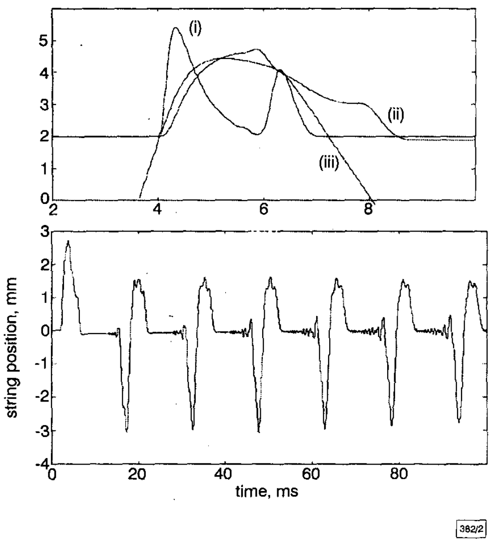

Fig. 2 *Simulation results for WD model of piano interaction*
(i) force ($N \times 10^{-1}$)
(ii) string position (mm)
(iii) hammer position (mm)
Lower trace: string position at bridge

Example of application: As an example of application of the above approach, we present the synthesis of a model of the hammer-string interaction in an acoustic piano. The implementation, entirely involving WDF and DWG elements, includes a mechanical model of the hammer and its interaction with a physical model of a string (which includes stiffness and distributed losses) connected to a DWG soundboard model. The physical model of the piano and its WD implementation are shown in Fig. 1. As the string is a decoupling two-port, the soundboard can be connected to many hammer-string models. Starting from an appropriate semantic description of the building blocks of the system and their topology of interconnection and interaction with the user, the system automatically built C++ simulation source code that produced the results shown in Fig. 2.

The computational complexity of the resulting algorithm mostly depends on the complexity of the resonating structure. For example, the WD model of an electro-mechanical piano (e.g. Wurlitzer or Fender-Rhodes) can easily run with full polyphony on a Pentium-based PC platform.

Conclusion: We have described our approach to the problem of the automatic synthesis of nonlinear wave digital structures. Our implementation proved effective for the automatic and modular synthesis of a wide class of physical structures encountered in musical acoustics.

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References

1 BORIN, G., DE POLI, G., and SARTI, A.: 'Musical signal synthesis' *in* ROADS, C., POPE, S.T., PICCIALLI, A., DE POLI, G.: (Eds.): 'Musical signal processing' (Swets and Zeitlinger, 1997), pp. 5–30
2 FETTWEIS, A.: 'Wave digital filters: theory and practice', *Proc. IEEE*, 1986, **74**, (2), pp. 270–327
3 SMITH, J.O.: 'Acoustic modeling using digital waveguides' *in* ROADS, C., POPE, S.T., PICCIALLI, A., DE POLI, G.: (Eds.): 'Musical signal processing' (Swets and Zeitlinger, 1997)
4 SARTI, A., and DE POLI, G.: 'Toward nonlinear wave digital filters', *IEEE Trans.*, 1999, **SP-47**, (6), pp. 1654–1668

Capacity of slotted ALOHA under Nakagami fading with near-far and shadowing effects

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The throughput performance of slotted ALOHA in radio environments characterised by Nakagami fading and the user spatial distribution is analysed. The effect of shadowing on Nakagami fading is also evaluated and generalised throughput expressions are presented.

Introduction: The performance of slotted ALOHA has been shown to benefit from propagation impairments such as fading and shadowing. Models for analysing the performance of slotted ALOHA in the presence of Rayleigh fading have been widely researched [1]. Its performance under Nakagami fading also has been evaluated recently [2, 3]. An investigation into the performance of slotted ALOHA under Nakagami fading with identical fading parameters and received power levels has been presented in [2]. The results were then extended to a more general case of the fading parameters, and correlated shadowing and near-far effects were also considered [3]. A uniform model for the spatial distribution was assumed. Moreover, perfect correlation among the interfering packets was assumed. This means that a single interfering packet was assumed to represent the behaviour of all the interfering packets.

In this Letter, the analysis is carried out assuming the more commonly used quasi-uniform spatial distribution [4]. A general expression showing the impact of the user spatial distribution around the central base station is presented. Moreover, uncorrelated shadowing along with Nakagami fading is considered. This represents a more realistic assumption since the shadowing on the interfering packets will behave differently.

System model: In a Nakagami fading environment, the signal magnitude of the *i*th packet is statistically characterised as

$$f_{R_i}(r) = \frac{2}{\Gamma(m_i)} \frac{m_i}{\bar{P}_i} r^{2m_i-1} \exp\left(-\frac{m_i r^2}{\bar{P}_i}\right) \quad (1)$$

where $\Gamma(\cdot)$ is the Gamma function, $\bar{P}_i = E[R_i^2]$ and m_i are the *i*th packet average power and fade figure, respectively. It is assumed that the fading statistics are identical for all the packets (i.e. $m_i = m$). All terminals are assumed to transmit their packets to a central station in packets of duration T. The number of generated packets is assumed to be Poisson distributed with parameter λ. Therefore, the probability that n packets arrive in a slot is $R_n(G) = (G^n/n!)\exp(-G)$, where $G = \lambda T$.

The probability of capture of a test packet in the presence of n interfering packets can be expressed as

$$P_{capt}(n) = \Pr\left(\frac{P_s}{P_n} > z_o\right) \quad (2)$$