Electromagnetic Inversion in Monostatic Ground Penetrating Radar: TEM Horn Calibration and Application

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Abstract—A comprehensive analysis of electromagnetic (EM) inversion applied to pavement profiling by using a monostatic ground penetrating radar (GPR) is presented. Since in GPR systems using transfer EM (TEM) horns, the antenna is positioned close to the the investigated medium and a strong EM interaction occurs. This effect is taken into account by modeling the antenna with equivalent sources placed on the aperture section and by using an accurate EM modeling within the inversion loop in order to account for the contribution of near-field effects.

Experimental results on the antenna modeling procedure and on a pavement profile estimation validate the feasibility of both the calibration and the inverse problem proposed.

Index Terms—Electromagnetic scattering of inverse problems, electromagnetic scattering by nonhomogeneous media, radar applications.

I. INTRODUCTION

GROUND penetrating radar (GPR) is a remote sensing system used to measure short-pulse electromagnetic (EM) reflections due to electric-type and magnetic-type inhomogeneities of the medium. The GPR system is mainly devoted to the detection and/or estimation of objects and interfaces. It represents a remote sensing transducer useful in multiple application areas such as geophysics, geology, hydrology, environmental engineering, civil and transportation engineering, mining, and archeology (see [1] and [2]). The work discussed in this paper has been motivated by the need to recover asphalt or concrete thicknesses in road and highway pavements. The purpose for transportation engineers is to assess the pavement condition with a nondestructive test in order to help carry out a maintenance plan [1].

In the frequency range (100 MHz–2 GHz) of the monostatic GPR system used here, ground materials (asphalt and concrete with different mixtures) can be characterized by low losses with a $Q$ factor (a global parameter that includes all the losses in the medium) approximately constant [4], [5]. For these materials, the attenuation constant $a$ is linearly increasing with frequency $f$

$$\alpha \simeq \frac{\pi \sqrt{\varepsilon \mu c}}{\alpha Q} f$$

(1)

where $c$ is the speed of light in air, $\varepsilon_r$ and $\mu_r$ are the relative (subscript $r$) dielectric and magnetic constants of the media. In the far-field, the range resolution $\Delta R_r$ of a pulse radar system is proportional to the pulse width $T_w$ (or equivalently inversely proportional to its bandwidth)

$$\Delta R_r \simeq \frac{c T_w}{2 \sqrt{\varepsilon_r \mu_r}}.$$  

(2)

The choice of the pulse width in GPR systems is a tradeoff between penetration (large $T_w$) and resolution (small $T_w$) [6]. GPR for road and highway pavements have $T_w \simeq 1$ ns and range resolution (for the asphalt $\varepsilon_r \simeq 6$) $\Delta R_r \simeq 6$ cm.

The EM properties of the medium, when measured with a GPR system, appear averaged over the bandwidth of the excitation pulse. The media under investigation are largely heterogeneous and difficult to model. In fact, each layer of asphalt and/or concrete is basically a mixture of particles embedded in a homogeneous matrix [7]. Since the particle size is small compared to the antenna beamwidth (e.g., for normal operations, the footprint, or spatial resolution, is approximately $30 \times 30$ cm$^2$) and range resolution of the GPR system, each layer can be described by an equivalent complex dielectric constant [5]. Furthermore, owing to the gentle variations of the EM properties along the profile, one is allowed to assume a parallel-layers model within the antenna beamwidth.

The purpose of the present work is to estimate the permittivity profile of the investigated medium, assuming that all loss coefficients are known. These can be globally represented in the frequency domain by a complex dielectric constant $\varepsilon = \varepsilon' + \varepsilon''$ (assuming a time dependency $e^{-\alpha t}$, the loss factor being $Q = \varepsilon'' / \varepsilon'$). Although in general the complex dielectric constant is frequency dependent, in this work we can assume that both $\varepsilon'$ and $\varepsilon''$ are independent of frequency (at least within the radar bandwidth) [4], [5].

The antenna commonly used in GPR systems for pavement profiling is a broadband transverse EM (TEM) horn. The modeling and calibration of the system is therefore referred here to this case. The GPR system is mounted on a moving vehicle (shown in Fig. 1) to make continuous surveys at the speed of 50–60 km/h with accurate measurements of survey-distance. The performance of GPR systems, in terms of noise, radar-target interaction, and penetration, are optimized when the antenna is placed in the proximity of the investigated medium [8] at heights.
$h$ ranging from 15 cm up to 25 cm (this is lower than the dominant wavelength $\lambda_d = cT_{\text{rad}} = 30 \text{ cm}$ when $T_{\text{rad}} = 1 \text{ ns}$).

Under these conditions, frequencies typically used in GPR applications call for a realistic modeling of the interaction of the GPR antenna with the investigated medium, including phase front curvature and near-field effects both on the scatterer and on the received signal. In order to take into account the near-field effects, we propose to adopt an EM model of the antenna-scatterer interaction well suited for dielectric constant estimation by EM inversion with a TEM horn antenna. The model makes use of the formally exact representation of the EM field for a stratified medium in terms of Sommerfeld integrals [9]. The model is revised in Appendix A.

The aperture field has been modeled by using equivalent point sources positioned at the antenna aperture section, and the optimum sources distribution is determined by a calibration procedure in which a known scatterer (perfect conducting plate) is employed. Both the aperture-field description of the scattering problem and its equivalent representation in terms of point sources yield an overall modelization of the antenna–scatterer interaction that is virtually independent (at least in some aspects) of the specific antenna used in the experiment. The flexibility in the modelization allows us to employ the overall antenna model with different antenna types. All that is needed is the system recalibration. The calibration procedure is presented in Section II, together with examples on a simulated TEM horn antenna and on the actual GPR system in Fig. 1.

The EM inversion is discussed in Section III. The permittivity profile is estimated by minimizing the functional based on the difference between measured and simulated signals in every spatial location (continuity of permittivity profile along the acquisition survey is not considered here; see [7]). Gauss-Newton method is used for iterative optimization of the error function.

The layered medium modeling and the equivalent antenna representation both simplify the updating relationships still taking into account near-field effects (Appendix B).

The EM inversion has been carried out both on synthetic (in [10]) and real data acquired with a monostatic GPR system. Some results, in terms of permittivity profile estimation and simulated signals, are discussed in Section IV.

**Basic notation:** $\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y + A_z \mathbf{e}_z$ is a vector field, $(A_x, A_y, A_z)$ are the field components, and $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ are the unit amplitude vectors. Uppercase (lowercase) bold denotes matrix (column matrix), $(\cdot)^*$ and $(\cdot)^H$ stand for complex conjugate and Hermitian transpose, $\text{Re}\{\cdot\}$ selects the real part of a complex valued number, $||\mathbf{x}||^2 = \mathbf{x}^H \mathbf{x}$ is the Euclidean norm of $\mathbf{x}$, and $||\mathbf{x}||_C^2 = \mathbf{x}^H \mathbf{C} \mathbf{x}$ is the Euclidean norm of $\mathbf{x}$ weighted by any positive definite matrix $\mathbf{C}$. $\mathcal{F}\{\cdot\}$ denotes the Fourier transform (or DFT) and $\mathcal{F}^{-1}\{\cdot\}$ its inverse (or IDFT). $f$ and $\omega = 2\pi f$ stand for frequency and angular frequency. For the sake of simplicity in the notation, the explicit dependency of variables is indicated only when necessary.

**II. Equivalent Model of a TEM Horn Antenna**

For modeling and inversion purposes, one can consider the transfer function of the radar system as the ratio between the received signal $V_R(\omega) = \mathcal{F}\{v_R(t)\}$ and the transmitted signal $V_T(\omega) = \mathcal{F}\{v_T(t)\}$. We therefore consider the TEM horn antenna terminal section as an aperture with an impressed electric/magnetic field. The basic building block describing the interaction of the antenna with the scatterer is the field produced by a point source (either an electric or a magnetic dipole) in the presence of a layered medium. As shown in Fig. 2, we consider a medium composed of N parallel homogeneous layers, each characterized by a dielectric constant $\varepsilon_i$, a magnetic constant $\mu_i$ (free space magnetic permeability), an equivalent conductivity...
σ_l = ωε_l and a thickness h_l. The sources are represented by electric and magnetic dipoles positioned in air and oriented parallel to the layers interfaces. The scattered field is expressed in terms of Sommerfeld integrals (Appendix A).

The TEM horn antenna has been modeled by using a set of equivalent point sources (electric and magnetic dipoles) placed on the antenna aperture. The purpose of this model is to represent the TEM horn antenna behavior in a simple equivalent form. The use of a rather general description (an array of dipoles) allows a great flexibility in the modeling process. Different TEM antennas used in the GPR system only require a new calibration procedure (see Section III-C), but the consistency of the model is preserved. The equivalent model is particularly suitable for EM inversion purposes, mainly because of its computational efficiency.

In the framework of the GPR system characterization, the model describes the behavior of the antenna as a transducer operating the transformation from the incident EM field to the received voltage. This is a fundamental step in the solution of the forward problem and in the EM inversion algorithm, since it allows a realistic simulation of the received signals and therefore, an immediate comparison between measured and simulated echoes.

Features of the model developed are summarized as follows.

1) The set of point sources is a frequency independent set. This hypothesis is supported by the fact that we deal with a properly terminated TEM horn antenna whose terminal section is comparable in size to the operating dominant wavelength of the radar.

2) The set of point sources represents (in equivalent form) the aperture field of the antenna when pointing toward free space, and multiple reflections between the antenna terminal section and the target are neglected. This last hypothesis implies that the target does not modify the aperture field associated with forward waves. Its contribution to the total aperture field is simply due to a single backscattered wave.

3) The set of point sources can be complex. This allows an equivalent description of constant phase shifts in the transfer function of the radar system (see Section II-C).

We describe below the main steps to build the transfer function in terms of incident and scattered fields.

A. Model of the Transmitting/Receiving Antenna

By the reciprocity theorem, a reaction-type integral can be established. Using the transfer function between the received \( V_R(\omega) \) and the transmitted voltage \( V_T(\omega) \) [12], we have

\[
2Y_c V_T(\omega)V_R(\omega) = - \int_{S_A} \left[ \vec{E}_T(\omega) \times \vec{H}_R(\omega) \right] \cdot \vec{n} \, dS
\]

(3)

where \( \vec{E}_T(\omega) \), \( \vec{H}_T(\omega) \) is the aperture field, \( \vec{E}_R(\omega) \), \( \vec{H}_R(\omega) \) is the scattered field, both evaluated in correspondence of the aperture with the antenna terminals in open circuit, \( S_A \) is the surface corresponding to the antenna aperture, and \( Y_c \) is the characteristic impedance of the line feeding the antenna. In (3), both \( \vec{E}_T(\omega) \), \( \vec{H}_T(\omega) \) and \( \vec{E}_R(\omega) \), \( \vec{H}_R(\omega) \) are proportional to \( V_T(\omega) \). In addition, both near-field (first layer) and far-field (bottom layers) contributions are inherently taken into account.

B. Equivalent Point Sources

The antenna aperture field has been represented by \( N_S \) equivalent point sources. To simplify the modeling procedure, we have chosen to use Huygens sources. In this case, the relationship between the electric part \( \vec{J}_e(\omega) = \sum_{i=1}^{N_S} \vec{J}_{ei}(\omega) \) and the magnetic part \( \vec{J}_m(\omega) = \sum_{i=1}^{N_S} \vec{J}_{mi}(\omega) \) is given by

\[
\vec{J}_m(\omega) = \eta_0 (\vec{z} \times \vec{J}_e(\omega)).
\]

(4)

The \( i \)-th electric current density is located in \( \vec{r}_i \) and is given by

\[
\vec{J}_e(\omega) = [I_{xe,i}(\omega) \vec{x}^e + I_{ye,i}(\omega) \vec{y}] \delta(\vec{r} - \vec{r}_i)
\]

(5)

(\( I_{xe,i}(\omega), I_{ye,i}(\omega) \)) being complex components.

Using a normalized form for the received fields at the antenna aperture

\[
\vec{E}_R(\omega) = \vec{z} \times \vec{H}_T(\omega);
\]

\[
\vec{H}_R(\omega) = \vec{H}_T(\omega)
\]

the following equation is finally obtained from (3) by summing all contributions:

\[
V_R(\omega) = - \frac{V_T(\omega)}{2Y_c} \sum_{i=1}^{N_S} \int_{S_A} \left[ \vec{J}_{ei}(\omega) \times \vec{z} \right] \cdot \vec{n} \, dS
\]

(8)

where \( Y_c \) is the characteristic admittance of the TEM line, and \( \eta_0 \) is the free space impedance.

The received signal \( V_R(t) = \mathcal{F}^{-1}\{V_R(\omega)\} \) is a sum of responses weighted by the spectrum of the transmitted waveform \( V_T(\omega) \) and the distribution of the electric currents \( \vec{J}_e(\omega) \). Since for a monocyclic impulse, the spectrum \( V_T(\omega) \) is characterized by a pick around \( f_o \approx 1/T_o \) (from experimental data \( f_o \approx 850 \) MHz), the received signal is mostly influenced by the frequency content around this maximum. The frequency-variation of each source \( \vec{J}_e(\omega) \) can be approximated by the expansion around \( f_o \)

\[
\vec{J}_e(\omega) \approx \left[ \vec{J}_{xe}(\omega_o) \exp(-j\omega \tau_{xe}) \vec{x}^e + \vec{J}_{ye}(\omega_o) \exp(-j\omega \tau_{ye}) \vec{y} \right] \delta(\vec{r} - \vec{r}_i)
\]

(9)
for each component $I_{x}(\omega)$, and $I_{y}(\omega)$ amplitude variations are neglected (or approximately constant), while the expansion is carried out with respect to phase only

$$I(\omega) \approx |I(\omega_0)| \exp[j(2I(\omega_0) + 2I(\omega_0)\gamma(\omega - \omega_0))]
= I(\omega_0)|\exp[-j\omega\tau].$$

(10)

$I_{x}(\omega_0)$ and $I_{y}(\omega_0)$ are complex valued, as their phases take into account the waveform distortion that occurs for near-field effects. The Hilbert transform is implicitly performed when making the computation on half the frequency spectrum for real-valued signals (in practice: $\psi_R(t) = \text{Re}\{F^{-1}\{V_R(\omega)\}\}$ as $V_R(\omega) \equiv 0$ for every $\omega < 0$). $\tau_{x\gamma}$ and $\tau_{y\gamma}$ are the differential delays for the wideband waveform. From a practical point of view (when there are few experimental measurements obtained from the wideband pulse system) and during the calibration, the simulated and measured signals are temporally aligned so as to avoid the delay calibration for the estimation of the set.

**Remark:** The approximation depicted here (frequency-independent point sources) holds when the frequency-variation is rather smooth within the bandwidth of the waveform and when the wavefront is rather “compact” (as for pure-TEM field or for any field with phase that varies smoothly with frequency). In particular, the assumption of frequency-independent point sources is suitable to characterize TEM horn antennas, since the electromagnetic field is mainly a pure-TEM field, and the antenna terminal cross-section is not large with respect to the dominant radar wavelength. Small differences with respect to pure-TEM field (i.e., phase front shape, localized reactive energy) are represented by the set of point sources in equivalent form. When localized reactive energy contributions dominate in the aperture field distribution, a frequency-dependent set of point sources should be employed. This latter aspect is not considered in this work, since in the manufacturing of TEM horns, resistive sheets are used in order to avoid strong edge currents and thus reduce the presence of localized reactive energy [13].

### C. Antenna Calibration

The strengths of the point sources are determined by means of a calibration procedure. Calibration is carried out as an optimization problem, where the reference signals are the backscattered echoes $\psi_{\text{abs}}(t; h_1)$ measured with the antenna positioned at different heights ($h_1$) from a perfect conducting plate (also referred to as flat metal plate calibration). In the experiments considered here, the calibration has been performed at nine different heights ($h_1 = \{17, 23, 28, 33, 38, 47, 52, 58, 74\}$ cm), and the set $h$ has been chosen in order to obtain different operating conditions for the system with respect to the dominant frequency $f_d$ (the near-field contributions cannot be neglected at low heights, while increasing the antenna height the far-field dominates). The choice of the perfect conducting plate is motivated by the fact that it represents a scatterer whose characteristics are known with excellent accuracy (e.g., one can assume $\sigma = \infty$). A similar calibration procedure has also been applied on a simulated TEM horn, for which reference data are obtained by a full-wave numerical analysis (see Table IV-A).

The equivalent sources are parameterized only by the frequency-independent set $J = \{\tilde{J}_1, \ldots, \tilde{J}_{N_S}\}$ (locations $\tilde{t}_i$ are chosen according to the antenna symmetries and aperture field as sketched in Fig. 3). We seek for the optimum set $J$ that optimizes the energy difference between the simulated signals $\psi_{\text{R}}(t; h_1, J)$ and those measured with the GPR system. The optimization is carried out with respect to the $N_{\text{par}}$ independent parameters of the $N_S$ point sources. Due to antenna symmetries, the number of free parameters exploited by optimization is $N_{\text{par}} \leq 4N_S$. Several calibrations procedures have been carried out by varying $N_S$, as this allows us to analyze how the accuracy of the simplified antenna model depends on the number of point sources. Some comparisons between $\psi_{\text{abs}}(t; h_1)$ (for the actual GPR system in Fig. 1) and the simulated signals $\psi_{\text{R}}(t; h_1, J_{\text{opt}})$ (for optimized values of the sources $J = J_{\text{opt}}$) are shown in Fig. 3 for varying $N_S$ and $h_1$. Obviously, by increasing $N_S$, the differences become negligible. In addition, delay mismatch arising from the simplified model becomes negligible for $N_S \geq$...
TABLE I

<table>
<thead>
<tr>
<th>Ns</th>
<th>Nsim</th>
<th>Simulated TEM Horn</th>
<th>Experimental data (GPR system in Fig. 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>3.57%</td>
<td>24.77%</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>3.33%</td>
<td>9.41%</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>2.71%</td>
<td>7.28%</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>1.90%</td>
<td>5.41%</td>
</tr>
</tbody>
</table>

As shown in Table I, using $N_S = 4$ and 5 equivalent sources, the mean squared error (MSE) for actual system rapidly decreases from 25% down to 9%. A further increase of the number of sources slightly improves the accuracy (up to 5% for 16 dipoles) but it heavily increases the computing time. The modeling of the antenna by choosing $N_S = 5$ equivalent point sources seems to be a reasonable tradeoff between accuracy and computational efficiency. This is the configuration chosen for EM inversion. The decreasing behavior of MSE for increasing $N_S$ is also confirmed by a similar numerical experiment with a simulated TEM horn antenna (see Section IV).

III. ELECTROMAGNETIC INVERSION

For a specified location $\ell$, the EM inversion estimates the permittivity by minimizing the $l_2$ norm of the difference between the observed GPR signals $v_{\text{obs}}(t;\ell)$ and the signals $v_{\text{sim}}(t;\ell) = v(t;\ell;\hat{m}(\ell),h_{11},J_{\text{eq}})$ simulated using the specified antenna model (the antenna height $h_{11} = 21$ cm and $N_S = 5$ sources with $J_{\text{eq}}$ are used). The inversion result is the permittivity profile whose parameters are summarized by the column matrix $\hat{m}(\ell)$. For the stability purposes of the optimization algorithm, it is convenient to parameterize the permittivity profile by using a basis of (simple) functions [7]. Here we have chosen a description in terms of staircase functions

$$\epsilon(z,\hat{m}(\ell)) = \sum_{k=1}^{M} m_k(\ell) \ell_k(z/\Delta z - k).$$

(12)

$\ell(\varphi) = 1$ for $0 \leq \varphi < 1$ and $\ell(\varphi) = 0$ elsewhere. The permittivity changes in depth $z$ occur at $k\Delta z$, and the discretization interval in depth $\Delta z$ is chosen to be smaller than pulse resolution ($\Delta z < \Delta R_i$). The column matrix $\hat{m}(\ell) = [m_1(\ell), m_2(\ell), \ldots, m_M(\ell)]^T$ represents, for each location $\ell$, the unknown model parameters to be estimated.

In the frequency domain, $V_{\text{obs}}(\omega;\ell)$ denotes the measured GPR signals ($V_{\text{obs}}(\omega;\ell) = F\{v_{\text{obs}}(t;\ell)\}$ at a specified location (since we choose to estimate the permittivity independently on each location, this is understood in the notation) and for $N_F$ frequency samples

$$V_{\text{obs}} = [v_{\text{obs}}(\omega_1;\ell), v_{\text{obs}}(\omega_2;\ell), \ldots, v_{\text{obs}}(\omega_{N_F};\ell)]^T.$$  \hspace{1cm} (13)

$V(m)$ is the column matrix of data simulated according to the simplified antenna model (Section II) and permittivity model parameters $m$

$$V(m) = [V(\omega_1;m), V(\omega_2;m), \ldots, V(\omega_{N_F};m)]^T; \hspace{1cm} (14)$$

(by exploiting the symmetries of $F\{\cdot\}$ for real signals here, $N_F$ is lower or equal to half the number of time samples). The inverse problem consists in finding the set of model parameters $m$ that minimizes the $l_2$ norm.

$$Q(m) = ||V_{\text{obs}} - V(m)||^2.$$  \hspace{1cm} (15)

According to Parseval’s relations, the minimization can be carried out either in time or in the frequency domain. In this work, the frequency domain has been preferred for computational convenience since this is directly related to the forward modeling approach (Appendix A).

Owing to the nonlinear dependence of the simulated signals $V(m)$ on the model parameters, the minimization of $Q(m)$ is a nonlinear problem. Different minimization strategies can be exploited. Here we propose the use of the iterative Gauss-Newton method that, for each step, linearizes $V(m)$ around the current model $m^{(i)}$

$$V(m) \approx V^{(i)} + A \cdot (m - m^{(i)}).$$

where $[A]_{jk} = \frac{\partial V(\omega_i;m)}{\partial m_k}|_{m=\text{arr}^{(i)}}$.  \hspace{1cm} (16)

The model updating for the $(i+1)$-th iteration is obtained by backprojecting the residual of the previous iteration $\Delta V^{(i)} = V_{\text{obs}} - V^{(i)}$ onto the model space

$$m^{(i+1)} = m^{(i)} + \text{Re}^{-1} \{A^HA\} \cdot \text{Re}\{A^H\Delta V^{(i)}\}. \hspace{1cm} (17)$$

The optimum permittivity model $\hat{m}_{\text{opt}}$ is the one that lets the iterations converge to a minimum of $Q(m)$: $\hat{m}_{\text{opt}} = m^{(\infty)}$ (or when the residual is small enough). Because of the nonlinear dependence of $V(m)$ on the model parameters, the optimization technique converges to a solution directly dependent upon the algorithm initialization $m^{(0)}$. To avoid the convergence to local minima, the starting model $m^{(0)}$ is derived from a layer-stripping approach, which provides a first rough estimation of the permittivity profile through a measure of the amplitude and delay of the backscattered echoes [7], [14].

The matrix of partial derivatives $A$ depends on the parameterization chosen to describe the permittivity and on the forward modeling. For the EM field scattered by the layered medium, the matrix $A$ can be efficiently evaluated by exploiting the recursive structure of the reflectivity functions (the complete derivation is in Appendix B).
A. Structured a Priori Information (Gaussian Model)

Resolution and SNR of measured data $V_{\text{obs}}$ decrease with depth and thus, the inversion algorithm turns unstable unless a different objective function is considered. The main drawback of the objective function (15) is that there is no use of the knowledge of noise level (or SNR) or any a priori information. Here we propose to estimate the permittivity profile by maximizing the a posteriori probability density for the model $\mathbf{m}$. Simplicity and robustness suggest here the use of a Gaussian model (see [15] for a complete discussion).

Under Gaussian assumption, the a posteriori density can be obtained by combining the measurement errors and modelization errors. According to the a posteriori model, the objective function can be rewritten as a weighted sum of two terms

$$Q(\mathbf{m}) = \| V_{\text{obs}} - V(\mathbf{m}) \|^2_{C_{\text{obs}}^{-1}} + \| \mathbf{m} - \mathbf{m}_{\text{ap}} \|^2_{C_{\text{m}}^{-1}}.$$  \hfill (18)

The covariance matrix $C_{\text{obs}}$ takes into account Gaussian noise in measurements. The range of admissible solutions is around the a priori permittivity model $\mathbf{m}_{\text{ap}}$, while the covariance matrix $C_{\text{m}}$ represents the a priori model uncertainties. The iterative search (17) can be modified to accommodate such a priori information [15]

$$\mathbf{m}^{(i+1)} = \mathbf{m}^{(i)} + \frac{1}{\mathbf{C}_{\text{m}}} \left[ \mathbf{A}^T \left( \mathbf{C}_{\text{obs}}^{-1} \right)^{-1} \mathbf{A} \mathbf{m}^{(i)} - \mathbf{m}_{\text{ap}} + \mathbf{C}_{\text{m}}^{-1} \Delta \mathbf{v}^{(i)} \right].$$  \hfill (19)

In practice, the choice of a priori information ($\mathbf{m}_{\text{ap}}$ and $C_{\text{m}}$) and the covariance matrix $C_{\text{obs}}$ is not always straightforward. The covariance matrix $C_{\text{obs}}$ can be diagonal (i.e., in the frequency domain, the noise samples are uncorrelated), and the simple choice preferred here is the white noise model $C_{\text{obs}} = \sigma_{\text{obs}}^2 \mathbf{I}$. In this paper, we also propose to assign the a priori information by using a structured model as this simplifies the choice of $\mathbf{m}_{\text{ap}}$ and $C_{\text{m}}$.

The contributions of the a priori information can be separated into two structured terms as follows:

$$\| \mathbf{m} - \mathbf{m}_{\text{ap}} \|^2_{C_{\text{m}}^{-1}} = \| \mathbf{m} - \mathbf{m}_{\text{ap}} \|^2_{C_{\text{m}}^{-1}} + \Delta \mathbf{v}^2 \| \frac{\partial \mathbf{v}(\mathbf{m})}{\partial \mathbf{z}} \|^2 \| \mathbf{C}_{\text{m}}^{-1} \mathbf{C}_{\text{m}}^{-1} \mathbf{D}.$$  \hfill (20)

The first term is related to the a priori knowledge of the range of admissible solutions around $\mathbf{m}_{\text{ap}}$, where covariance matrix $C_{\text{m}}$: $\| \partial \mathbf{v}(\mathbf{m}) / \partial \mathbf{z} \|^2$ is related to the variations of the vertical permittivity. To have the a priori information structured as in (18), the covariance matrix and the permittivity profile must be set as

$$C_{\text{m}} = \left( \mathbf{C}_{\text{m}}^{-1} + \mathbf{D}^T \mathbf{C}_{\text{m}}^{-1} \mathbf{D} \right)^{-1}$$

$$\mathbf{m}_{\text{ap}} = \left( \mathbf{C}_{\text{m}}^{-1} + \mathbf{D}^T \mathbf{C}_{\text{m}}^{-1} \mathbf{D} \right)^{-1} \mathbf{C}_{\text{m}}^{-1} \mathbf{m}_{\text{ap}}.$$ \hfill (21)

$$\mathbf{m}_{\text{ap}} = \left( \mathbf{C}_{\text{m}}^{-1} + \mathbf{D}^T \mathbf{C}_{\text{m}}^{-1} \mathbf{D} \right)^{-1} \mathbf{C}_{\text{m}}^{-1} \mathbf{m}_{\text{ap}}.$$ \hfill (22)

The equivalences in (20) can be proved by substitution and hold (only) for the purpose of minimizing the objective function (18).

The choice

$$\mathbf{D} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}.$$ \hfill (23)

is the discrete version of the first-order derivative ($\mathbf{D} \cdot \mathbf{m} = [m_2 - m_1, m_3 - m_2, \ldots, m_N - m_{N-1}]^T$) and represents an approximation of the vertical permittivity gradient (sampling $\Delta \mathbf{v}$ is included in $C_{\mathbf{D}}$). To simplify the choice of the covariance matrices, it is useful to consider a diagonal matrix for the a priori model $C_{\text{m}} = \sigma_{\text{m}}^2 \mathbf{I}$ (i.e., the samples of the a priori model are uncorrelated), while the correlation between the samples of the permittivity model can be incorporated into the penalty term for the vertical permittivity variations in (20). The covariance matrix $C_{\mathbf{D}} = \sigma_{\mathbf{D}}^2 \mathbf{I}$ can be similarly chosen as diagonal. In this case, the second term in the a priori (20) becomes $\sigma_{\mathbf{D}}^2 \| \mathbf{m} - \mathbf{m}_{\text{ap}} \|^2_{C_{\mathbf{D}}^{-1}} = \sigma_{\mathbf{D}}^2 \| \mathbf{D} (m_n - m_{n-1}) \|^2$. For these choices, matrix $C_{\mathbf{m}}^{-1}$ is tridiagonal, and the values depend on the relative weight of each term in the structured a priori information (20).

Remark 1: In the experiments (Section V), the a priori model has been chosen by assuming for $\mathbf{m}_{\text{ap}}$ the permittivity profile obtained from the initialization $\mathbf{m}_{\text{ap}} = \mathbf{m}^{(0)}$. It is understood that inversion can be better constrained by core samples, if available. Selection of a priori parameters can be simplified by choosing $\eta_{\mathbf{D}} = \sigma_{\mathbf{m}}^2 / \sigma_{\mathbf{D}}^2$ and $\eta_{\mathbf{m}} = \sigma_{\text{m}}^2 / \sigma_{\mathbf{m}}^2$; the iterative model updating (19) can be modified accordingly as

$$\mathbf{m}^{(i+1)} = \mathbf{m}^{(i)} + \left( \frac{1}{\mathbf{C}_{\text{m}}} \left[ \mathbf{A}^T \left( \mathbf{C}_{\text{obs}}^{-1} \right)^{-1} \mathbf{A} \mathbf{m}^{(i)} - \mathbf{m}_{\text{ap}} + \mathbf{C}_{\text{m}}^{-1} \Delta \mathbf{v}^{(i)} \right] + \eta_{\mathbf{m}} \left( \mathbf{m}^{(i)} - \mathbf{m}_{\text{ap}} \right) \right).$$  \hfill (24)

The choice of $\eta_{\mathbf{D}}$ depends on the relative importance between the a priori solution (small $\eta_{\mathbf{D}}$) and the smoothness of the permittivity profile (large $\eta_{\mathbf{m}}$). $\eta_{\mathbf{m}}$ is a tradeoff between prevalence of the structured a priori information (large $\eta_{\mathbf{m}}$) and good quality (high SNR) in experimental measurements (small $\eta_{\mathbf{m}}$). Note that the dimensionality should be included in these terms.

Remark 2: Under the Gaussian model, the a posteriori maximization is equivalent to the least-square approach with regularization [15]. The a priori constraints are degrees of freedom that, after some practice, can be exploited to reduce the probability that the iterative scheme converges to a “meaningless” solution (e.g., we experienced that the a priori solution $\mathbf{m}_{\text{ap}}$ can be released after few iterations). In any case, the starting model $\mathbf{m}^{(0)}$ and the a priori parameters $\{ \eta_{\mathbf{D}}, \eta_{\mathbf{m}}, \mathbf{m}_{\text{ap}} \}$ are essential to tune-up the optimization strategy to the specific example. The a priori information structured differently from (20) has not been considered for this application.

IV. EXPERIMENTAL RESULTS

Some experimental results are now discussed to verify the accuracy of the method presented in this work. They refer to two cases. In the first one, the modelization, calibration, and inversion schemes are applied to a simulated experiment obtained with a commercial finite element simulator (HFSS™ program
by Hewlett Packard). This experiment allows us to evaluate the limits and the errors obtained by using the method proposed in this work on both the received signals and the dielectric profile. This ensures that small errors in the received signals correspond to small errors in the dielectric constant estimation. In the second example, the method is applied to an experimental set of data obtained in a pavement profiling data acquisition campaign with the system in Fig. 1.

A. Comparison With Finite Elements Results

A TEM horn antenna has been simulated by using the HFSS™ program. Owing to the overwhelming memory demand and computation time for the simulation of the horn, an optimized design of the TEM horn was not feasible with the program. The horn was designed with two facing electric walls and two facing magnetic walls. The cross section was varied gradually in order to reduce the abrupt discontinuity between the input waveguide and the antenna aperture section and thus to present a small reflection coefficient throughout the operation bandwidth of the GPR system.

As discussed in Section III, a 5-dipole model was set up and the sources coefficients $J_{0E}$ were determined in the calibration phase (an ideal metallic reflector was used as a reference). With the sources distribution determined during calibration, a full inversion on a layered dielectric was carried out. The dielectric was set at $h_1 = 17$ cm from the antenna terminal section and it was composed of a layer having $\varepsilon_r = 6$ and thickness $h_2 = 6$ cm, terminated by air ($\varepsilon_r = 1, \varepsilon_3 = \infty$). The results obtained by the inversion algorithm in terms of relative permittivity are $\mathbf{m}^{(\infty)} = \{1, 5.96, 0.98\}$ (the initialization profile is $\mathbf{m}^{(0)} = \{1, 4, 3\}$). Although this experiment was controlled, some errors are still present, such as numerical accuracy of the HFSS™ simulator, imperfect absorbing boundary conditions, edge diffraction from the finite-size dielectric media (infinity extent dielectric are not implemented in the simulator) and from the antenna terminal section. In spite of that, a remarkable accuracy in the dielectric constant profile was obtained, even if the unconstrained inversion ($\eta_{D} = \eta_{M} = 0$) lets the solution converge to a relative dielectric constant lower than unity (a priori constraints, if used, can easily avoid meaningless solutions as $\varepsilon_r < 1$ for the half-space).

B. Experiments on Pavements Profiling

In the experimental measurements acquired with the GPR system in Fig. 1, the antenna is positioned at height $h_1 = 21$ cm from the ground. Distance between scans is about $\Delta l = 30$ cm. The transmitted signal, shown in Fig. 4, is a monocyclic impulse with a pulse-width $T_{ip} = 1$ ns. The resolution range is $\Delta R_z \approx 6$ cm. Since the quantitative analysis of GPR measurements is sensitive to the nonideality of the acquisition, additional processing is needed before EM inversion in order to compensate for vehicle vibration and antenna height fluctuations [7].

Here we show the results of the EM inversion performed on two different real data set, each composed of about 70 neighboring scans, corresponding to approx. a 20 m distance of survey along the highway-road. The model in the EM inversion algorithm has been chosen with a discretization interval $\Delta z = 4$ cm (lower than the resolution range), a number of model parameters (or layers) $M = 12$, and a corresponding maximum investigated range $z_{\text{max}} = (M - 1)\Delta z = 44$ cm.

For each dataset, the measured signals $v_{\text{obs}}(l, t)$ are displayed. Using the same scale, the residual $\delta t(l, t; \mathbf{m}_{\text{obs}}) = v_{\text{obs}}(l, t) - v(l, t; \mathbf{m}_{\text{obs}})$ is also shown. The MSE value indicates the energy of $\delta t(l, t; \mathbf{m}_{\text{obs}})$ divided by the energy of the measured signals $MSE = \sum_{l,t} \delta t(l, t; \mathbf{m}_{\text{obs}})^2 / \sum_{l,t} v_{\text{obs}}(l, t)^2$. In Fig. 5 (left), the measured data of the first dataset are shown. It is possible to distinguish three main reflections: the first at 3.7 ns associated with the air-asphalt interface, the second at 4.8 ns associated with the interface between two slightly different asphalt layers, and the third at 9.2 ns associated with the asphalt-concrete interface. In the same figure (right), the obtained residual is shown, corresponding to $MSE = 4.5\%$. The recovered permittivity profile is shown in Fig. 6.
V. CONCLUSION

A novel approach has been proposed for EM inversion applied to GPR pavements profiling. The approach can model the interaction of a TEM horn GPR antenna with a scatterer, also taking into account near-field effects. The medium is assumed (locally) layered, and losses within the layers are assumed to be known. The TEM horn antenna has been modeled by a distribution of equivalent point (Huygens) sources useful in describing the antenna-medium interaction. For the actual GPR system, the calibration results show that a five-point sources model represents a satisfactory tradeoff between accuracy (MSE ≈ 9%) and computing time. In fact, an increase in the number of equivalent point sources slightly improves the accuracy (up to 5% for 16 dipoles) but heavily degrades the computational efficiency of the algorithm.

The permittivity model is estimated as a result of the minimization of the energy of the error between simulated and measured signals. Iterative minimization is based on Gauss-Newton method. Computational efficiency of iterative minimization is improved by the representation of the EM field for layered media (Sommerfeld type integrals). The a priori information useful to improve the stability of the inversion algorithm can be easily (and efficiently) structured.

The EM inversion has been performed on real monostatic GPR data with MSE ranging from 2% to 4%. Data with strong lateral variations can have an MSE ≈ 7%. In such cases, the horizontally layered one-dimensional (1-D) model loses its validity in part, but it is still a good approximation for most practical purposes.

In any case, for a good inversion accuracy, it is necessary that the experimental setup during data acquisitions be carefully controlled as compared to normal operation. The solution is to control (or at least monitor) all the fluctuations of the system parameters (e.g., antenna height, system gain, and pulse shape stability).

APPENDIX A

SOMMERFELD-TYPE REPRESENTATION OF THE ELECTROMAGNETIC FIELD

The electromagnetic field scattered by a layered medium (Fig. 2) can be obtained in the form of Sommerfeld-type integrals. The field within the \( i \)-th layer \( \vec{E}_i \), \( \vec{H}_i \) can be expressed as a superposition of a transverse electric (TE) field and a transverse magnetic (TM) field (see [9])

\[
\vec{E}_i = \vec{E}^{\text{TE}}_i + \vec{E}^{\text{TM}}_i, \quad \vec{H}_i = \vec{H}^{\text{TE}}_i + \vec{H}^{\text{TM}}_i
\]

(vector phasors notation is used within the Appendix).

The TE and TM fields are conveniently represented, respectively, by a single-component electric potential vector \( \vec{F}_i = F_{z,i} \hat{z} \) and by a single-component magnetic potential...
vector $\vec{A}_0 = A_{0z} \hat{Z}$, both solutions of Helmholtz equation in each layer. Sommerfeld-type integrals for the potentials in layer 1 (where the sources are located) are

$$
\left[ \begin{array}{c}
F_{1z}^2 \\
F_{1\phi}^2
\end{array} \right] = \left[ \begin{array}{c}
P_x \sin \phi \\
-P_y \cos \phi
\end{array} \right] \frac{k_0^2}{4\pi \omega} \int_{0}^{\infty} R^{TE} J_1(k_0 \rho \rho) \, d\rho,$n

$$
(26a)
A_{1z}^2 = A_{1\phi}^2 = \left[ \begin{array}{c}
P_x \cos \phi \\
M_y \sin \phi
\end{array} \right] \frac{\mu}{4\pi} \int_{0}^{\infty} R^{TM} J_1(k_0 \rho \rho) \, d\rho,$n

$$
(26b)
for an electric dipole and

$$
\left[ \begin{array}{c}
F_{1z}^M \\
F_{1\phi}^M
\end{array} \right] = \left[ \begin{array}{c}
M_x \cos \phi \\
M_y \sin \phi
\end{array} \right] \frac{\epsilon_1}{4\pi} \int_{0}^{\infty} R^{TE} J_1(k_1 \rho \rho) \, d\rho,$n

$$
(26c)
A_{1z}^M = A_{1\phi}^M = \left[ \begin{array}{c}
-M_x \sin \phi \\
M_y \cos \phi
\end{array} \right] \frac{k_0^2}{4\pi \omega} \int_{0}^{\infty} R^{TM} J_1(k_1 \rho \rho) \, d\rho,$n

$$
(26d)
for a magnetic dipole. Here

$$
R^{TE} = R_1^{TE} e^{jk_1 z_1} (2h_1 - z),
$$

$$
(27)
R^{TM} = R_1^{TM} e^{jk_1 z_1} (2h_1 - z),
$$

$$
(28)
da_{R_1}^{TE}$ and $R_1^{TM}$ are, respectively, the TE and TM Fresnel reflection coefficients of the layered medium as seen from the first interface $J_0(\cdot)$ and $J_1(\cdot)$ are Bessel functions of first kind of order zero. $P_x, P_y, M_x, M_y$ are, respectively, the components of the electric and magnetic dipole moment of the sources. Finally, $k_{z_1} = \sqrt{k_0^2 - k_1^2}$, where $k_0^2 = j \omega \mu (\epsilon_0 - \dot{\epsilon} \epsilon_i).

From the knowledge of vector potentials, the searched expressions of the reflected EM field ($\vec{E}_R$ and $\vec{H}_R$) in layer 1 can be obtained as

$$
\vec{E}_R = \vec{E}_1^{TE} + \vec{E}_1^{TM}
$$

$$
(29)
= \frac{1}{\epsilon_1} \nabla \times F_{1z} + j \omega \frac{\epsilon_1^2 + \nabla \partial}{\nabla \zeta} A_{1z} \nabla \times \frac{\epsilon_1^2 + \nabla \partial}{\nabla \zeta} A_{1z}
$$

$$
(30)
\vec{H}_R = \vec{H}_1^{TE} + \vec{H}_1^{TM}
$$

The computing time to evaluate the scattered field mainly depends on the algorithm used to compute Sommerfeld integrals ((26a)–(26d)). In the literature, this problem has been the subject of several studies (see [11]). For high values of $k_\rho$, the integrand functions decrease exponentially. This allows us to easily truncate the numerical integration. The poles of the integrand functions are located in proximity to the real axis, in the interval $[k_0 \sqrt{\epsilon_{\text{min}}}, k_0 \sqrt{\epsilon_{\text{max}}}]$, where

$$
\epsilon_{\text{min}} = \min (\epsilon_1, \epsilon_2, \ldots, \epsilon_N)
$$

$$
(31)
\epsilon_{\text{max}} = \max (\epsilon_1, \epsilon_2, \ldots, \epsilon_N)
$$

$$
(32)
A convenient path in the complex $k_\rho$ plane that avoids poles can be used. This leads to a smoother integrand function and to considerable time saving in the computation. Several paths can be used [9], the best choice being a compromise between the smoothness of the function and the path length. Time reduction with respect to simple real axis integration schemes can be as high as 10–20.

**APPENDIX B**

**GRADIENT OF THE REFLECTIVITY FUNCTION**

Elements of matrix of partial derivatives $A$ can be evaluated with the chain rule of partial differentiation

$$
[A]_{j,k} = \frac{\partial N(\omega; \mathbf{m}(\rho))}{\partial \epsilon_k},
$$

$$
(33)
where $\mathbf{m}$ is the complex dielectric medium.

For the specific parameterization chosen here, the elements of $A$ can be evaluated from the gradient of the reflectivity function $R_1(\omega; \mathbf{m})$ at the first interface (TE and TM). One finds

$$
\frac{\partial N(\omega; \mathbf{m}(\rho))}{\partial \epsilon_j} = -2 \nu \sum_{z=1}^{N_z} \nabla \times \frac{\vec{J}_z \cdot \vec{\mathbf{k}}}{\epsilon_j} - \eta k \times \vec{J}_z \cdot \frac{\partial N(\omega; \mathbf{m}(\rho))}{\partial \epsilon_j},
$$

$$
(34)
For simplicity, only the electric field equations are presented in a more detailed form (the magnetic field equations are quite analogous and similar expressions are obtained)

$$
\frac{\partial \vec{H}_z}{\partial \epsilon_j} = \frac{\partial \vec{H}_z}{\partial \epsilon_j} + \frac{\partial \vec{H}_z}{\partial \epsilon_j}
$$

$$
= \frac{1}{\epsilon_1} \nabla \times \frac{\epsilon_1^2 + \nabla \partial}{\nabla \zeta} A_{1z}
$$

$$
(35)
where

$$
\frac{\partial A_{1z}}{\partial \epsilon_j} = \frac{\partial A_{1z}}{\partial \epsilon_j} + \frac{\partial A_{1z}}{\partial \epsilon_j}
$$

$$
= \frac{P_{xz} \cos \phi}{P_{xy} \sin \phi}
$$

$$
(36a)
$$
$$
(36b)
Now let $R_1(\omega; \mathbf{m})$ denote $R^{TE}$ or $R^{TM}$. In (36a) and (36b), the gradient $\partial R_1(\omega; \mathbf{m})/\partial \epsilon_j$ of the reflectivity function $R_1(\omega; \mathbf{m})$ can be evaluated recursively. The recursive relationships are (for the sake of conciseness, frequency and model dependency is understood)

$$
R_{N-1} = \Gamma_{N-1} R_{N-1} + \Gamma_{N-1} R_{N-1} R_{N-1} R_{N-1},
$$

$$
(37)
\Gamma_{N-1} = \frac{R_{N-1} R_{N-1} R_{N-1}}{R_{N-1} R_{N-1} R_{N-1}},
$$

$$
(38)
\Gamma_{N-1} = \frac{R_{N-1} R_{N-1} R_{N-1}}{R_{N-1} R_{N-1} R_{N-1}}.
$$

Here, $\Gamma_{N-1}$ denotes the reflectivity coefficient associated with the interface between the $i$-th and $(i-1)$-th layers. $h_i$ and $k_{z_i}$ are, respectively, the thickness and the vertical wavenumber associated with the $i$-th layer.
The gradients in (36a)–(36b) need to be evaluated from the bottom layers up to the first layer. Let the layered medium be composed of the last two layers whose reflectivity function is the reflection coefficient associated with the interfaces. In this case, the expression of the partial derivatives of $R_{N-1}$ is straightforward

\[
\frac{\partial R_{N-1}}{\partial \epsilon_j} = 0 \quad 1 \leq j < N - 1
\]  
\[
\frac{\partial R_{N-1}}{\partial \eta_j} = \frac{\partial R_{N-1}}{\partial \epsilon_j} = 0 \quad 1 \leq j < N - 1
\]  
\[
\frac{\partial R_{N-1}}{\partial \epsilon_N} = \frac{\partial R_{N-1}}{\partial \epsilon_j} = 0
\]

(38a)  
(38b)  
(38c)

Obviously, the reflectivity function for the $i$-th layer depends only on the parameters of all the layers underneath. The relationships (38a)–(38c) can be iterated by evaluating $\frac{\partial R_{N-2}}{\partial \epsilon_j}$

\[
\frac{\partial R_{N-2}}{\partial \epsilon_j} = 0 \quad 1 \leq j < N - 2
\]

(38d)

\[
\frac{\partial R_{N-2}}{\partial \eta_j} = \frac{\partial R_{N-2}}{\partial \epsilon_j} = 0 \quad 1 \leq j < N - 2
\]

(38e)

\[
\frac{\partial R_{N-2}}{\partial \epsilon_N} = \frac{\partial R_{N-2}}{\partial \epsilon_j} = 0
\]

(38f)

\[
\frac{\partial R_{N-2}}{\partial \epsilon_N} = \frac{\partial R_{N-2}}{\partial \epsilon_j} = 0
\]

(38g)

\[
\frac{\partial R_{N-2,1}}{\partial \epsilon_j} = \frac{\partial R_{N-2,1}}{\partial \epsilon_j} = 0
\]

(38h)

where $\frac{\partial R_{N-1,1}}{\partial \epsilon_j}$ (for $j = 1, \ldots, N$) are available from the previous step. The other partial derivatives can be easily calculated. The method can be iterated to the $i$-th layer

\[
\frac{\partial R_{i-1}}{\partial \epsilon_j} = 0 \quad 1 \leq j < i - 1
\]

(38i)

\[
\frac{\partial R_{i-1}}{\partial \epsilon_i} = \frac{\partial R_{i-1}}{\partial \epsilon_i} = 0
\]

(38j)

\[
\frac{\partial R_{i-1}}{\partial \epsilon_j} = \frac{\partial R_{i-1}}{\partial \epsilon_j} = 0
\]

(38k)

and the terms $\frac{\partial R_{i}}{\partial \epsilon_j}$ are available from the $(i - 1)$-th step.

The gradient of the reflection coefficients with respect to the permittivity model are (distinguishing between TM and TE modes)

\[
\frac{\partial \Gamma_{TM,i-1,i}}{\partial \epsilon_i} = \frac{\epsilon_i - 1}{\epsilon_i \epsilon_0 k_{z,i-1}^2} 2 \epsilon_i k_{z,i} k_{z,i-1}^2 1 - \Gamma_{N-2,N-1} k_{z,i}^2
\]

(39)

\[
\frac{\partial \Gamma_{TE,i-1,i}}{\partial \epsilon_i} = \frac{\epsilon_i - 1}{\epsilon_i \epsilon_0 k_{z,i-1}^2} 2 \epsilon_i k_{z,i} k_{z,i-1}^2 1 - \Gamma_{N-2,N-1} k_{z,i}^2
\]

(40)

\[
\frac{\partial \Gamma_{TM,i-1,i}}{\partial \epsilon_j} = \frac{\epsilon_i - 1}{\epsilon_i \epsilon_0 k_{z,i-1}^2} 2 \epsilon_i k_{z,i} k_{z,i-1}^2 1 - \Gamma_{N-2,N-1} k_{z,i}^2
\]

(41)

\[
\frac{\partial \Gamma_{TE,i-1,i}}{\partial \epsilon_j} = \frac{\epsilon_i - 1}{\epsilon_i \epsilon_0 k_{z,i-1}^2} 2 \epsilon_i k_{z,i} k_{z,i-1}^2 1 - \Gamma_{N-2,N-1} k_{z,i}^2
\]

(42)

Following this recursive procedure, it is possible to obtain the desired expression of the gradient of the reflectivity function associated with the layered medium.

ACKNOWLEDGMENT

The authors wish to thank two former students, S. Agosti and M. Pazzi, who devoted much of their efforts to software developments and to the experimental activity described here. They also wish to thank an anonymous reviewer for stimulating comments that helped us modify the manuscript.

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