Input/output Impedances and Gain Calculations

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Outline

• Choice of a suitable test source
• Ex: Differential stage
• Impedance reconstruction
• Ex: Buffer stage
• Compensation
Ideal case (current source)

\[ V^- = 0 \Rightarrow Z_{id} = \frac{V^-}{I_s} = 0 \Rightarrow Z_{in} = \frac{Z_{OL}}{1 - G_{loop}} \]
\[ Z_{OL} \text{ and } G_{loop} \]

\[ Z_{OL} = R_i \parallel R \quad G_{loop} = -A(s) \frac{R_i}{R + R_i} \]
Final result

\[ Z_{in} = \frac{Z_{OL}}{1 - G_{loop}} = \frac{RR_i}{R + Ri} = \frac{RR_i}{R + Ri + AR_i} \]

Side note: another way to look at the circuit is to consider \( R_i \) in parallel to the rest. The remaining circuit has \( Z_{OL} = R \) and \( G_{loop} = -A \), so that

\[ Z_{in} = R_i \parallel \frac{R}{1 + A} = \frac{RR_i}{R + Ri + AR_i} \]
Ideal case (voltage source)

\[ V_o = -\infty \Rightarrow I_s = \infty \Rightarrow Z_{id} = \frac{V^{-}}{I_s} = 0 \]

\[ \Rightarrow Z_{in} = \frac{Z_{OL}}{1 - G_{loop}} \]
$Z_{OL}$ and $G_{loop}$

$Z_{OL} = R_i \parallel R$

$G_{loop} = 0$
Comments

• Feedback circuit theory works if the loop is not broken, i.e., if $G_{loop} \neq 0$

• If you use the «wrong» source, you end up with $\infty$ in voltages and/or currents, i.e., with inconsistencies in the equations (e.g., biasing a virtual ground)

• Practically
  – Low Z (e.g., virtual ground, inv. input of OAs) $\Rightarrow$ current source
  – High Z (e.g., non inv. input of OAs) $\Rightarrow$ voltage source

• Of course, if you are NOT using the feedback theory but just solve the network, you get the correct result whatever source you use
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Differential stage

\[ V_o = \frac{V_1}{R_1} R_2 - \frac{V_2}{R_3} R_4 \]

\[ R_i = 2 \, \text{M}\Omega \]
\[ R_o = 75 \, \Omega \]
\[ GBWP = 1 \, \text{MHz} \]
\[ A_0 = 126 \, \text{dB} \]

1. Size the resistors to achieve a gain of 10
2. Compute input and output resistances
Resistor sizing

\[ V_o = -\frac{R_2}{R_1} V_1 + \frac{R_2}{R_1} \frac{1 + R_1/R_2}{1 + R_3/R_4} V_2 \]

\[ \frac{R_2}{R_1} = \frac{R_4}{R_3} = G \]

We pick \( R_1 = R_3 = 10 \text{ kΩ} \); \( R_2 = R_4 = 100 \text{ kΩ} \)
\[ Z_1 = R_1 + Z' \]

\[ Z_{id}' = 0 \text{ (virtual ground)} \]

\[ Z' = \frac{Z'_{OL}}{1 - G'_{loop}} \]

\[ Z'_{OL} = (R_i + R_3 \parallel R_4) \parallel (R_2 + R_0) = 95.2 \text{ kΩ} \]

Turn off VCVS to compute \( Z'_{OL} \)
$G'_{\text{loop}}$

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Impedance reconstruction unnecessary
Result

\[
G'_{loop} = -A(s) \frac{R_i}{R_i + R_3 \parallel R_4 + R_2 + R_o} = -0.95 A(s)
\]

At low frequencies, \( A(s) = A_0 = 2 \times 10^6 \Rightarrow G'_{loop} = 1.9 \times 10^6 \) and \( Z' = 0.05 \Omega = 50 \text{ m}\Omega \)

\[
Z_1 = R_1 + Z' = R_1 = 10 \text{ k}\Omega
\]
\[ Z_2 = R_3 + R_4 \parallel Z'' \]

\[ Z_{id} = \infty \ (V_d = 0) \]

\[ Z'' = Z_{OL}'(1 - G_{loop}) \]

\[ Z_{OL}'' = R_i + R_1 \parallel \]

\[ (R_2 + R_o) \approx R_i = 2 \ \text{M}\Omega \]

Turn off VCVS to compute \( Z_{OL}'' \)
$G''_{\text{loop}}$
Result

\[ G'''_{loop} = -A(s) \frac{R_1 \parallel R_i}{R_1 \parallel R_i + R_2 + R_o} \]

\[ = -0.09 \, A(s) \]

At low frequencies, \( A(s) = A_0 = 2 \times 10^6 \Rightarrow \)

\[ G'''_{loop} = 1.81 \times 10^5 \text{ and } Z'' = 3.6 \times 10^{11} \, \Omega = 360 \, \text{G} \Omega \]

\[ Z_2 = R_3 + R_4 \parallel Z'' = R_3 + R_4 = 110 \, \text{k} \Omega \]
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Breaking the loop

• In a block scheme, it makes no difference where you choose to break the loop
• In a real circuit, you must terminate the loop end with the impedance that existed before you broke the loop
• In reality, things are even more complex because you must be careful not to modify the DC operating point
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1. Evaluate the loop gain using A, B and C as breakpoints
2. Compensate if needed (negl. \( R_i \))
Loop breaking – case A

\[ Z = R + \frac{1}{sC} \parallel R_i = \frac{(R + R_i) \frac{1}{1 + sC(R \parallel R_i)}}{1 + sCR_i} \]
Calculations – case A

\[ G_{\text{loop}}^A = - \frac{1}{sC} \parallel R_i \frac{A(s)}{R + \frac{1}{sC} \parallel R_i} \frac{Z}{Z + R_o} \]

\[ = -A(s) \frac{1}{sC} \parallel R_i \frac{1}{R + R_o + \frac{1}{sC} \parallel R_i} \]
Loop breaking – case B

\[ Z = R_i \]
Calculations – case B

\[ G_{loop}^B = -A(s) \frac{1}{sC \parallel Z} \frac{1}{R + R_o + \frac{1}{sC} \parallel Z} \]

\[ = -A(s) \frac{1}{sC \parallel R_i} \frac{1}{R + R_o + \frac{1}{sC} \parallel R_i} \]
Loop breaking – case C

\[ Z = R_o + R + R_i + \frac{1}{sC} R_i \]

(but it is not needed for the calculations!)
Calculations – case C

\[ G_{\text{loop}}^C = -A(s) \frac{1}{sC} \parallel R_i \frac{1}{R + R_o + \frac{1}{sC} \parallel R_i} \]

\[ = -\frac{A_0}{1 + s\tau} \frac{R_i}{R + R_o + R_i} \frac{1}{1 + sC(R_i \parallel (R + R_o))} \]

\[ \approx -\frac{A_0}{1 + s\tau} \frac{1}{1 + sC(R + R_o)} \]

10 Hz \thinspace \thinspace 8.74 kHz
Bode plot

\[ G_1 = 100 \]

\[ G_{loop} \text{ } dB \]

\[ f_{p1} = 10 \]

\[ f_{p2} = 8.74k \]

\[ G_2 \]
\[ G_1 f_{p_1} = G_2 f_{p_2} \Rightarrow G_2 = G_1 \frac{f_{p_1}}{f_{p_2}} = 114 = 41 \text{ dB} \]

\[ G_2 f_{p_2}^2 = f_{0dB}^2 \Rightarrow f_{0dB} = f_{p_2} \sqrt{G_2} = 93.2 \text{ kHz} \]

\[ \phi_m = 180 - 90 - \arctan \left( \frac{f_{0dB}}{f_{p_2}} \right) = 5^\circ \]
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Compensation (negl. $R_i$)

$$G_{\text{loop}} = -A(s) \frac{sC}{R_o + R \parallel \frac{1}{sC_c} + \frac{1}{sC}}$$
Effect of $C_c$

\[
G_{\text{loop}} = -\frac{A_0}{1 + s\tau} \frac{1 + sC_cR}{1 + s(CR + CR_o + C_cR) + s^2CC_cRR_o}
\]

1. A zero is introduced in $G_{\text{loop}}$
2. A second pole also arises
3. The poles time constants are entangled
Sizing of $C_c - 1$

- Approximate values of pole frequencies:
  - $f_{p1} = \frac{1}{2\pi C (R + R_o)} = 8.74 \text{ kHz} \ (C_c = \text{open})$
  - $f_{p2} = \frac{1}{2\pi C_c (R \parallel R_o)} \ (C = \text{short}; \text{high-frequency})$

- To achieve a phase margin of 45° we then need $f_z = f_{0dB}$, i.e.
  $$\frac{1}{2\pi C_c R} = 93.2 \times 10^3 \Rightarrow C_c = 1.88 \text{ nF}$$

- The 2\textsuperscript{nd} pole falls at 8.47 MHz
Polynomial roots

• Given a 2\textsuperscript{nd} order polynomial

\[ as^2 + bs + 1 \]

The approximate values of the (real) roots are

\[ S_{LF} \approx -\frac{1}{b} \]

\[ S_{HF} \approx -\frac{b}{a} \]

• The method works fine for well-separated roots
Pole position

- Pole frequencies become:

\[-f_{p1} = \frac{1}{2\pi(C(R+R_o)+C_cR)}\]
\[-f_{p2} = \frac{C(R+R_o)+C_cR}{2\pi C_cRR_0}\]

- Values are similar to the previous ones if $C_c \ll C$. With $C_c = 1.88$ nF we get

\[f_{p1} = 8.0 \text{ kHz}; \ f_{p2} = 9.36 \text{ MHz}\]
Sizing of $C_C - 2$

$$\begin{align*}
G_0 f_{p0} &= G_1 f_{p1} \\
G_1 f_{p1}^2 &= f_z^2 \quad \Rightarrow \quad f_z^2 &= G_0 f_{p0} f_{p1}
\end{align*}$$

$$\left( \frac{1}{C_c R} \right)^2 = \frac{A_0}{\tau (CR + CR_0 + C_c R)}$$

- If $C_c \ll C$ we neglect the last term, achieving $C_c = \sqrt{\frac{\tau (CR + CR_0)}{A_0 / R}} = 1.88$ nF (same expression as before)

- The correct solution gives $C_c = 1.98$ nF
Sizing of $C_C - 3$

• For $f > f_{p1}$ the loop gain becomes

$$G_{loop} \approx \frac{A_0}{s^2\tau (CR + CR_0 + C_c R)}$$

• We put $|G_{loop}| = 1$ for $s = 1/C_c R$, obtaining

$$\left( \frac{1}{C_c R} \right)^2 = \frac{A_0}{\tau (CR + CR_0 + C_c R)}$$

Same equation as before!
Homework

1. Compute the output impedance for the differential stage in slide #10
2. Compute the differential input impedance for the differential stage in slide #10
3. Compute the loop gain for the buffer stage (slide #20) using a current source
4. Compensate the buffer stage using a resistor rather than a capacitor