Frequency behavior, stability and compensation

Alessandro Spinelli
Phone: (02 2399) 4001
alessandro.spinelli@polimi.it
home.deib.polimi.it/spinelli
Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes.
Outline

• Frequency response of feedback amplifiers
• Stability of feedback amplifiers
• Compensation
• Appendix
Frequency behavior

\[ G_{\text{loop}}(s) = -\frac{G_{\text{open}}(s)}{G_{id}(s)} \Rightarrow \]

\[ |G_{\text{loop}}(s)|_{dB} = |G_{\text{open}}(s)|_{dB} - |G_{id}(s)|_{dB} \]

\[ G = \frac{G_{\text{open}}(s)}{1 - G_{\text{loop}}(s)} = \frac{G_{id}(s)}{1 - 1/G_{\text{loop}}(s)} \]

\[ G_{\text{loop}} \gg 1 \Rightarrow G \approx G_{id} \]

\[ G_{\text{loop}} \ll 1 \Rightarrow G \approx G_{\text{open}} \]
Frequency behavior

\[ G_L(s) \]
\[ G_{open}(s) \]
\[ G_{loop}(s) \]
\[ G_{id}(s) \]

\[ G(s) \]

\[ |\cdot|_{dB} \]

\[ \log f \]
Single-pole amplifier

$G_{open}(s) = A(s)$

$G_{loop}(s)$

$G_{id}(s)$

$G(s)$

$log f$

$f_T = \frac{GBWP}{G_{id}}$

$\frac{A_0}{2\pi \tau} = GBWP$
Analytical solution

\[ G = \frac{G_{id}}{1 - \frac{1}{G_{loop}(s)}} = \frac{G_{id}}{1 + \frac{R_1 + R_2}{A(s)R_1}} \]

The pole position is

\[ \left( R_1 + R_2 \right) \left( 1 + s\tau \right) \frac{A_0 R_1}{A(s)R_1} = -1 \]

\[ s = -\frac{1}{\tau} \left( 1 + \frac{A_0 R_1}{R_1 + R_2} \right) = -\frac{1}{\tau} \left( 1 - G_{loop}(0) \right) \]
Gain-bandwidth product

• For a non-inverting amplifier we have

\[ f_p \approx -\frac{1}{2\pi\tau G_{id}} \Rightarrow |f_p|G_{id} = \frac{A_0}{2\pi\tau} = GBWP \]

• In an inverting configuration we should write

\[ |f_p|(1 - G_{id}) = GBWP, \]

which becomes the same for high gains

• The feedback loop reduces the (open-loop) gain by \(1 - G_{\text{loop}}(0)\) and widens the bandwidth by the same factor
Stability of feedback systems

- Stability only depends on \( G_{\text{loop}} \)
- The critical condition is \( G_{\text{loop}} = 1 \), i.e., \( -G_{\text{loop}} = -1 \)
Outline

• Frequency response of feedback amplifiers
• Stability of feedback amplifiers
• Compensation
• Appendix
Bode stability criterion (1945)

• If
  – $G_{loop}(s)$ only has poles in LHP (or in $s = 0$)
  – There is only one critical frequency $f_{180}$ where the phase of $-G_{loop}$ is $180^\circ$ ($\pm$ multiples of $360^\circ$)
  – $|G_{loop}(f_{180})| < 1$
• Then, the system is stable
Gain and phase margins

\[ G_m = \frac{1}{|G_{\text{loop}}(f_{180})|} \quad G_m\big|_{dB} = -G_{\text{loop}}(f_{180})\big|_{dB} \]

\[ \varphi_m = 180 + \angle \left( -G_{\text{loop}}(f_{0dB}) \right) \]

- \( G_m \) and \( \varphi_m \) represent how much increase in gain or phase lag the system can withstand before becoming unstable.

- Important in real systems, where transfer functions are subjected to tolerances.
Simplified Bode criterion

If poles and zeros are in LHP, stability can be inferred from Bode plot.
How much phase margin?

- Step response
- \( \phi_m = 75^\circ \)
- \( \phi_m = 60^\circ \)
- \( \phi_m = 45^\circ \)
- \( \phi_m = 30^\circ \)

- Overshoot [%]

- Phase margin [°]
Outline

• Frequency response of feedback amplifiers
• Stability of feedback amplifiers
• Compensation
• Appendix
Frequency compensation of OAs

• Is the tailoring of $G_{\text{loop}}(s)$ in order to improve the circuit stability

• Most OAs are «internally compensated» for easing their use with resistive feedback, and have a single pole above 0dB

• For frequency-dependent feedback, stability must be checked and compensation applied (if needed)
Uncompensated OAs
Dominant pole compensation

\[ |G_{\text{loop}}(s)|_{\text{dB}} \]
Pole-zero compensation

$|G_{\text{loop}}(s)|_{dB}$

log $f$
Input capacitance

\[ G_{\text{loop}}(s) = -A(s) \frac{Z_1}{Z_1 + R_2} \]

\[ = -A(s) \frac{R_1}{R_1 + R_2} \frac{1}{1 + sC_i (R_1 \parallel R_2)} \]
Compensation of input capacitance

\[ G_{\text{loop}} = -A(s) \frac{R_1}{R_1 + R_2} \frac{1 + sC_2R_2}{1 + s(C_i+C_2)(R_1 \parallel R_2)} \]
Resulting loop gain

\[ f_p = \frac{1}{2\pi (C_i + C_2) (R_1 \parallel R_2)} \]

\[ f_z = \frac{1}{2\pi C_2 R_2} \]

\[ f_p < f_z \]
Notes...

• $C_2$ modifies the closed-loop gain $\Rightarrow$ stability is traded off against bandwidth (now given by $f_z$)

• Another possibility is $C_2 R_2 = C_i R_1$ (pole-zero cancellation), but keep in mind that $C_i$ is never constant in reality...

• In differential amplifiers, use symmetric compensation
Lag network \((f_p < f_z)\)

\[
G_{loop} = -A(s) \frac{R_1}{R_1 + R_2} \frac{1 + sC_cR_c}{1 + sC_c(R_c + R_1 \parallel R_2)}
\]

Does not affect \(G_{id}\), but can degrade \(Z_{in}\) in NI amplifiers
A simple resistor $R_c$ between the OA inputs can be effectively used for compensation.

$$G_{loop} = -A(s) \frac{1}{1 + sR(C + C_i)}$$
Yet another compensation

LF:

HF:
Poles and zeros

\[ f_z = \frac{1}{2\pi C R_c} \]

\[ f_{p1} \approx \frac{1}{2\pi C (R_c + R)}; \quad f_{p2} \approx \frac{1}{2\pi C_i (R_c \| R)} \]

- \( f_{p2} \) is usually at high frequency and can be neglected
- Lag network \((f_{p1} < f_z)\) can be used for compensation
- Closed-loop gain bandwidth limited to \( \frac{1}{2\pi R_c C} \)
Additional pole in $G_{loop}$ must be above $GBWP$ (say, $10GBWP$) $\implies$ maximum load capacitance is

$$C_L \approx \frac{1}{2\pi R_o (10 \ GBWP)}$$
Compensation – 1

\[
f_z = \frac{1}{2\pi C_L R_C} \quad f_p \approx \frac{1}{2\pi C_L (R_c + R_O)}
\]

If \( R_1 + R_2 \gg R_c, R_o \)
Outline

• Frequency response of feedback amplifiers
• Stability of feedback amplifiers
• Compensation
• Appendix
Poles and zeros

• If $R_c, R_o \ll R_1 + R_2$

\[
\begin{align*}
fp_1 & \approx \frac{1}{2\pi C_L(R_c + R_o)}; & fz_1 & \approx \frac{1}{2\pi C_L R_c} \\
fp_2 & \approx \frac{1}{2\pi C_c(R_1 || R_2)}; & fz_2 & \approx \frac{1}{2\pi C_c R_2}
\end{align*}
\]

• Pole-zero cancellation leads to

\[
R_c = R_o \frac{R_1}{R_2}; \quad C_c = C_L R_o \frac{R_1 + R_2}{R_2^2}
\]