Frequency behavior, stability and compensation

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Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes.
Outline

• Frequency response of feedback amplifiers
• Stability of feedback amplifiers
• Compensation
• Appendix
Frequency behavior

\[ G_{loop}(s) = - \frac{G_{OL}(s)}{G_{id}(s)} \Rightarrow \]

\[ |G_{loop}(s)|_{dB} = |G_{OL}(s)|_{dB} - |G_{id}(s)|_{dB} \]

\[ G = \frac{G_{OL}(s)}{1 - G_{loop}(s)} = \frac{G_{id}(s)}{1 - 1/G_{loop}(s)} \]

\[ G_{loop} \gg 1 \Rightarrow G \approx G_{id} \]

\[ G_{loop} \ll 1 \Rightarrow G \approx G_{OL} \]
Frequency behavior

\[ |\cdot|_{dB} \]

\[ G_{loop}(s) \]

\[ G_{OL}(s) \]

\[ G_{id}(s) \]

\[ G(s) \]

\[ \log f \]
Single-pole amplifier

\[ G_{OL}(s) = A(s) \]

\[ G_{loop}(s) \]

\[ G_{id}(s) \]

\[ G(s) \]

\[ f_T = \frac{GBWP}{G_{id}} \]

\[ \frac{A_0}{2\pi\tau} = GBWP \]
Analytical solution

\[ G = \frac{G_{id}}{1 - \frac{1}{G_{loop}(s)}} = \frac{G_{id}}{1 + \frac{R_1 + R_2}{A(s) R_1}} \]

The pole position is

\[ s = -\frac{1}{\tau} \left( 1 + \frac{A_0 R_1}{R_1 + R_2} \right) = -\frac{1}{\tau} \left( 1 - G_{loop}(0) \right) \]
Gain-bandwidth product

• For a non-inverting amplifier we have

\[ f_p \approx -\frac{1}{2\pi \tau} \frac{A_0}{G_{id}} \Rightarrow |f_p|G_{id} = \frac{A_0}{2\pi \tau} = GBWP \]

• In an inverting configuration we should write

\[ |f_p|(1 - G_{id}) = GBWP, \]

which becomes the same for high gains

• The feedback loop reduces the (open-loop) gain by \(1 - G_{loop}(0)\) and widens the bandwidth by the same factor
Stability of feedback systems

• Stability only depends on $G_{loop}$
• The critical condition is $G_{loop} = 1$, i.e., $-G_{loop} = -1$
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Bode stability criterion (1945)

- If
  - $G_{\text{loop}}(s)$ only has poles in LHP (or in $s = 0$)
  - There is only one critical frequency $f_{180}$ where the phase of $-G_{\text{loop}}$ is $180^\circ$ ($\pm$ multiples of $360^\circ$)
  - $|G_{\text{loop}}(f_{180})| < 1$

- Then, the system is stable
Gain and phase margins

\[ G_m = \frac{1}{|G_{\text{loop}}(f_{180})|} \quad G_m|_{dB} = -|G_{\text{loop}}(f_{180})|_{dB} \]

\[ \varphi_m = 180 + \angle \left( -G_{\text{loop}}(f_{0dB}) \right) \]

- \( G_m \) and \( \varphi_m \) represent how much increase in gain or phase lag the system can withstand before becoming unstable
- Important in real systems, where transfer functions are subjected to tolerances
Simplified Bode criterion

If poles and zeros are in LHP, stability can be inferred from Bode plot

\[ \varphi_m \approx 90^\circ \]

\[ \varphi_m \approx 0^\circ \]

\[ \varphi_m \approx 45^\circ \]
How much phase margin?

![Graph showing step response and overshoot with phase margin as parameter.](image)
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Frequency compensation of OAs

• Is the tailoring of $G_{loop}(s)$ in order to improve the circuit stability

• Most OAs are «internally compensated» for easing their use with resistive feedback, and have a single pole above 0dB

• For frequency-dependent feedback, stability must be checked and compensation applied (if needed)
Uncompensated OAs

\[ G_{OL}(s) \]
\[ G_{id}(s) \]
\[ G_{\text{loop}}(s) \]
\[ G(s) \]

\[ |\cdot|_{dB} \]
\[ \log f \]
Dominant pole compensation

\[ |G_{\text{loop}}(s)|_{\text{dB}} \]

\[ \log f \]
Pole-zero compensation

\[ |G_{\text{loop}}(s)|_{\text{dB}} \]
Input capacitance

\[ G_{\text{loop}} = -A(s) \frac{Z_1}{Z_1 + R_2} \]

\[ = -A(s) \frac{R_1}{R_1 + R_2} \frac{1}{1 + sC_i(R_1 \parallel R_2)} \]
Compensation of input capacitance

\[ G_{\text{loop}} = -A(s) \frac{R_1}{R_1 + R_2} \frac{1 + sC_2R_2}{1 + s(C_i+C_2)(R_1 \parallel R_2)} \]
Resulting loop gain

\[ f_p = \frac{1}{2\pi(C_i + C_2)(R_1 \parallel R_2)} \]

\[ f_z = \frac{1}{2\pi C_2 R_2} \]

\[ f_p < f_z \]
Notes...

- $C_2$ modifies the closed-loop gain $\Rightarrow$ **stability is traded off against bandwidth** (now given by $f_z$)
- Another possibility is $C_2 R_2 = C_i R_1$ (pole-zero cancellation), but keep in mind that $C_i$ is never constant in reality...
- In differential amplifiers, use symmetric compensation
Lag network \((f_p < f_z)\)

\[
G_{loop} = -A(s) \frac{R_1}{R_1 + R_2} \frac{1 + sC_cR_c}{1 + sC_c(R_c + R_1 \parallel R_2)}
\]

Does not affect \(G_{id}\), but can degrade \(Z_{in}\) in NI amplifiers.
Differentiator

\[ G_{\text{loop}} = -A(s) \frac{1}{1 + sR(C + C_i)} \]

A simple resistor \( R_c \) between the OA inputs can be effectively used for compensation.
Yet another compensation

LF:

HF:
Poles and zeros

\[ f_z = \frac{1}{2\pi C R_c} \]

\[ f_{p1} \approx \frac{1}{2\pi C (R_c + R)}; \quad f_{p2} \approx \frac{1}{2\pi C_i (R_c \| R)} \]

- \( f_{p2} \) is usually at high frequency and can be neglected
- Lag network \((f_{p1} < f_z)\) can be used for compensation
- Closed-loop gain bandwidth limited to \( \frac{1}{2\pi R_c C} \)
Capacitive load

Additional pole in $G_{loop}$ must be above $GBWP$ (say, $10GBWP$) $\Rightarrow$ maximum load capacitance is

$$C_L \approx \frac{1}{2\pi R_o (10 \text{ GBWP})}$$
Compensation – 1

\[ f_z = \frac{1}{2\pi C_L R_c} \]

\[ f_p \approx \frac{1}{2\pi C_L (R_c + R_o)} \]

If \( R_1 + R_2 \gg R_c, R_o \)
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Compensation – 2
Poles and zeros

- If $R_c, R_o \ll R_1 + R_2$
  
  $$f_{p1} \approx \frac{1}{2\pi C_L (R_c + R_o)}; \quad f_{z1} \approx \frac{1}{2\pi C_L R_c}$$
  $$f_{p2} \approx \frac{1}{2\pi C_c (R_1 \parallel R_2)}; \quad f_{z2} \approx \frac{1}{2\pi C_c R_2}$$

- Pole-zero cancellation leads to
  
  $$R_c = R_o \frac{R_1}{R_2}; \quad C_c = C_L R_o \frac{R_1 + R_2}{R_2^2}$$