HP filters
and Baseline restorers

Alessandro Spinelli
Tel. (02 2399) 4001
alessandro.spinelli@polimi.it
home.deib.polimi.it/spinelli
Slides are supplementary material and are NOT a replacement for textbooks and/or lecture notes
The problem

• Up to now, we have mainly considered white- or large-bandwidth noise, but LF noise (e.g., flicker) can also become an issue

• Previous filters work on HF noise components and are not effective in these cases

• In the time domain, LF noise has long correlation time ⇒ it cannot be eliminated by averaging
• Weighting function

\[ h(t) = \delta(t) - \frac{1}{T} e^{-\frac{t}{T}} u(t) \]

• Transfer function

\[ H(s) = \frac{sT}{1 + sT} = 1 - \frac{1}{1 + sT} \]
Weighting function autocorrelation

\[ k_{hh}(\tau) = \int h(t)h(t + \tau)dt \]

\[ = - \int_0^\infty \left( \delta(t) - \frac{1}{T}e^{-\frac{t}{T}} \right) \frac{1}{T}e^{-\frac{t+\tau}{T}} dt \]

\[ = - \frac{e^{-\frac{\tau}{T}}}{T} + \frac{e^{-\frac{\tau}{T}}}{T^2} \int_0^\infty e^{-\frac{2t}{T}} dt = - \frac{e^{-\frac{\tau}{T}}}{2T} \Rightarrow - \frac{e^{-\frac{\vert\tau\vert}{T}}}{2T} \]

\[ k_{hh}(\tau) = \delta(\tau) - \frac{e^{-\frac{\vert\tau\vert}{T}}}{2T} \]
Output rms noise

\[ \overline{n^2_y} = \int R_{xx}(\tau)k_{hh}(\tau)d\tau \]

\[ = R_{xx}(0) - \frac{1}{2T} \int R_{xx}(\tau)e^{-|\tau|/T}d\tau \]
Effect on noise

- If we consider a rectangular $R_{xx}$ we obtain
  \[
  \overline{n_y^2} = \overline{n_x^2} - \frac{1}{T} \int_0^{T_n} \overline{n_x^2 e^{-\frac{\tau}{T}}} d\tau = \overline{n_x^2 e^{-\frac{T_n}{T}}}
  \]

- For white or non-correlated noise, $T_n$ is small and $\ll T \Rightarrow$ the filter has little effect

- For LF noise ($T_n \gg T$) the filter is effective

- In the frequency domain the filter rejects the components below $1/2\pi T$
Effect on signal

From [1]

Let’s look in detail the effect of a high-pass filter ($RC = T_F$) on a pulse signal

**INPUT**

$$V_p = \frac{A}{T_p}$$

$pulse\ area$  

$$A = V_p T_p$$

**OUTPUT**

$$V_p = \frac{A}{T_p}$$

$$T_F = RC$$

View on SHORT TIME scale

View on LONG TIME scale

Long tail  

$$- \frac{A}{T_F} e^{-\frac{t}{T_F}}$$

NB: DC transfer of CR is zero  →  net area of the output signal is zero
Effect on pulse sequence

From [1]

A pulse that follows a previous one within a fairly short time interval \( T_D < 5 \ T_F \) steps on the slow tail of the first pulse. Therefore, it starts from a down-shifted baseline, so that the amplitude measured for it is smaller than the true one.

For periodic pulses with fairly short repetition period \( T_R \ll T_F \), the superposition of slow pulse-tails shifts down the baseline by a \( V_S \) that makes zero the net area of the output signal.

Repetition-rate-dependent baseline-shift \( V_S = V_P \frac{T_P}{T_R} = A f_R \)
A better choice

Baseline (LF noise, drift, offset,...)

1) Measure the baseline without the signal
2) Subtract the baseline from the signal

Obviously, a time-variant filter!
Baseline Restorer

1) Baseline measurement ⇒ switch closed
2) BL subtraction from signal ⇒ switch open
Weighting function

Remember that $w$ is the system response at time $t$ to a delta-function applied in $\tau$
Effect on signal

From [1]
Weighting function autocorrelation

\[ k_{hh}(\tau) \]

\[ \frac{1}{2T} \]

\[ t_0 \]

\[ -t_0 \]

\[ T \gg t_0 \]

\[ T \ll t_0 \]
Effect on noise

$$n_y^2 = \int R_{xx}(\tau)k_{hh}(\tau)d\tau$$

- For white noise $n_y^2$ is even larger than $n_x^2$, as we subtract uncorrelated samples
- For LF noise ($T_n \gg t_o$) the negative areas of $k_{hh}$ reduce $n_y^2$ and the filter is effective
Frequency domain

- We set $t = 0$ for simplicity and we recall the time-reversal property, obtaining:
  $$W(s) = 1 - \frac{e^{-st_0}}{1 - sT}$$

- At LF $e^{-st_0} \approx 1 - st_0$
  $$W(f) \approx -j2\pi f(T - t_o)$$
Intrinsic HP filtering

From [2]

- In all real cases, even with DC coupled electronics: weighting is inherently NOT extended down to zero frequency, because an intrinsic high-pass filtering is present in any real operation.

- The intrinsic filtering action arises because:
  
  a) operation is **started at some time before** the acquisition of the measure
  b) operation is **started from zero** value

- EXAMPLE: measurement of amplitude of the output signal of a DC amplifier. **Zero-setting is mandatory:** the baseline voltage is preliminarily adjusted to zero, or it is measured, recorded and then subtracted from the measured signal. It may be done a long time before the signal measurements (e.g. when the amplifier is switched on) or repeated before each measurement; it may be done manually or automated, but it must be done anyway. Zero-setting produces a high-pass filtering: let us analyze why and how
Correlated double sampling

\[ w(t, \tau) = \delta(\tau) - \delta(\tau + t_s) \iff W(s) = 1 - e^{st_s} \]

\[ |W(f)| = \sqrt{2(1 - \cos(2\pi ft_s))} \]

- At LF \(|W(f)| \approx 2\pi ft_s\)
- Further calculations can be found in [2]
References