Automatic Control (MSc in Engineering Physics)
Lecturer: Andrea Zanchettin
Date: 2016.07.05

Given and family names .................................................................
Student ID number ......................... Signature .........................

- Duration: 120 minutes
- Number of exercises/questions: 5
- Maximum score: 32 pts.
- All solutions must be written (either in English or Italian) within the available blank space (and not on additional paper sheets)
- It is forbidden to use any electronic device, apart from a non programmable and non graphic calculator
- Books and lecture material are not allowed
- It is not allowed to leave the class room within the first 30 minutes
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Question 1

Given the nonlinear system \( \dot{x} = \sin(x) + u \), compare the two controllers \( u = -2x \) and \( u = -x - \sin(x) \) in terms of stability (asymptotic or not, global or local) of the origin of the corresponding closed-loop system.

Solution:

Exercise 2

The following linear system \( \dot{x} = -x + 2u, y = 5x \), at rest in the origin, is fed with the input \( u(t) = \exp(t), t \geq 0 \). Compute the corresponding motion and discuss the validity (true/false and why) of following statements:

1. as the output will diverge, the system is unstable;

2. the phase margin of the associated transfer function is positive, hence the system is asymptotically stable.

Solution:
Consider the following closed-loop system, where $G(s) = \frac{1-s}{s^2}$. Assume $n = 0$ and design the transfer function $R(s)$ in order to achieve the following requirements:

1. zero steady state error for a step disturbance signal, i.e. $e_\infty = 0$ when $w(t) = \text{step}(t)$;
2. a minimum phase margin of 70 degrees, i.e. $\psi_m \geq 70\,\text{deg}$;
3. a crossover frequency of at least 0.1 rad/s, i.e. $\omega_c \geq 0.1\,\text{rad/s}$.

For the given controller, evaluate the minimum attenuation of the disturbance $n$ on the output $y$ within the bandwidth $[10\omega_c, 100\omega_c]$.

Finally, select a proper sampling time $T_s$ for the digital implementation of the controller.

**Suggestion:** carefully select the type of the controller.

**Solution:**
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Exercise 4

Given the following linear discrete-time system:

\[ A = \begin{bmatrix} 0 & 1 \\ \rho & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ \rho \end{bmatrix}, C = [1 \ 0] \]

evaluate the values of \( \rho \) such that the system is: 1) asymptotically stable, 2) (fully) observable, and 3) (fully) reachable.

Assume that \( \rho = 0.5 \) and that the system is controlled (in closed-loop with negative feedback) with \( R(z) = 1/(z - 1) \). Analyse the stability of the closed-loop system.

Solution:
Exercise 5

For \( k \geq 0 \), discuss the global asymptotic stability of the origin of the nonlinear system

\[
\begin{align*}
\dot{x}_1 &= -x_1 + x_2, \\
\dot{x}_2 &= -x_2 + k \sin(x_1) x_1
\end{align*}
\]

**Suggestion:** try to apply the circle criterion to a properly defined system.

**Solution:**
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